Name:	
Student ID#:	
Section.	

Midterm Exam 1 Wednesday, February 5

MAT 21C, Temple, Winter 2020

Print names and ID's clearly, and have your student ID ready to be checked when you turn in your exam. Write the solutions clearly and legibly. Do not write near the edge of the paper or the stapled corner. Show your work on every problem. Correct answers with no supporting work will not receive full credit. Be organized and use notation appropriately. No calculators, notes, books, cellphones, etc. may be used on this exam.

Problem	Your Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
Total		100

Problem #1 (20pts): State the definition of $\lim_{n\to\infty} a_n = L$, and use it to prove that $a_n = 2 + \frac{(-1)^n}{n}$ converges to 2.

Problem #2 (20pts): Recall the sum of a geometric series: $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$.

(a) Use this to find a fraction equal to the decimal .8888....

(b) By the same method we used for summing the geometric series, derive a formula for the limit of the geometric series $\sum_{n=N}^{\infty} r^n$ when |r| < 1, applicable for any starting index N.

Problem #3 (20pts): Use a geometrical depiction of the integral test to show:

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

(b) $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

Problem #4 (20pts): Recall that the Taylor polynomial of order n for cos(x) expanded about x = 0 is given by

$$P_{2n}(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!}$$

Assume |x| < 1.

(a) Find the absolute value of the error determined from the Taylor Remainder term $R_{2n}(x)$ by estimating the (n + 1)'st derivative of cos(x) by 1.

(b) Find the error implied by the Alternating Series Theorem.

(c) Determine which estimate is smaller (and hence better).

Problem #5 (20pts):

(a) Use the Root Test to determine the interval of convergence of the modified power series:

$$\sum_{n=1}^{\infty} \frac{(5x-3)^n}{n^2}$$

(b) Use the Ratio Test to determine the radius of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{n^n (x-2)^n}{n!}$$