Online Final Exam

Friday, March 20, 6-8pm

MAT 21C, Temple, Winter 2020

Please enter all answers into Canvas:
Login: `https://canvas.ucdavis.edu/courses/432246/quizzes`.
Choose all *correct* answers. (There could be more than one.)
No partial credit. Each problem worth 4pts.
You may use your book and notes.
By submitting this exam you swear that this work is yours alone, and that you received no help, in any form, from any other living person.

1. Let $\mathbf{A} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{B} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ be vectors. Find $\mathbf{A} \cdot \mathbf{B}$

(a) $\mathbf{i}$ **(b)** 3 **(c)** 5 **(d)** -5
(e) None of the above

2. Let $\mathbf{A} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{B} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ be vectors. Find $\mathbf{A} \times \mathbf{B}$

(a) $3\overrightarrow{\langle -1, -1, 1\rangle}$ **(b)** $3\overrightarrow{\langle 1, 1, -1\rangle}$ **(c)** $\overrightarrow{\langle -3, 3, 3\rangle}$ **(d)** $\overrightarrow{\langle -3, -2, 3\rangle}$
(e) None of the above

3. Let $w = f(x, y, z) = 3 - 2x + 3y - z$ be the temperature at (x, y, z), and let $\mathcal{S}$ be the level surface of a function $g(x, y, z) = 0$, $\nabla g \neq 0$. Then a relative maximum of temperature on $\mathcal{S}$ occurs at a point where

(a) $\nabla g = 0$ **(b)** $\nabla g = \lambda\overrightarrow{\langle 3, -2, 3\rangle}$ **(c)** $\nabla g = \lambda\overrightarrow{\langle -2, 3, -1\rangle}$ **(d)** $\nabla f = g$
(e) None of the above

4. The critical points of function $f(x, y) = x(y - 2)^2$ consist of:

(a) one rel/max **(b)** one rel/min **(c)** one saddle **(d)** only inflection points
(e) None of the above

5. Let $w = f(x, y, z) = 2xy^2 - z$. Then the derivative with respect to arclength $\frac{dw}{ds}$ in direction $v = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ at point $P = (1, -1, 1)$ is:

(a) $5/\sqrt{6}$ **(b)** $-5/\sqrt{6}$ **(c)** $7/\sqrt{3}$ **(d)** $11/\sqrt{6}$ **(e)** None of the above

6. Assume the approximation that all planets move along circular orbits $\vec{\mathbf{r}}(t) = R\{\cos\omega t\mathbf{i} + \sin\omega t\mathbf{j}\}$, so $\omega T = 2\pi$ where T is the time of one orbit. Assume Newton's inverse square force law

$$\vec{\mathbf{a}} = -G\frac{\vec{\mathbf{r}}}{R^3}.$$

Then Kepler's third law $T^2/R^3 = H$ holds with:

(a) $H = G$ **(b)** $H = \frac{G}{4\pi^2}$ **(c)** $H = \frac{4\pi^2}{G}$ **(d)** $\frac{\pi^2 G}{4}$
(e) None of the above

7. Let $w = f(x, y, z) = x^2y + \sin z$, and $\vec{\mathbf{r}}(t) = t^2\mathbf{i} + t^2\mathbf{j} + \pi t\mathbf{k}$. Then $\frac{dw}{dt} = \frac{d}{dt}(f \circ \vec{\mathbf{r}})(t)$ at $t = 1$ equals:

(a) $3 - \pi$ **(b)** $3 + \pi$ **(c)** 6π **(d)** $6 - \pi$
(e) None of the above

8. Let $f(x) = \sum_{k=3}^{\infty} 2x^k$. Then $f(-1/2)$ is equal to:

(a) 1/2 **(b)** -1/2 **(c)** -1/6 **(d)** 1/4
(e) None of the above

9. The repeating decimal $12.121212\cdots$ represents which fraction?

(a) $\frac{1202}{99}$ **(b)** $\frac{1212}{99}$ **(c)** $\frac{1299}{99}$ **(d)** $\frac{1200}{99}$
(e) None of the above

10. Let $w = f(x,y) = 2xy - x^2y - xy^2 + 2$. Then the critical point $(x,y) = (2/3, 2/3)$ is:

(a) rel/max **(b)** rel/min **(c)** saddle point **(d)** inflection point
(e) None of the above

11. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(n)^n (2x-3)^{n+1}}{n!}$

(a) 3/2 **(b)** -3/2 **(c)** 1/2e **(d)** 2/3
(e) None of the above

12. For which x does the series $\sum_{n=1}^{\infty} (-1)^n \frac{x}{n}$ converge:

(a) $|x| < 1$ **(b)** $|x| \leq 1$ **(c)** $|x| < \infty$ **(d)** $x = 0$
(e) None of the above

13. For which x does the series $\sum_{n=1}^{\infty} (-1)^n \frac{x}{n}$ converge absolutely:

(a) $|x| < 1$ **(b)** $|x| \leq 1$ **(c)** $|x| < \infty$ **(d)** $x = 0$
(e) None of the above

14. A cannonball is shot in the direction of the wind, starting with initial velocity $\vec{\mathbf{v}} = \overrightarrow{(1,2)}$ miles per hour. Assume it takes trajectory $\vec{\mathbf{r}}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, starts at $x = 0$, $y = 0$, and $\ddot{y} = -g$, $\ddot{x} = h$, where g is the acceleration of gravity and h is the acceleration due to the wind. What *greater* distance d will the cannonball fly due to the effect of the wind?

(a) $d = \frac{2h}{g}$ **(b)** $d = 2hg$ **(c)** $d = \frac{8h}{g^2}$ **(d)** $d = 4\frac{g}{h}$
(e) None of the above

15. Find the area of the parallelogram spanned by the two vectors $A = (1,-1,2)$ and $B = (1,0,1)$.

(a) $\sqrt{3}$ **(b)** $\sqrt{5}$ **(c)** 3 **(d)** 1
(e) None of the above

16. Find $Proj_B A$ for the two vectors in problem (15).

(a) $\frac{3\sqrt{2}}{2}$ **(b)** $\overrightarrow{(3/2, 0, -3/2)}$ **(c)** $\overrightarrow{(3, 0, 3)}$ **(d)** $-\frac{3\sqrt{2}}{2}$
(e) None of the above

17. Find the equation for the plane through $P = (1, -1, 2)$ normal to the vector $\mathbf{n} = (1, 0, -1)$.

(a) x-z=-1 **(b)** x-y+2z=-1 **(c)** x-y=2 **(d)** x-y+2z=2
(e) None of the above

18. Find the distance from the point $Q = (1, 3, -1)$ to the plane through $P = (1, -1, 2)$ normal to $\vec{\mathbf{n}} = (1, 0, -1)$.

(a) 4 **(b)** $4\sqrt{2}$ **(c)** $\frac{3}{\sqrt{2}}$ **(d)** $4\sqrt{2}$
(e) None of the above

19. If we approximate $e^x \approx 1 + x + \frac{x^2}{2!} + E$, what is the *best* estimate for the error E in this approximation implied by Taylor's theorem for $0 \leq x \leq 2$.

(a) $\frac{4}{3}$ **(b)** $\frac{ex^3}{6}$ **(c)** $\frac{ex^3}{6}$ **(d)** $\frac{e^2x^3}{6}$
(e) None of the above

20. A particle traverses the path $\vec{\mathbf{r}}(t) = 2\{\cos t\mathbf{i} + \sin t\mathbf{j}\} + 3t\mathbf{k}$. Find the distance the particle travels between $t = 0$ and $t = 10$.

(a) $5\sqrt{5}$ **(b)** $5\sqrt{13}$ **(c)** $10\sqrt{13}$ **(d)** $10\sqrt{5}$
(e) None of the above