Name: ______________________

Student ID#: ______________________

Section: ______________________

Midterm Exam 1

Wednesday, April 20

MAT 21D, Temple/Romik, Spring 2016

Show your work on every problem. Correct answers with no supporting work will not receive full credit. Be organized and use notation appropriately. No calculators, notes, books, cellphones, etc. may be used on this exam. Please write legibly. Please have your student ID ready to be checked when you turn in your exam.

Exam duration: 50 minutes

Problem	Your Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
Total		100

Problem #1 (20pts): (a) Sketch and find the AREA of the region of integration $\mathbf{R}_{xy}$ determined by the following iterated integral. (You do not need to evaluate this integral.)

$$\int_0^{\pi} \int_{-\sin x}^{\sin x} \ln\left(x^2 y^4\right) dy\, dx. \tag{1}$$

(b) Rewrite this iterated integral as an iterated integral with the order of integration reversed that produces the same value. (Again, you do not need to evaluate the integrals.)

$$\int_0^{1} \int_{2x^3}^{2x} \ln\left(x^2 y^4\right) dy\, dx. \tag{2}$$

Problem #2 (20pts): (a) Use polar coordinates to evaluate the integral

$$\iint_{\mathcal{R}_{xy}} e^{-r^2} \, dx \, dy,$$

where $\mathcal{R}_{xy}$ is the region inside the circle of radius R centered at $(0,0)$.

(b) Since in the limit $R \to \infty$ you are integrating over all of $\mathcal{R}^2$, take the limit $R \to \infty$ in (a) to obtain a value for $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} \, dx \, dy$.

(c) Iterate the integral to show $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,dx\,dy = \left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)^2$.

(d) Use (b) and (c) to evaluate the famous integral $\int_{-\infty}^{\infty} e^{-x^2}\,dx$ (the *Gaussian*).

Problem #3 (20pts): Consider a triangular metal plate with three corners $(0,0)$, $(2,0)$, $(0,2)$ meters and density $\delta(x,y) = xy\ kg/m^2$.

(a) Find the mass of the plate. (Put in units.)

(b) Find the center of mass $(\bar{x}, \bar{y})$. (Put in units. Note by symmetry $\bar{x} = \bar{y}$.)

Problem #4 (20pts): Evaluate $\iiint_D x\,dV$, where D is the region bounded by the planes $x = 0$, $y = 0$ and $z = 2$, and the surface $z = x^2 + y^2$ and lying in the quadrant $x \geq 0$, $y \geq 0$. Sketch the region.

Problem #5 (20pts): Let R_{xy} denote the rectangle

$$a \le x \le b,\ c \le y \le d.$$

Consider R_{xy} as a thin plate with uniform density δ. Derive the formula expressing the kinetic energy obtained by rotating R_{xy} at an angular velocity of ω radians per second about the x-axis as a double integral, by using the definition of the integral as a limit of Riemann sums.

Guidance. Approximate the plate as a collection of N (=some large integer) point masses each having mass $m = \delta \times \Delta A_k$ centered at points $(x_1, y_1), \dots, (x_N, y_N)$ (where ΔA_k is the area of a small rectangle near (x_k, y_k)). Use the Newtonian relation $KE = \frac{1}{2}mv^2$ for the kinetic energy of each point mass, and interpret the total energy as a Riemann sum. Explain each step. You do not need to evaluate the double integral.