Name: ______________________

Student ID#: ______________________

Section: ______________________

Final Exam

Thursday December 11th

MAT 21D, Temple, Fall 2008

Show your work on every problem. Correct answers with no supporting work will not receive full credit. Be organized and use notation appropriately. No calculators, notes, books, cellphones, etc. Please write legibly. Please have your student ID ready to be checked when you turn in your exam.

Problem	Your Score	Maximum Score
1		30
2		30
3		28
4		24
5		30
6		30
7		28
Total		200

Problem #1: Evaluate

$$\int_{-1}^{1}\int_{y^2}^{1} \sqrt{x}y^2 \cos x^2 dxdy$$

(Hint: Change the order of integration.)

Problem #2: Use the transformation $u = x + y$, $v = 2y$, to evaluate the integral

$$\int\int_{\mathcal{R}} \sin{(x+y)^2}\, dA,$$

where $\mathcal{R}$ is the region in the xy-plane bounded below by $y = 0$ and on the sides by the two lines $y = x$ and $y = \sqrt{\pi/2} - x$.

Problem #3: Let

$$\mathbf{F}(x, y, z) = 2xyz\mathbf{i} + (x^2z + 2yz)\mathbf{j} + (x^2y + y^2 + 2z)\mathbf{k}.$$

Find f such that $\mathbf{F} = \nabla f$.

Problem #4: Let $\mathbf{v}(x, y, z)$ be the velocity in m/s of a fluid

$$\mathbf{v}(x, y, z) = x^2yz\mathbf{i} + (x^2 + y^2)\mathbf{k}$$

(a) Find **Curl v**.

(b) Find the axis around which the circulation is maximal at the point $(1, 1, -1)$.

(c) Find the circulation per area about the axis $\mathbf{N} = 2\mathbf{i} + \mathbf{k}$ at the point $(1, 1, -1)$.

Problem #5: Consider a fluid flow with $\delta(x, y, z)$the density in $\frac{kg}{m^3}$, $\mathbf{v}(x, y, z)$ the velocity in $\frac{m}{s}$, and let $\mathbf{F} = \delta\mathbf{v}$ be the mass flux vector, given by $\mathbf{F}(x, y, z) = x^2yz\mathbf{i} + (x^2 + y^2)\mathbf{k}$.

(a) Find $\mathbf{Div}\,\mathbf{F}$.

(b) Find the rate at which mass is leaving the point $(1, 1, -1)$ per volume of fluid.

(c) Find the rate at which mass is passing upward through the disk $x^2+y^2 \leq 1,\ z = 0$.

Problem #6: Verify Stokes Theorem

$$\int_{\mathcal{C}} \mathbf{F} \cdot \mathbf{T}\, ds = \int \int_{\mathcal{S}} \mathbf{Curl} \mathbf{F} \cdot \mathbf{n}\, d\sigma$$

where $\mathcal{S}$ is the upper half of the sphere $x^2 + y^2 + z^2 = 4$ above the xy-plane, and F is the vector field

$$\mathbf{F}(x, y, z) = \{y\mathbf{i} - x\mathbf{j} + \mathbf{k}\}$$

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Problem #7: Let $\mathbf{F}$ be a smooth vector field in the xy-plane,

$$\mathbf{F}(x,y) = M(x,y)\,\mathbf{i} + N(x,y)\,\mathbf{j},$$

and assume $F = \nabla f$ for some scalar function $f(x,y)$.

(a) Prove that $\int_C \mathbf{F}\cdot\mathbf{T}\;ds = f(B) - f(A)$ for every smooth curve $\mathcal{C}$ taking A to B.

(b) Prove that $\mathbf{Curl}\,\mathbf{F}{=}0$.