12.5 #14

\[ \hat{\mathbf{a}} = -2 - \frac{3}{5}, \hat{\mathbf{b}} = 2 \mathbf{2} + 3 \frac{3}{5} \]

area = \left| \begin{vmatrix} -1 & -1 \\ 2 & 3 \end{vmatrix} \right| = |-3 + 2| = |-1| = 1

12.5 #16

Use Thm 2

Let \( \mathbf{B} = \mathbf{A} \) with the first and third columns exchanged.

By Thm 2, \(|\mathbf{B}| = -|\mathbf{A}|\)

By our assumption, \(|\mathbf{B}| = |\mathbf{A}|\)

\( \Rightarrow |\mathbf{A}| = 0 \)
\[
\begin{vmatrix}
    c_1 & c_2 & c_3 \\
    b_1 & b_2 & b_3 \\
    a_1 & a_2 & a_3 \\
\end{vmatrix}
\begin{vmatrix}
    b_2 & b_3 \\
    a_2 & a_3 \\
\end{vmatrix}
- \begin{vmatrix}
    a_1 & a_3 \\
    c_1 & c_3 \\
\end{vmatrix}
+ \begin{vmatrix}
    b_1 & b_2 \\
    a_1 & a_3 \\
\end{vmatrix} \\
= a_3 b_2 c_1 - a_2 b_3 c_1 - a_3 b_1 c_2 + a_1 b_3 c_2 + a_3 b_1 c_2 \\
- a_1 b_2 c_3 \\
\]

\[
\begin{vmatrix}
    a_1 & a_2 & a_3 \\
    b_1 & b_2 & b_3 \\
    c_1 & c_2 & c_3 \\
\end{vmatrix}
= a_1 \begin{vmatrix}
    b_2 & b_3 \\
    c_2 & c_3 \\
\end{vmatrix}
- a_2 \begin{vmatrix}
    b_1 & b_3 \\
    c_1 & c_3 \\
\end{vmatrix}
+ a_3 \begin{vmatrix}
    b_1 & b_2 \\
    c_1 & c_2 \\
\end{vmatrix} \\
= a_1 b_2 c_3 - a_1 b_3 c_2 - a_2 b_1 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 \\
- a_3 b_2 c_1 \\
= \begin{vmatrix}
    c_1 & c_2 & c_3 \\
    b_1 & b_2 & b_3 \\
    a_1 & a_2 & a_3 \\
\end{vmatrix}
\]
12.5)
2) \[ |a, b| = ad - bc. \] So \( \begin{vmatrix} 3 & 3 \\ 2 & 2 \end{vmatrix} = 3 \cdot 2 - 3 \cdot 2 = 0. \)

We could've seen this since \( \begin{vmatrix} 3 & 3 \\ 2 & 2 \end{vmatrix} \) has two columns equal.

6) \[ \begin{vmatrix} 0 & 0 & 0 \\ 1 & 5 & 9 \\ 3 & -1 & 2 \end{vmatrix} = 0. \begin{vmatrix} 5 & 9 \\ 2 & 1 \end{vmatrix} + 0. \begin{vmatrix} 1 & 5 \\ 3 & 2 \end{vmatrix} = 0 \]

Note there is a zero row. This implies \( \begin{vmatrix} 0 & 0 & 0 \\ 1 & 5 & 9 \\ 3 & -1 & 2 \end{vmatrix} = 0. \)

12) \[ \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 2 \cdot 6 - 3 \cdot 4 = 12 - 12 = 0. \)

Note \( [4, 6] = 2 \cdot [2, 3] \). Hence \( \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0. \)

21) \[ \begin{vmatrix} a_1 + kb_1 & a_2 + kb_2 \\ b_1 & b_2 \end{vmatrix} \\
= (a_1 + kb_1)b_2 - b_1(a_2 + kb_2) \\
= a_1b_2 + k b_1 b_2 - b_1 a_2 - b_1 k b_2 \\
= a_1b_2 - b_1 a_2 = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \]