Name: ______________________

Student ID#: ______________________

Section: ______________________

Midterm Exam 2

Wednesday November 19th

MAT 21D, Temple, Fall 2008

Show your work on every problem. Correct answers with no supporting work will not receive full credit. Be organized and use notation appropriately. No calculators, notes, books, cellphones, etc. may be used on this exam. Please write legibly. Please have your student ID ready to be checked when you turn in your exam.

Problem	Your Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
Total		100

Problem #1 (20pts): A particle of mass $m = 3$ moves along a trajectory given by

$$\mathbf{r}(t) = 2\sin t\ \mathbf{i} - 2\cos t\ \mathbf{j} + t\ \mathbf{k}.$$

At each time t find:

(a) The velocity vector $\mathbf{v}$

(b) The speed v

(c) The acceleration vector $\mathbf{a}$

(d) The unit tangent vector $\mathbf{T}$

(e) The force $\mathbf{F}$ on m

(f) The unit normal $\mathbf{N}$

(g) The arclength of the trajectory from $t = 2$ to $t = 3$.

Problem #2 (20pts): Let

$$f(x, y, z) = x^2 y \sin z,$$

and

$$\mathbf{F}(x, y, z) = 2xy \sin z \mathbf{i} + x^2 \sin z \mathbf{k} + x^2 y \cos z \mathbf{k}.$$

Find:

(a) Div $\mathbf{F}$

(b) Curl $\mathbf{F}$

(c) ∇ f

(d) Evaluate $\int_C \mathbf{F} \cdot \mathbf{T}\ ds$ where C is the *hypo-geometric-meso-cycloid*, a smooth curve that takes $A = (1, -1, \pi/4)$ to $B = (-2, 1, \pi/3)$.

Problem #3 (20pts): Let

$$\mathbf{F}(x,y,z) = \left(\frac{-y}{x^2+y^2}\right)\mathbf{i} + \left(\frac{x}{x^2+y^2}\right)\mathbf{k} + z\mathbf{k}.$$

(a) Evaluate $\int_C \mathbf{F}\cdot\mathbf{T}\,ds$ where C is the unit circle $x^2+y^2=1$ at $z=1$.

(b) A calculation would show that Curl $\mathbf{F}=0$. Explain why $\int_C \mathbf{F}\cdot\mathbf{T}\,ds \neq 0$.

Problem #4 (20pts): Let $\delta = x \frac{kg}{meter^2}$ be the density$=\frac{mass}{area}$ of a fluid moving in the (x, y)-plane with velocity $\mathbf{v} = x\ \mathbf{i} - y\ \mathbf{j}$ meters per second. Find the rate at which mass is moving into the box $1 \leq x \leq 2$, $0 \leq y \leq 1$. Justify each step.
(For reference, the two forms of Green's Theorem are:

$$\int_C \mathbf{F} \cdot \mathbf{T} ds = \int \int_R N_x - M_y \ dA, \qquad \int_C \mathbf{F} \cdot \mathbf{n}\ ds = \int \int_R M_x + N_y \ dA.)$$

Problem #5 (20pts): Use the Leibniz substitution principle to show that in general, if $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ is a parameterization of curve C for $t_a \leq t \leq t_b$, and $F = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is a vector field, then the following are equal:

$$\int_C \mathbf{F} \cdot \mathbf{T}\ ds = \int_C \mathbf{F} \cdot \mathbf{dr} = \int_C \mathbf{F} \cdot \mathbf{v}\ dt = \int_C M dx + N dy + P dz.$$