Name: ______________________________

Student ID#: ________________________

Section: ________________________

Midterm Exam 2

MAT 25–Temple

Wednesday, March 1, 2017

Show your work on every problem. Correct answers with no supporting work will not receive full credit. Be organized and use notation appropriately. No calculators, notes, books, cellphones, etc. may be used on this exam. Please write legibly. Please have your student ID ready to be checked when you turn in your exam.

Problem	Your Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
Total		100

Problem #1 (20pts): Short Answer: (**a**) Define what it means for a sequence of real numbers s_n to be Cauchy. Then state the Cauchy Equivalence of Convergence Theorem.

(**b**) Define $\underline{s}_N$, $\bar{s}_N$, liminf(s_n), and limsup(s_n).

(**c**) Write the correct inequalities that hold between $\underline{s}_N$, $\bar{s}_N$, liminf(s_n) and limsup(s_n). (You need not justify your answers.)

Problem #2 (20pts): Short Answer:

(**a**) A sequence is convergent if:

$$\exists\ s_0 \in \mathcal{R}\ st\ \forall \epsilon > 0\ \exists\ N \in \mathcal{N}\ st\ \forall\ n > N,\ |s_n - s_0| < \epsilon.$$

Write the statement which asserts the sequence does *not* converge.

(**b**) State the Bolzano-Weierstrass Theorem:

(**c**) Define what it means for a set to be (sequentially) *closed*.

Problem #3 (20pts): (**a**) Give the definition of $s_n \to -\infty$.

(**b**) Prove directly that if $s_n \to -\infty$, then every subsequence $s_{n_k} \to -\infty$.

Problem #4 (20pts): Assume $s_n \to s_0$ converges to a real number s_0, and let t_n be a bounded sequence. Prove

$$\limsup\,(s_n t_n) = s_0 \limsup\,(t_n).$$

Problem #5 (20pts): Let s_n be a bounded sequence. We call a term s_N in the sequence *diminutive* if it is smaller than all the terms in the sequence which follows it, i.e., $s_N \leq s_n$ for all $n \geq N$. Prove directly that if a sequence has only a finite number of diminutive terms, then it contains a monotone subsequence.