Spivak Vol II

Supplement (2)

Chapter 4. The Curvature of Higher Dimensional Manifolds

A. An Inaugural Lecture

On June 10, 1854 the faculty of Göttingen University heard a lecture entitled Über die Hypothesen, welche der Geometrie zu Grunde liegen (On the Hypotheses which lie at the Foundations of Geometry). This lecture was delivered by Georg Friedrich Bernhard Riemann, who had been born just a year before Gauss' paper of 1827. Although the lecture was not published until 1866, the ideas contained within it proved to be the most influential in the entire history of differential geometry. To be sure, mathematicians had not neglected the study of surfaces in the meantime; in fact, Gauss' work had inspired a tremendous amount of work along these lines. But the results obtained in those years can all be proved with much greater ease after we have followed the long series of developments initiated by the turning point in differential geometry which Riemann's lecture provided.

A short account of the life and character of Riemann can be found in the biography by Dedekind* which is included in Riemann's collected works (published by Dover). His interest in many fields of mathematical physics, together with a demand for perfection in all he did, delayed until 1851 the submission of his doctoral dissertation Grundlagen für eine allegemeine Theorie der Functionen einer veränderlichen complexen Grösse (Foundations for a general theory of functions of a complex variable). Gauss' official report to the Philosophical Faculty of the University of Göttingen stated "The dissertation submitted by Herr Riemann offers convincing evidence of the author's thorough

*Even for those who can only plod through German, this is preferable to the account in E. T. Bell's Men of Mathematics, which is hardly more than a translation of Dedekind, written in a racy style and interladen with supercilious remarks of questionable taste.

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Riemann was now qualified to seek the position of Privatdocent (a lecturer who received no salary, but was merely forwarded fees paid by those students who elected to attend his lectures). To attain this position he first had to submit an "inaugural paper" (Habilitationsschrift). Again there were delays, and it was not until the end of 1853 that Riemann submitted the Habilitationsschrift, Über die Darstellbarkeit einer Function durch eine trigonometrische Reihe (On the representability of a function by a trigonometric series). Now Riemann still had to give a probationary inaugural lecture on a topic chosen by the faculty, from a list of three proposed by the candidate. The first two topics which Riemann submitted were ones on which he had already worked, and he had every reason to expect that one of these two would be picked; for the third topic he chose the foundations of geometry. Contrary to all traditions, Gauss passed over the first two, and picked instead the third, in which he had been interested for years. At this time Riemann was also investigating the connection between electricity, magnetism, light, and gravitation, in addition to acting as an assistant in a seminar on mathematical physics. The strain of carrying out another major investigation, aggravated perhaps by the hardships of poverty, brought on a temporary breakdown. However, Riemann soon recovered, disposed of some other work which had to be completed, and then finished his inaugural lecture in about seven more weeks.

Riemann hoped to make his lecture intelligible even to those members of the faculty who knew little mathematics. Consequently, hardly any formulas appear and the analytic investigations are completely suppressed. Although

Dedekind describes the lecture as a masterpiece of exposition, it is questionable how many of the faculty comprehended it. In making the following translation,* I was aided by the fact that I already had some idea what the mathematical results were supposed to be. The uninitiated reader will probably experience a great deal of difficulty merely understanding what Riemann is trying to say (the proofs of Riemann's assertions are spread out over the next several Chapters). We can be sure, however, that one member of the faculty appreciated Riemann's work. Dedekind tells us that Gauss sat at the lecture "which surpassed all his expectations, in the greatest astonishment, and on the way back from the faculty meeting he spoke to Wilhelm Weber, with the greatest appreciation, and with an excitement rare for him, about the depth of the ideas presented by Riemann."

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*The original is contained, of course, in Riemann's collected works. Two English translations are readily available, one in Volume 2 of Smith's Source Book in Mathematics (Dover), and one in Clifford's Mathematical Papers (Chelsea).

(10). As described above, the severely opisthopubic condition of their pelvis is consistent with the notion that these birds roosted in trees. In contrast, based primarily on disputed measurements of claw curvature, *Archaeopteryx* has been interpreted as adapted primarily for a terrestrial rather than an arboreal existence (18). However, as in the enantiornithines, the morphology of *Archaeopteryx*'s pelvis is best interpreted as adapted for a largely, if not exclusively, arboreal existence.

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4. S. F. Perry, *Adv. Anat. Embryol. Cell Biol.* **79**, 1 (1983); in *Oxygen Transport in Biological Systems*, S. Egginton and H. F. Ross, Eds. (Society for Experimental Biology Seminar Series 51, Cambridge Univ. Press, Cambridge, 1992), pp. 57–77.
5. Maximal rates of lung O_2-CO_2 exchange are limited primarily by hydrostatic pressure constraints on pulmonary blood flow (19). Thus, in order for a modern reptile (the active lizard *Varanus*, for example) with a bellowslike septate lung to attain endothermlike rates of maximal oxygen consumption {about 10 times those of active ectotherms [A. F. Bennett and J. A. Ruben, *Science* **206**, 649 (1979)]}, maximal pulmonary blood flow would have to be accelerated by about 10 times, or about 5 times if blood oxygen carrying capacity were to approximate that in many mammals {20 volume % rather than the actual 10 volume % in modern lizards [A. F. Bennett, *J. Comp. Biochem. Physiol.* **46**, 673 (1973)]}. In either case, because pulmonary hydrostatic pressure is largely a product of blood flow rate, pulmonary capillary pressures would be far in excess of dangerous levels [~ 45 millimeters of mercury (mmHg)], approaching at least 100 mmHg if not far higher {based on the observed similarity of resting mean pulmonary arterial pressure in mammals and normal *Varanus* (about 20 mm Hg at a body temperature of 35°C) [A. Ischimatsu, J. W. Hicks, N. Heisler, *Respir. Physiol.* **71**, 83 (1988)] and the assumption that (i) pulmonary capillary recruitment is maximal in exercising tetrapods and (ii) that mean arterial pressure during intense exercise in normal *Varanus* is actually broadly equivalent to that in mammals (about 35 mm Hg) (19)}. Hypothetically, these pressure constraints on the bellowslike septate lung might be overcome either by increasing the magnitude of lung vascularization (thus decreasing pulmonary capillary resistance to blood flow) or by increasing total lung volume by a factor of at least 5. However, a substantial increase in lung vascularization would necessarily restrict the volume of nonvascularized portions of the lung, thereby reducing capacity for lung ventilation. Alternately, an increase by a factor of 5 in total lung volume would leave little, if any, space in the visceral cavity for organs other than the lung.
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12. Rather than representing primitive archosaurian structures, it is probable that the hepatic-piston diaphragm systems in crocodilians and theropods are convergently derived. Pelvic anatomy in early "protodinosaurs" such as *Lagosuchus*, as well as in all ornithischian dinosaurs, shows no evidence of the pubis having served as a site of origin for similar diaphragmatic musculature (pubic bones are comparatively less well developed, and in ornithischian dinosaurs a pubic symphysis is absent). See R. Carroll, *Vertebrate Paleontology and Evolution* (Freeman, New York, 1988).
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16. A few theropod dinosaurs [for example, *Segnosaurus* and *Adasaurus* (9)] possess a moderately opisthopubic pelvis, but the distal pubis remains ventrally situated and the degree of dorsal rotation of the pubis does not approximate that in *Archaeopteryx* and the enantiornithine birds.
17. The position of the pubis in *Archaeopteryx* has occasionally been interpreted as having been vertical rather than severely opisthopubic [for example, J. H. Ostrom, *Biol. J. Linn. Soc. London* **8**, 91 (1976)]. However, the overall similarity of the pelvis of *Archaeopteryx* to those of the enantiornithine birds, especially the presence of the hypopubic cup, as well as the morphology of the London and Berlin *Archaeopteryx* specimens, offer support for our interpretation of the pelvic structure of these early birds [L. D. Martin, in *Origin of the Higher Groups of Tetrapods*, H. P. Schultze and L. Trueb, Eds. (Cornell Univ. Press, Ithaca, NY, 1991), pp. 485–540.
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22. Figure 6B was modified from (13) and A. Feduccia, *The Origin and Evolution of Birds* (Yale Univ. Press, New Haven, CT, 1996).

23. We thank J. Baumel, A. Bennett, P. Dodson, J. Farlow, A. Feduccia, J. Hicks, L. Martin, S. Perry, L. Witmer, and G. Zug for invaluable discussions and constructive criticisms. We offer special thanks to D. Bubier and D. Wolberg (Philadelphia Academy of Sciences) for *Sinosauropteryx* photos. R. Elsie provided alligator specimens. Figures 3, 4, and 6 were drawn by R. Jones. This work was supported by NSF grant IBN-9420290 to W.J.H. and J.A.R.

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Belated Decision in the Hilbert-Einstein Priority Dispute

Leo Corry, Jürgen Renn,* John Stachel

According to the commonly accepted view, David Hilbert completed the general theory of relativity at least 5 days before Albert Einstein submitted his conclusive paper on this theory on 25 November 1915. Hilbert's article, bearing the date of submission 20 November 1915 but published only on 31 March 1916, presents a generally covariant theory of gravitation, including field equations essentially equivalent to those in Einstein's paper. A close analysis of archival material reveals that Hilbert did not anticipate Einstein. The first set of proofs of Hilbert's paper shows that the theory he originally submitted is not generally covariant and does not include the explicit form of the field equations of general relativity.

It took Einstein 8 years, from 1907 to 1915, to complete the general theory of relativity, based on the field equations

$$R_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu}T\right) \qquad (1)$$

where $g_{\mu\nu}$ is the metric tensor representing the gravitational potentials, $R_{\mu\nu}$ is the Ricci tensor, κ is a constant, $T_{\mu\nu}$ is the stress-energy tensor of matter, and T is its trace. The principal difficulty he had to overcome was finding the right balance between the mathematical implications of a generalized principle of relativity and physical requirements such as the existence of a Newtonian limit (1, 2). Hilbert, on the other hand, only began to work seriously on gravitation in mid-1915. Concerning physics, his interests had focused since the end of 1912 on the structure of matter, and in particular, since mid-1913, on Gustav Mie's special-relativistic electromagnetic theory of matter. Then, after Einstein's visit to Göttingen in the summer of 1915, Hilbert attempted to forge a synthesis between Mie's theory and Einstein's approach to gravitation based on $g_{\mu\nu}$ (3, 4).

A recent comprehensive Einstein biography, which shows promise of becoming the standard reference, offers a succinct

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summary of the presently accepted account of the almost simultaneous formulation of the field equations by Einstein and Hilbert (5):

> In the decisive phase [of work on general relativity] Einstein even had a congenial colleague, though this caused him more annoyance than joy, as it seemed to threaten his primacy. "Only one colleague truly understood it, and he now tries skillfully to 'nostrify' [that is, appropriate] it," he complained to [Heinrich] Zangger about what he evidently regarded as an attempt at plagiarism. This colleague was none other than David Hilbert. . . . What must have irritated Einstein was that Hilbert had published the correct field equations first—a few days before Einstein. [. . .]
>
> In November, when Einstein was totally absorbed in his theory of gravitation, he essentially corresponded only with Hilbert, sending Hilbert his publications and, on November 18, thanking him for a draft of his article. Einstein must have received that article immediately before writing this letter. Could Einstein, casting his eye over Hilbert's paper, have discovered the term which was still lacking in his own equations [the trace term $-1/2g_{\mu\nu}R$ (6)], and thus 'nostrified' Hilbert?

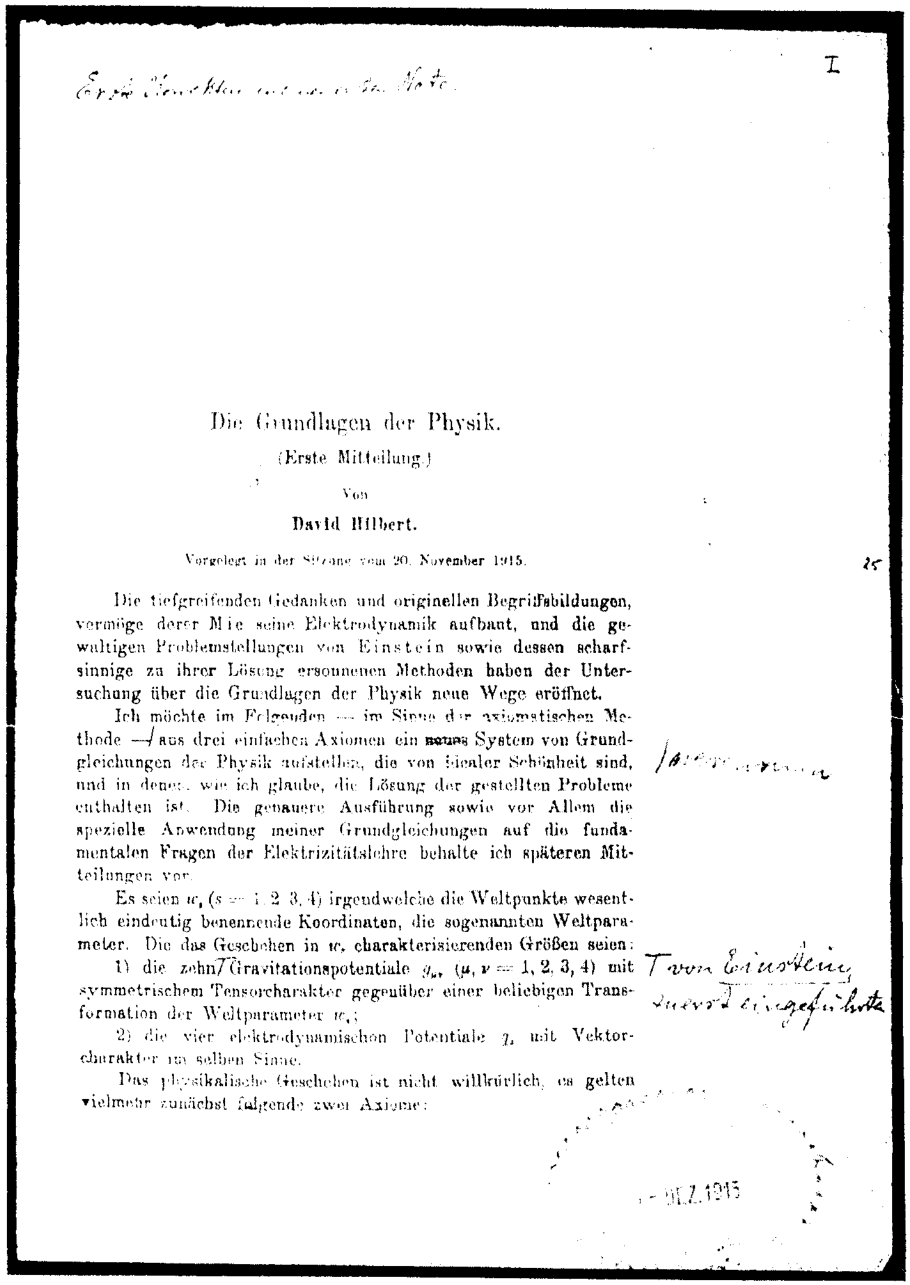
Fölsing is convinced, in agreement with the presently accepted view among physicists and historians of science, that Einstein's and Hilbert's achievements were actually parallel and independent, with the priority in submitting the field equations in their final form going to Hilbert (7). If, however, the standard account were correct, it would seem quite possible that indeed Einstein "nostrified" from Hilbert the critical trace term, still missing from the field equations in the paper he submitted on 11 November (8). Hilbert's published paper is mathematically complex and might have been difficult for Einstein to fully digest so quickly, but it does clearly display the field equations of general relativity, including the critical trace term [(3), p. 404]. Although this possible conclusion from the accepted view usually is not drawn, the arguments by which Einstein is exculpated are rather weak, turning on his slowness in fully grasping Hilbert's mathematics (5).

In the course of a project on the history of general relativity at the Max Planck Institute for the History of Science, archival work by Corry brought to light a hitherto unnoticed set of proofs of Hilbert's paper (9). Detailed analysis and comparison of these proofs with both published versions of Hilbert's paper (10) and with Einstein's papers on gravitation from 1913 to 1915 (2) enabled us to construct an account of the crucial weeks in November 1915 that radically differs from the standard view, excludes the possibility that Einstein plagiarized from Hilbert the last crucial step in completing general relativity and sheds new light on Einstein's complaint of "nostrification" by Hilbert.

Both the proofs and the final version of Hilbert's first communication (3) are dated "submitted on 20 November 1915," presumably referring to the original manuscript. A copy of the proofs, preserved in his archives and marked in his own hand "First proofs of my first note," bears a printer's stamp dated 6 December 1915 (Fig. 1). However, the cover of the issue in which the heavily revised published version appeared is dated 31 March 1916. Its first note cites Einstein's conclusive paper, in which he reached the final form of his generally covariant theory (11), submitted on 25 November 1915 and published on 2 December 1915. Thus, Hilbert could have revised his paper in response to Einstein's work.

Differences between the proofs and this published version of Hilbert's paper confirm this view. Two of the differences, in particular, are fundamental.

1) In the proofs, Hilbert asserts that his theory cannot be generally covariant. In addition to 10 generally covariant equations, there must be four additional noncovariant equations to guarantee causality (12):



I

Die Grundlagen der Physik.

(Erste Mitteilung.)

Von

David Hilbert.

Vorgelegt in der Sitzung vom 20. November 1915.

Die tiefgreifenden Gedanken und originellen Begriffsbildungen, vermöge derer Mie seine Elektrodynamik aufbaut, und die gewaltigen Problemstellungen von Einstein sowie dessen scharfsinnige zu ihrer Lösung ersonnenen Methoden haben der Untersuchung über die Grundlagen der Physik neue Wege eröffnet.

Ich möchte im Folgenden — im Sinne der axiomatischen Methode — aus drei einfachen Axiomen ein neues System von Grundgleichungen der Physik aufstellen, die von idealer Schönheit sind, und in denen, wie ich glaube, die Lösung der gestellten Probleme enthalten ist. Die genauere Ausführung sowie vor Allem die spezielle Anwendung meiner Grundgleichungen auf die fundamentalen Fragen der Elektrizitätslehre behalte ich späteren Mitteilungen vor.

Es seien w_s $(s = 1, 2, 3, 4)$ irgendwelche die Weltpunkte wesentlich eindeutig benennende Koordinaten, die sogenannten Weltparameter. Die das Geschehen in w_s charakterisierenden Größen seien:

1) die zehn Gravitationspotentiale $g_{\mu\nu}$ $(\mu, \nu = 1, 2, 3, 4)$ mit symmetrischem Tensorcharakter gegenüber einer beliebigen Transformation der Weltparameter w_s;

2) die vier elektrodynamischen Potentiale q_s mit Vektorcharakter im selben Sinne.

Das physikalische Geschehen ist nicht willkürlich, es gelten vielmehr zunächst folgende zwei Axiome:

Fig. 1. The first page of a set of proofs of Hilbert's first communication, with Hilbert's handwritten corrections and a printer's stamp, dated 6 December 1915. [Reproduced with permission by the Staats- und Universitätsbibliothek Göttingen (Handschriftenabteilung), Germany]

Since our mathematical theorem shows that the previous axioms I and II can only provide ten essentially independent equations for the 14 potentials [of gravitation and electromagnetism]; and further, maintaining general covariance makes quite impossible more than ten essentially independent equations for the 14 potentials $g_{\mu\nu}$, q_s; then, in order to keep the deterministic characteristic of the fundamental equations of physics, in correspondence with Cauchy's theory of differential equations [that is, to have a well-posed Cauchy problem], the requirement of four further non-invariant equations to supplement [the generally covariant gravitational equations] is unavoidable. In order to find these equations I start out by setting up a definition of the concept of energy.

After setting up an equation that he calls "the energy theorem," Hilbert introduces a third axiom (*13*):

Axiom III (axiom of space and time). The space-time coordinates are those specific world parameters for which the energy theorem . . . is valid.

Using this axiom, space and time provide in fact a labelling of the world points for which the energy principle is valid.

The validity of [the energy] equation . . . is a consequence of axiom III: these four differential equations . . . supplement the [generally covariant] gravitational equations . . . to yield a system of 14 equations for the 14 potentials $g^{\mu\nu}$, q_s: *the system of the fundamental equations of physics*.

Note that Hilbert distinguishes here between the world parameters, which are arbitrary, and the space-time coordinates, which are not. Here he follows Einstein's earlier argument against general covariance from his papers of 1913 to 1915. Einstein then justified his theory's lack of general covariance and the need to select "adapted coordinate systems" on the same grounds of causality and energy-momentum conservation.

Hilbert abandoned this entire argument in the published version of his first communication. In a letter to Felix Klein (*14*), he remarked with regard to the role of the energy theorem in the proof version of his theory, "But I later suppressed the whole thing because the thing did not appear mature to me." In his second communication, published in 1917 (*15*), Hilbert gave a radically different definition of causality for a generally covariant theory, essentially the one accepted today (*16*). He there critically notes the noncovariant nature of Einstein's earlier work, characteristic also of his own version in the proofs: "In his original, now abandoned theory Einstein indeed postulated four non-invariant equations for the $g_{\mu\nu}$ in order to save the causality principle in its old form" (*15*). Hilbert's revised definition of causality for generally covariant theories led him to explicitly reject the possibility of the coordinates being physically significant. In the 1924 version of his first communication, he described axiom II, the requirement of general covariance, for the first time as "the simplest mathematical expression for the requirement that the coordinates in themselves have no sort of physical significance . . ." [(*10*), p. 4].

2) In the proofs of his first communication, Hilbert's world function includes a gravitational term $\sqrt{g}K$ (*17*) and indicates that the gravitational part of the field equations takes the form of the variational derivative of the gravitational term with respect to the metric. Hilbert does not, however, give the explicit form of this gravitational part of the field equations. In the published version, on the other hand, he explicitly writes down the expression for the variational derivative [(*3*), p. 404]

$$[\sqrt{g}K]_{\mu\nu} = \sqrt{g}\cdot\left(K_{\mu\nu} - \frac{1}{2}Kg_{\mu\nu}\right) \tag{2}$$

justifying his expression by the argument [(*3*), pp. 404–405]

. . .which follows easily without calculation from the fact that, except for $g_{\mu\nu}$, $K_{\mu\nu}$ [the Ricci tensor] is the only tensor of second order and K [the trace of the Ricci tensor] is the only invariant that can be constructed from only the $g^{\mu\nu}$ and its first and second order partial derivatives. . . .

This argument is, however, untenable because there are many other tensors of second order and many other invariants that can be constructed from the Riemann tensor; even if one requires linearity in the Ricci tensor, the crucial coefficient of the trace term remains undetermined by this argument. In the 1924 republication, he dropped this argument, replacing it with an outline of how to calculate the gravitational term [(*10*), p. 7]:

In order to determine the expression $[\sqrt{g}K]_{\mu\nu}$ one first specializes the coordinate system in such a way that at the world point being considered all the $g_s^{\mu\nu}$ [the derivatives of the metric tensor] vanish. One finds in this way [our Eq. 2].

To summarize: Initially, Hilbert did not give the explicit form of the field equations; then, after Einstein had published his field equations, Hilbert claimed that no calculation is necessary; finally, he conceded that one is. Taken together, this sequence suggests that knowledge of Einstein's result may have been crucial to Hilbert's introduction of the trace term into his field equations.

In the light of this analysis of Hilbert's work, we can now better understand the exchange between Hilbert and Einstein in the crucial days of November 1915. On the 14th, Hilbert wrote to Einstein, inviting him to come to Göttingen 2 days later, when Hilbert intended to lecture on "my axiomatic solution of your great problem." In a postscript, he added, "Insofar as I understand your new paper, the solution given by you is completely different *from* mine . . . " (*18*). Hilbert is referring to Einstein's communication of 4 November to the Prussian Academy of Sciences. On the 15th, Einstein replied (*19*), excusing himself from coming on grounds of being overworked, expressing great interest in Hilbert's work and asking for a copy of Hilbert's paper as soon as possible "to satisfy my impatience."

Hilbert must have sent the requested copy or a summary of his paper immediately because, on the 18th, Einstein replied, reacting sharply to Hilbert's claim of originality. Far from thanking him for sending his communication, as Fölsing claims, Einstein began his letter by denying the novelty of Hilbert's approach: "The system given by you agrees—as far as I can see—exactly with that which I found in recent weeks and submitted to the Academy" (*20*). In order to claim his priority, he explained to Hilbert that he had "considered the only possible generally covariant field equations three years earlier." He also insinuated that Hilbert had not even discussed the fundamental physical problems raised by these equations (*20*):

The difficulty was not to find generally covariant equations for the $g^{\mu\nu}$; this is easy with the help of the Riemann tensor. What was difficult instead was to recognize that these equations form a generalization, and, that is, a simple and natural generalization of Newton's law.

Einstein's claim is understandable. In his 4 November paper, he had announced with a flourish his return to the Riemann tensor as the appropriate starting point for a theory of gravitation (*21*). Although the theory in this paper is different from the version of Hilbert's theory that he saw, as well as from the version of the theory Einstein developed by the 18th, they are all based on the metric tensor and the only generally covariant tensor that can be built from it, the Riemann tensor, as their common foundation.

It was after this exchange with Hilbert that Einstein wrote a friend charging Hilbert with "nostrification." Einstein's letter of 18 November may have been the motive for a reputed apologetic letter (now lost) by Hilbert to Einstein [(*7*), p. 261] and Hilbert's handwritten note, added to the proofs of 6 December, supplementing his initial reference to the gravitational potentials $g_{\mu\nu}$ with the phrase "first introduced by Einstein" (Fig. 1).

In any case, by 20 December 1915, that is, before the appearance of Hilbert's final version, Einstein's anger had subsided to the point that he offered to Hilbert a reconciliation (22):

> There has been a certain resentment between us, the cause of which I do not want analyze any further. I have fought against the feeling of bitterness associated with it, and with complete success. I again think of you with undiminished kindness and I ask you to attempt the same with me. It is objectively a pity if two guys that have somewhat liberated themselves from this shabby world are not giving pleasure to each other.

In the printed version of his paper, Hilbert added a reference to Einstein's conclusive paper and a concession of the latter's priority: "The differential equations of gravitation that result are, as it seems to me, in agreement with the magnificent theory of general relativity established by Einstein in his later papers" [(3), p. 404]. If Hilbert had only altered the dateline to read "submitted on 20 November 1915, revised on [any date after 2 December 1915, the date of publication of Einstein's conclusive paper]," no later priority question could have arisen.

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6. Hilbert used the trace of the Ricci tensor (*3*), whereas Einstein used the trace of the matter tensor (see Eq. 1); but these two forms are essentially equivalent.
7. See, for example, A. Pais, *"Subtle is the Lord. . .": The Science and the Life of Albert Einstein* (Oxford Univ. Press, New York, 1982), pp. 257–261.
8. A. Einstein, *Sitzungsber. Preuss. Akad. Wiss.* **1915**, 799 (1915).
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10. Hilbert republished his paper (*3*) with extensive changes in *Math. Ann.* **92**, 1 (1924).
11. Einstein's paper (*1*) is cited on p. 395 of (*3*), as noted by Guth (*23*).
12. Pages 3 and 4 of the proofs (*9*); translation of this and the following quotations is by the authors.
13. Page 7 of the proofs (*9*).
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24. The authors wish to thank G. Castagnetti for his cooperation in the archival work, especially in the dating of the central documents. They also thank P. Damerow, M. Janssen, U. Majer, D. Rowe, and T. Sauer for helpful discussions. We also thank Z. Rosenkranz and the Einstein Archives, The Hebrew University of Jerusalem, Israel, for permission to quote from Einstein's letters, and the Staats- und Universitätsbibliothek Göttingen (*Handschriftenabteilung*) for the kind permission to reproduce Fig. 1.

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Warming Early Mars with Carbon Dioxide Clouds That Scatter Infrared Radiation

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Geomorphic evidence that Mars was warm enough to support flowing water about 3.8 billion years ago presents a continuing enigma that cannot be explained by conventional greenhouse warming mechanisms. Model calculations show that the surface of early Mars could have been warmed through a scattering variant of the greenhouse effect, resulting from the ability of the carbon dioxide ice clouds to reflect the outgoing thermal radiation back to the surface. This process could also explain how Earth avoided an early irreversible glaciation and could extend the size of the habitable zone on extrasolar planets around stars.

It is most likely that the martian atmosphere 3.8 billion years ago was composed primarily of CO_2, with a surface pressure ranging from a few hundred to several thousand millibars, and some H_2O (*1*). At that time, the solar luminosity was about 25% lower than it is at present. Under such conditions, calculations performed with a one-dimensional (1D) climate model by Kasting (*2*) showed that the atmospheric CO_2 should condense in the atmosphere for surface pressures larger than a few tens of millibars. Kasting found that the condensation of CO_2 decreases the atmospheric temperature lapse rate and reduces the magnitude of the greenhouse effect, making it impossible to warm the surface of Mars enough to allow the presence of fluid water together with a CO_2-H_2O gaseous atmosphere. Several alternative mechanisms such as geothermal heating (*3*), an early more massive sun (*4*), or the greenhouse effect of methane (*5*) and ammonia (*6*) have been considered but none has provided a likely solution to the early Mars climate enigma (*5*).

Another consequence of the condensation of CO_2 is the formation of CO_2 ice clouds. Because they are perfect scatterers at solar radiation wavelengths, the CO_2 ice particles should raise the planetary albedo.

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In the thermal infrared (IR), CO_2 ice is at least 500 times more transparent than water ice, except near 15 μm where the ν_2 absorption band is located and above 90 μm where two broad lattice vibration bands were measured (*7*). Thus, CO_2 ice clouds should not be able to contribute to an absorption-emission greenhouse effect as cirrus clouds on Earth do. On this basis, Kasting (*2*) estimated that CO_2 ice clouds should cool the planet through reflection of sunlight uncompensated by IR trapping.

We have studied the IR properties of the CO_2 ice clouds using a two-stream, hemispheric mean, source function code that allows for multiple scattering, absorption, and emission by atmospheric particles (*8*). The CO_2 ice particle single-scattering properties were obtained from the refractive index measured by Hansen (*7*), using Mie theory with a modified gamma size distribution of effective variance 0.1 (*9*). As expected by Kasting, a cloud composed of CO_2 ice particles smaller than a few micrometers should be almost transparent in the IR, except near 15 μm. However, larger particles can be expected in CO_2 ice clouds. Crystal size is determined by the time required for crystal growth versus the time it takes for the particles to fall out of a supersaturated layer (sedimentation). On Earth, despite the fact that the growth of water ice particles is limited by the diffusion of water vapor through air, particles 80 μm or larger are often observed in cirrus ice clouds, and the observed radiative properties of Earth's cirrus clouds can be fit by assuming equiv-

BOOK REVIEWS

A Biographical Bandwagon

Albert Einstein. A Biography. ALBRECHT FÖLSING. Viking, New York, 1997. xiv, 882 pp. + plates. $34.95 or C$45. ISBN 0-670-85545-6. Translated from the German edition (Frankfurt am Main, 1993) by Ewald Osers.

Einstein. A Life. DENIS BRIAN. Wiley, New York, 1996. xvi, 509 pp. + plates. $30, ISBN 0-471-11459-6; paper, $19.95 or C$27.95, ISBN 0-471-19362-3.

The publication of these two volumes brings to at least six the number of biographical works about Einstein appearing in the past five years (*1*), with others rumored to be in preparation. Einstein will, of course, and should continue to draw our interest. His development of the special and general theories of relativity, thereby laying the foundation for modern relativistic cosmology, his role in the development of the quantum theory, his contributions to our understanding of Brownian motion and the foundations of statistical physics, and his pursuit of the dream of a unified field theory mark him as the greatest scientist since Newton. He was an important figure in the political history of the 20th century through his work on behalf of pacifism, Zionism, and socialism, his role as an instigator of the American atomic bomb project, his post-war advocacy of the sharing of nuclear secrets, and his opposition to the McCarthyite persecutions of the late 1940s and early '50s. Einstein the human being also makes a fascinating study, for this modest and painfully self-aware intellectual combined a passionate love of humanity and a capacity for deep and enduring friendships with what often seemed a selfish and callous indifference to the feelings of some of those closest to him, especially his two wives and his children.

The recent spillage of ink over Einstein's life, however, is to be explained less by these perennial considerations than by lurid curiosity aroused by some new information uncovered by the editors of *The Collected Papers of Albert Einstein* in the course of preparing the first volumes in the series (*2*). Most notable in this regard are the so-called "love letters" between Einstein and his first wife, Mileva Marić, from which we learned of the birth of their illegitimate daughter, Lieserl, in 1902 and gleaned hints of previously unsuspected scientific collaboration between Albert and Mileva during their student years (*3*). Only a little less sensational were the early letters between Einstein and his second wife, Elsa, which revealed the development of their relationship well before Einstein's separation from Mileva and thus a powerful additional motivation for Einstein's move from Zurich to Berlin—Elsa's home—in 1914 (*4*). Background research for *The Collected Papers* has led to equally important discoveries on the scientific side, especially concerning the genesis of general relativity (*5*), but the new biographies have paid comparatively little attention to these less salacious matters.


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"A New Giant in World History . . . whose researches mean a complete overthrow of our views of nature," 1919. [From *Albert Einstein: A Biography*; Ullstein Bilderdienst]

Widespread interest in what we now see to be the more complicated human character of Einstein has created a climate in which authors and publishers are tempted to rush into print with inferior or at best premature efforts. In different ways, the biographies by Fölsing and Brian illustrate the problem. Both offer largely reliable guides to the basic facts about Einstein's life, if not also his scientific work (*6*). Moreover, both authors make extensive use of the materials now available in the Einstein Archive, though Fölsing does so to better advantage, and each author contributes some new information based on original research and the identification of new sources. But in addition to occasional technical or factual errors, both books evince serious flaws, ranging from lapses of style to problems of overall conception and balance. In the end, neither can be judged a significant contribution to the literature on Einstein or can be recommended unreservedly.

Of the two Fölsing's is, without question, the better. Indeed, the effort and ambition that have gone into the preparation of this massive book (*7*) could have made it a worthy successor to Ronald Clark's *Einstein: The Life and Times* (*8*), which was the first truly comprehensive biography of Einstein but, written just a few years before the Einstein Archive was made available and the Einstein Papers project got under way, was soon dated.

Fölsing is a science journalist whose work includes biographies of Galileo, Röntgen, and Hertz (*9*). In places, he makes a sincere effort to explain Einstein's scientific work, especially for the years through 1914, where editorial notes in the published volumes of *The Collected Papers* provide a guide, but the treatment is still superficial and insecure. Fölsing does a more thorough job with Einstein's private and public life. One might question, however, the relative attention paid to different periods. Like other authors following the lead of the early volumes of *The Collected Papers*, Fölsing devotes considerable attention to the years from Einstein's birth in 1879 to his 1914 move to Berlin, and his 19 years in Berlin receive even more intensive treatment, occupying 338 out of a total of 741 pages of text, whereas his 22 years in Princeton (1933–1955) receive a mere 63 pages. The child being the father of the man, it is necessary that we understand Einstein in those early years, which in any case include the *annus mirabilis* of 1905, in which Einstein's first papers on relativity, the quantum theory, and Brownian motion appeared; and it was during his Berlin years that he became the public figure whose every utterance made front-page news from Berlin to New York. Surely, however, Einstein's involvement in the politics of American atomic weapons development, his emergence as one of America's leading public intellectuals and an influential voice on issues ranging from social justice and racial discrimination to international understanding and Arab-Israeli relations, as well as the steady refinement of his critique of the quantum theory and his pursuit of a unified field theory, are all more important than the question whether, as the German government claimed at the time of his receipt of the Nobel Prize in 1922, he had actually become a German citizen upon accepting an appointment to the Prussian Academy of Sciences in 1914 or the even less interesting matter of his romantic liaisons with Berlin socialites.

It is the failings of the translation, however, that most clearly indicate indifference toward quality on the part of the publishers. Ewald Osers has produced an admirably fluid and colloquial translation of most of the text, excepting, surprisingly, the scientific material, where one finds an embarrassing number of errors, the worst being the laughable rendering of *Verjüngung*, the German term for the tensor operation of contraction, as "rejuvenation" (p. 648). One also puzzles over Osers's decision to translate afresh many passages from Einstein's papers and letters for which technically superior English translations have long been available. Related to this is the preservation of Fölsing's references to German editions of works that are widely available in English translation, as a result of which the very readers who have need for this translation of his biography will find the citations less helpful than they could have been.

One final reservation must be recorded. Fölsing pays considerable attention to Einstein's evolving attitude toward his Jewish identity and his involvement with the Zionist movement. One wants to commend this entirely appropriate emphasis but for the fact that Fölsing himself seems (to put it charitably) uncomfortable with the subject, as signaled, for example, by his habit of using a term, *Stammesgenossen*, that Einstein used occasionally for the purpose of referring to his fellow Jews by an ethnic, not religious, designation. Einstein's own innocent employment of a term that we now see as having racialist overtones does not justify a biographer's overuse of it. The problem is only compounded by Osers's translation of the term as "tribal companions," when "kindred" would be the more accurate equivalent (*10*).

When one turns to the biography by Denis Brian, one can only ask whether Wiley, a reputable science publisher, had any reason for publishing this book other than a desire to jump on the Einstein biography bandwagon. Brian's book tells us virtually nothing about Einstein's scientific work, aside from a few amateurish descriptions. What, for example, is "the quadruple [sic] formula for gravitational radiation" (p. 95)? Did Einstein really "perfect" the photoelectric effect (p. 105)? Did Otto Hahn really discover atomic fission in 1920 (p. 110)?

Brian concentrates instead on Einstein the person. But in spite of his promise that "this account will surprise those fixed on him as a secular saint," that its exploration of Einstein's private life will reveal him "with his halo slightly askew" (p. xi), Brian tells us virtually nothing that is not already well known in the literature, aside from a few quite unsensational stories gleaned from Princeton acquaintances.

Brian's style is breezy and journalistic, as is not surprising from an author whose previous books include biographies of Ernest Hemingway, Tallulah Bankhead, and the parapsychologist J. B. Rhine and a set of conversations with famous scientists (*10*). The book lacks any organizing idea other than the most linear narrative structure, and its efforts to set Einstein's life and work in a broader historical setting are uneven and superficial. Among the most annoying features of a most annoying book are the many forced, often silly transitions from one topic to another. My favorite is the sentence that carries us from discussions of racial discrimination, the atomic bomb, and the sad situation in Palestine in 1946 (all in three paragraphs, one might note) to remarks about a few inconsequential pieces of correspondence: "If Einstein needed to escape from the nightmare of a devastated world, he had merely to look through his mail from women" (p. 352). Should the reader be more offended by the implied insensitivity to the sufferings of Jews and Arabs in Palestine or by the trivialization of women as correspondents? Or should we be most offended by Wiley's failure to insist on a higher standard of editorial decency? That the latter is the root of the problem is suggested by the inclusion of a tasteless appendix that tells the pathetic tale of the removal of Einstein's brain and eyes by ghoulish, souvenir-hunting physicians before the body was cremated.

When major publishing companies are putting out this kind of biography, it is time for a moratorium. A definitive biography of Einstein cannot be written until the Einstein Papers project is completed, both because new material is still being discovered by the editorial staff and because an appreciation of Einstein's life, work, and legacy will require the perspective that only time and distance from a pandering press can give. Surely we owe him that much respect.

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REFERENCES AND NOTES

1. Others are: P. A. Bucky, in collaboration with A. Weakland, *The Private Albert Einstein* (Andrews and McMeel, Kansas City, MO, 1992); R. Highfield and P. Carter, *The Private Lives of Albert Einstein* (Faber & Faber, London, 1993); M. White and J. Gribbin, *Einstein: A Life in Science* (Simon and Schuster, London, 1993, and Dutton, New York, 1995); and A. Hermann, *Einstein: Der Weltweise und sein Jahrhundert* (Piper, Munich, 1994).
2. Published by Princeton University Press, Princeton, NJ, beginning in 1987; six volumes are now in print, covering the period from Einstein's birth to 1917.
3. The letters were first published in J. Stachel *et al.*, Eds., *The Collected Papers of Albert Einstein*, vol. 1, *The Early Years, 1879–1902* (Princeton Univ. Press, Princeton, NJ, 1987), and M. Klein *et al.*, Eds., *The Collected Papers of Albert Einstein*, vol. 5, *The Swiss Years: Correspondence, 1902–1914* (Princeton Univ. Press, Princeton, NJ, 1993). They have also been published in a separate edition: A. Einstein and M. Marić, *The Love Letters*, J. Renn and R. Schulmann, Eds. (Princeton Univ. Press, Princeton, NJ, 1992).
4. These letters were published in M. Klein *et al*, Eds., *The Collected Papers of Albert Einstein*, vol. 5, *The Swiss Years: Correspondence, 1902–1914* (Princeton Univ. Press, Princeton, NJ, 1993).
5. See especially J. Stachel, "Einstein's search for general covariance, 1912–1915," in *Einstein and the History of General Relativity*, D. Howard and J. Stachel, Eds. (Birkhäuser, Boston, 1989), pp. 63–100, and J. Norton, "How Einstein found his field equations, 1912–1915," *ibid.*, pp. 101–159.
6. For a technically sophisticated guide to Einstein's scientific work, A. Pais's *'Subtle is the Lord . . .': The Science and the Life of Albert Einstein* (Clarendon, New York, 1982) is by far the best bet. Those wishing a more popular but still knowledgeable and accurate account of Einstein's physics in the form of biography should consult B. Hoffmann and H. Dukas, *Albert Einstein: Creator and Rebel* (Viking, New York, 1972), or J. Bernstein, *Einstein* (Viking, New York, 1973).
7. At 882 pages, this is, in fact, a slightly abridged translation of the German original, *Albert Einstein: Eine Biographie* (Suhrkamp, Frankfurt am Main, Germany, 1993), though the fact that the translation omits a substantial portion of the German text is nowhere mentioned in the English edition.
8. F. Clark, *Einstein: The Life and Times* (World, New York, 1971).
9. *Galileo Galilei, Prozess ohne Ende: Eine Biographie* (Piper, Munich, 1983); *Wilhelm Conrad Röntgen: Aufbruch ins Innere der Materie* (Hanser, Munich, 1995); *Heinrich Hertz: Eine Biographie* (Hoffmann und Campe, Hamburg, 1997).
10. *Stamm* can be used to mean "tribe" in German, but it also means "stock," "lineage," or "family." Thus, a *Stammbaum* is a family tree.
11. *Tallulah, Darling: A Biography of Tallulah Bankhead* (Macmillan, New York, 1980); *The Enchanted Voyager: The Life of J. B. Rhine, an Authorized Biography* (Prentice-Hall, Englewood Cliffs, NJ, 1982); *The True Gen: An Intimate Portrait of Ernest Hemingway by Those Who Knew Him* (Grove, New York, 1988); *Genius Talk: Conversations with Nobel Scientists and Other Luminaries* (Plenum, New York, 1995). Curiously, the Bankhead and Rhine biographies are not mentioned in the author note on the dust jacket of the present work.