Quiz 1
VECTOR CALCULUS
MATH 21D, Sect 002, Winter Quarter, 2013
INSTRUCTOR: Blake Temple

1. Find the volume of the solid in the first octant bounded by the coordinate planes, the plane \( x = 3 \), and the parabolic cylinder \( z = 4 - y^2 \). (10 points)

2. Sketch the region bounded by the given lines and curves. Then express the region's area as an iterated double integral and evaluate the integral. (10 points)

The parabola \( x = -y^2 \) and the line \( y = x + 2 \)

1. Solution:
   
   first octant, \( x > 0, y > 0, z > 0 \)
   
   \[ V = \int_0^3 \int_0^2 (4 - y^2) \, dy \, dx \]
   
   \[ = \int_0^3 \left[ 4y - \frac{1}{3}y^3 \right]_0^2 \, dx \]
   
   \[ = 3 \times \left( 8 - \frac{8}{3} \right) = 16 \]

2. Solution:

   \( x = y - 2 \)
   \( x = -y^2 \)

   when \( y - 2 = -y^2 \),
   
   \[ y^2 + y - 2 = 0 \]
   \[ (y + 2)(y - 1) = 0 \]

   \( y_i = -2 \) and \( y_u = 1 \)

   \[ \int_{-2}^1 \int_{y-2}^{y^2} \, dx \, dy = \int_{-2}^1 (-\frac{1}{2}y^2 - \frac{1}{2}y + 2) \, dy \]

   \[ = \left. \left( -\frac{1}{2}y^2 - \frac{1}{3}y^3 + 2y \right) \right|_{y=2}^{y=1} = \frac{9}{2} \]