Cosmology

- Hubbles Law: The galaxies are receding from us at a rate proportional to the distance:

Observational Fact:

$$V = H L \qquad \text{(H)}$$

velocity at which galaxy is receding

Hubbles "Constant"

distance from us to galaxy

$$H \approx h_0 \cdot \frac{100\,km}{s} \frac{1}{Mpc} \qquad h_0 \in [.5, 1]$$

Quoted value: $h_0 = .55$ (Niel Cornish)

$Mpc = 10^6\,pc \qquad pc \cong 3.26\ lty$

"A galaxy 3.26 million ltys away is receding from us at about 55 km/s"

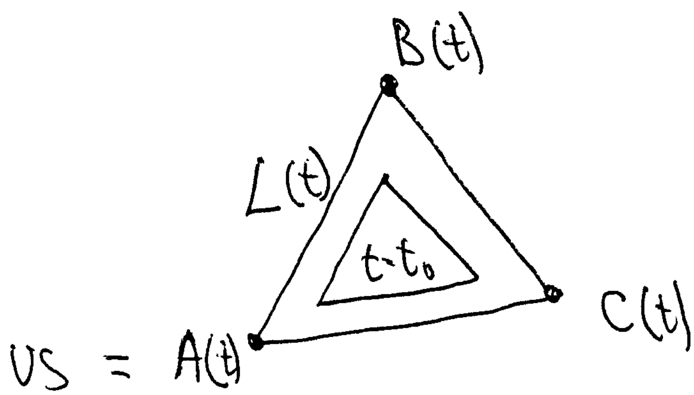
- <u>Note</u>: $[H] = \left[\frac{km}{s\ Mpc}\right] = \frac{1}{T}$

h_0 dimensionless constant

$$\frac{100\ km}{Mpc} \cong 3.24 \times 10^{-18} \quad \text{dimensionless}$$

- (H) is consistent with idea that the universe is uniformly expanding

<u>Ex</u>:



Assume all distances are increasing by a factor $R(t)$

$$L(t) = L_0\, R(t)$$

$R(t) =$ Cosmological scale factor

Then a galaxy at $B(t)$ will appear to recede from us at velocity

$$v = \frac{dL}{dt} = \frac{d}{dt}\left(R(t) L_0\right) = \dot{R} L_0 = \frac{\dot{R}}{R} L$$

<u>Conclude</u>: uniform expansion implies (H) if we take

$$H_0 = \frac{\dot{R}}{R}$$

<u>Note</u> if $\frac{\dot{R}}{R} = H_0 \equiv$ constant, then

$$\ln(R/R_0) = H_0 (t - t_0)$$

$$R = R_0 e^{H_0 (t - t_0)}.$$ (Steady state model of Cosmology)

- It is believed that $H = H(t)$ evolves with time.

- <u>Theorem</u>: The following metric defines a spacetime in which the 3-d space at $t = \text{const}$ uniformly expands according to the scale factor $R(t)$ ~~&~~ is symmetric homogeneous & uniform about ea pt (FRW)

$$ds^2 = -dt^2 + R(t)^2 \left\{ \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right\}$$

Friedmann-Robertson-Walker metric.

<u>Note</u> ① The space at $t = \text{const}$ with metric $ds^2 = R(t)^2 \left\{ \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right\}$ defines a 3-d space of constant scalar curvature: $\text{Sign}(k) = \text{Sign}(R^{\sigma\tau}{}_{\sigma\tau})$

$R^{\sigma\tau}{}_{\sigma\tau}$ (See Weinberg)

Note ②: By rescaling the radial coordinate by (chng of coords')

$$\bar{r} = \alpha r, \quad \alpha = \text{const}$$

we can rescale k to value $k = -1, 0, 1$ (FLP).

"Pf of Theorem" In the case $k = 0$, we obtain $ds^2 = dr^2 + r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2)$ from $dx^2 + dy^2 + dz^2$ by changing to spherical coordinates:

$$z = r\cos\theta$$
$$x = r\sin\theta\cos\varphi$$
$$y = r\sin\theta\sin\varphi$$

Thus the effect of $R(t)$ is to rescale

all lengths by factor of $R(t)$ ✓.

For $k \neq 0$ see Weinberg.

⊙ Assume the matter in universe has some average density $\rho(t)$ & pressure $p(t)$.

~~...~~

We look for metrics of form (FRW) that solve the Einstein equations

$$G = \frac{8\pi G}{c^4} T$$

where

$$T_{ij} = (\rho + p) u_i u_j + p g_{ij}$$

stress tensor for a perfect fluid.

- Assume (1) the galaxies are fixed relation to the spacial coordinate (r, θ, φ). This implies that the distance betw galaxies increase by factor $R(t)$:

$$L = L_0 R(t)$$

$L_0 \equiv$ distance apart when $R(t) = 1$.

(FIP) Show that this implies that galaxies follow geodesics of (FRW) metric $\Rightarrow$ they are in "freefall".

- Assume (1) $\Rightarrow$ $u^i = (1, 0, 0, 0)$ "no spatial motion".

- Our Equations are thus:

$$G_{ij}[g_{ij}] = \frac{8\pi G}{c^4}(\rho+p)u_i u_j + p g_{ij}$$

$$g_{ij} = \begin{bmatrix} -1 & & & 0 \\ & \frac{R(t)^2}{1-kr^2} & & \\ & & R(t)^2 r^2 & \\ 0 & & & R(t)^2 r^2 \sin^2\theta \end{bmatrix}$$

$$u_i = (-1, 0, 0, 0), \quad u^i = (1, 0, 0, 0)$$

Calculation (Wein Pg 472) $\Rightarrow$

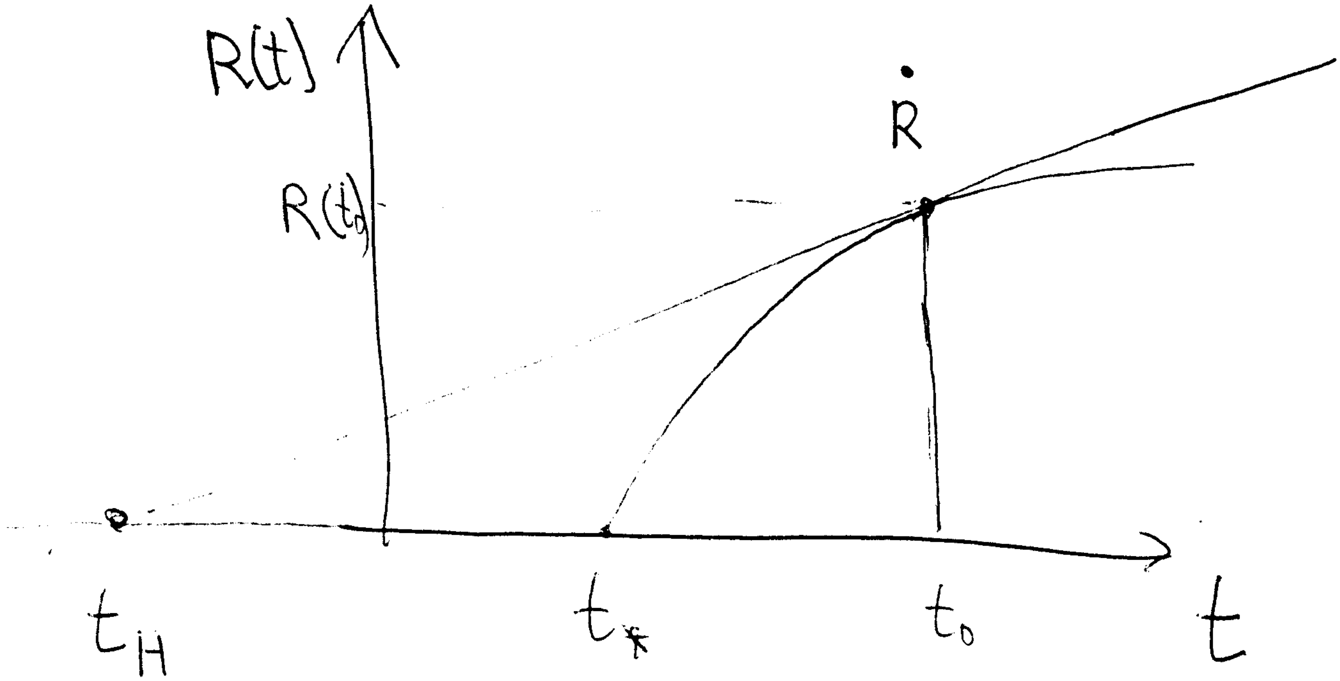
$$3\ddot{R} = -4\pi G(\rho+3p)R \qquad (1)$$

$$R\ddot{R} + 2\dot{R}^2 + 2k = 4\pi G(\rho-p)R^2 \qquad (2)$$

Eqn (1) $\Rightarrow$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) < 0$$

<u>Fact</u>: $R(t)$ is positive and concave down & increasing $\Rightarrow$ $R(t_*) = 0$ at some time in past $t_* \equiv$ "Big Bang"



$t_* > t_H$ $\quad \frac{R(t_0)}{t_0 - t_H} = \dot{R}(t_0) \quad H_0 = \frac{\dot{R}(t_0)}{R(t_0)} = \frac{1}{t_0 - t_H}$

<u>Conclude</u>: The universe began at some time $t = t_*$, and

$$\text{Age of Universe} = |t_0 - t_*| < |t_0 - t_H| = \frac{1}{H_0}$$

$$H_0 = h_0 \cdot \frac{100 \text{ km}}{\text{s Mpc}} = h_0 \cdot 3.24 \times 10^{-18} \text{ s}^{-1}$$

$h_0 \approx .55$

$$\frac{1}{H_0} = \frac{1}{h_0 3.24} \times 10^{18} \text{ s}$$

$$1 \text{ year} = 3.1558 \times 10^{7} \text{ s}$$

$$\frac{1}{H_0} = \frac{1}{(.55)(3.24)(3.1558)} \times 10^{11} \text{ years}$$

$$= .1778 \times 10^{11} \text{ years}$$

$$= 17.78 \times 10^{10} \text{ years} \approx 18 \text{ billion years}$$