Name: ____________________

Student ID#: ____________________

Section: ____________________

Midterm Exam 1

Friday, February 2

MAT 21D, Temple, Winter 2018

Show your work on every problem. Correct answers with no supporting work will not receive full credit. Be organized and use notation appropriately. No calculators, notes, books, cellphones, etc. may be used on this exam. Please write legibly. Please have your student ID ready to be checked when you turn in your exam.

Problem	Your Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
Total		100

Problem #1 (20pts): (a) Sketch the region of integration $\mathbf{R}_{xy}$ determined by the iterated integral

$$\int_0^2 \int_{x^3}^{4x} x^2 \sin(xy)\, dy dx. \tag{1}$$

(b) Rewrite (1) with order of integration reversed. (Do not evaluate)

Problem #2 (20pts): Consider a three dimensional metal object bounded on the sides by the coordinate planes, and bounded above by the plane that touches the three sides at $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$, where a, b, c are positive numbers. Let the density be $\delta = \delta(x, y, z)$ be general. Give formulas for the three coordinates of the center of mass involving only iterated integrals.

Problem #3 (20pts): (a) Set up the iterated integral in spherical coordinates for the total mass M of the "ice cream cone" shaped region D cut from the solid sphere $\rho \leq 2$ by the cone $\phi = \pi/3$, assuming the density $\delta(x, y, z) = y$. Do not evaluate.

(b) Set up the integral using cylindrical coordinates. Do not evaluate.

Problem #4 (20pts): The mapping $x = u^2 - v^2$, $y = \frac{1}{2}uv$ takes a domain R_{xy} in (x, y)-space to the rectangular region $0 \leq u \leq 1$, $1 \leq v \leq 2$. Using the change of coordinates method, (and letting $dA = dxdy$), evaluate

$$\int\int_{R_{xy}} y\, dxdy.$$

Problem #5 (20pts):

(a) Use polar coordinates (and the trick in class) to derive the value of $I = \int_{-\infty}^{+\infty} e^{-u^2} du$.

(b) The Gaussian with mean $\mu > 0$ and variance $\sigma^2 > 0$ is give by $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{\sqrt{2\sigma^2}}}$. Show that $\int_{-\infty}^{+\infty} f(x)dx = 1$ by reducing the case of general μ and σ^2 to the case (a). (σ is called the *standard deviation.*)