

Name: \_\_\_\_\_

Student ID#: \_\_\_\_\_

Section: \_\_\_\_\_

# Midterm Exam 1

Friday, February 1

MAT 21D, Temple, Winter 2013

Show your work on every problem. Correct answers with no supporting work will not receive full credit. Be organized and use notation appropriately. No calculators, notes, books, cellphones, etc. may be used on this exam. Please write legibly. Please have your student ID ready to be checked when you turn in your exam.

Problem	Your Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
Total		100

**Problem #1 (20pts):** (a) Sketch the region of integration  $\mathbf{R}_{xy}$  determined by the iterated integral

$$\int_0^2 \int_{x^4}^{4x^2} \tan(xy) \, dy \, dx. \quad (1)$$

(b) Rewrite (1) with order of integration reversed. (Do not evaluate)

**Problem #2 (20pts):** A metal plate of density  $1kg/m^2$  lies between the two curves  $y = x^4$  and  $y = 4x^2$ . Find the KE stored in rotating the plate about the  $y$ -axis at  $\omega = 2$  radians/sec. (Put in the units!)

**Problem #3 (20pts):** Integrate in spherical coordinates  $(\rho, \phi, \theta)$  to find the volume of the region obtained by removing the cone  $\phi \leq \pi/6$  from the sphere  $x^2 + y^2 + z^2 = 9$ .

**Problem #4 (20pts):** The change of variables formula for a general mapping  $x = g(u, v)$ ,  $y = h(u, v)$  is:

$$\int \int_{\mathcal{R}_{xy}} f(x, y) dx dy = \int \int_{\mathcal{R}_{uv}} f(g(u, v), h(u, v)) J du dv.$$

Give the formula for the amplification factor  $J$ , and use this to derive the amplification factor  $dx dy = r dr d\theta$  for the mapping to polar coordinates  $x = r \cos \theta$ ,  $y = r \sin \theta$ .

**Problem #5 (20pts):**

Use polar coordinates (and the trick in class) to evaluate  $I = \int_{-\infty}^{+\infty} e^{-x^2} dx$ , the area under the Gaussian.