- Wolfgang Bolyai 1832
- Nikolai Lobachevski 1829
- 1827 General Theory of surfaces
- 1870 Felix Klein "analytical constr of 2-d surf of const neg curvtr"
- 1846 John Adams Urbain Leverrier predict Neptune
- 1847 Leverrier 35"/century
- 1882 Simon Newcomb 42"/century

1895 "to drop these explorations as unsatisfactory, and to prefer provisionally the hypothesis that the sun's gravitation is not exactly inverse square"

"But the tale of history forms a very strong bulwark against the stream of time, and to some extent checks its irresistible flow, and, of all things done in it, as many as history has taken over, it secures and binds together, and does not allow them to slip away into the abyss of oblivion."
Anna Comnena, The Alexiad

I HISTORICAL INTRODUCTION

Physics is not a finished logical system. Rather, at any moment it spans a great confusion of ideas, some that survive like folk epics from the heroic periods of the past, and others that arise like utopian novels from our dim premonitions of a future grand synthesis. The author of a book on physics can impose order on this confusion by organizing his material in either of two ways: by recapitulating its history, or by following his own best guess as to the ultimate logical structure of physical law. Both methods are valuable; the great thing is not to confuse physics with history, or history with physics.

This book sets out the theory of gravitation according to what I think is its inner logic as a branch of physics, and not according to its historical development. It is certainly a historical fact that when Albert Einstein was working out general relativity, there was at hand a preexisting mathematical formalism, that of Riemannian geometry, that he could and did take over whole. However, this historical fact does not mean that the essence of general relativity necessarily consists in the application of Riemannian geometry to physical space and time. In my view, it is much more useful to regard general relativity above all as a theory of *gravitation*, whose connection with geometry arises from the peculiar empirical properties of gravitation, properties summarized by Einstein's Principle of the Equivalence of Gravitation and Inertia. For this reason, I have tried throughout this book to delay the introduction of geometrical objects, such as the metric, the affine connection, and the curvature, until the use of these objects could be motivated by considerations of physics. The order of chapters here thus bears very little resemblance to the order of history.

Nevertheless, because we must not allow the history of physics "to slip away into the abyss of oblivion," this first chapter presents a brief backward look at three great antecedents to general relativity—non-Euclidean geometry, the Newtonian theory of gravitation, and the principle of relativity. Their history is traced up to 1916, the year in which they were brought together by Einstein in the General Theory of Relativity.[1]

1 History of Non-Euclidean Geometry

Euclid showed in his *Elements*[2] how geometry could be deduced from a few definitions, axioms, and postulates. These assumptions for the most part dealt with the most fundamental properties of points, lines, and figures, and seem as self-evident to schoolboys in the twentieth century as they did to Hellenistic mathematicians in the third century B.C. However, one of Euclid's assumptions has always seemed a little less obvious than the others. The fifth postulate states

> "If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines if produced indefinitely meet on that side on which the angles are less than two right angles."

For two thousand years geometers tried to purify Euclid's system by proving that the fifth postulate is a logical consequence of his other assumptions. Today we know that this is impossible. Euclid was right, there is no logical inconsistency in a geometry without the fifth postulate, and if we want it we will have to put it in at the beginning rather than prove it at the end. However, the struggle to prove the fifth postulate is one of the great success stories in the history of mathematics, because it ultimately gave birth to modern non-Euclidean geometry.

The list of those who hoped to prove the fifth postulate as a theorem includes Ptolemy (d. 168), Proclos (410–485), Nasir al din al Tusi (thirteenth century), Levi ben Gerson (1288–1344), P. A. Cataldi (1548–1626), Giovanni Alfonso Borelli (1608–1679), Giordano Vitale (1633–1711), John Wallis (1616–1703), Geralamo Saccheri (1667–1733), Johann Heinrich Lambert (1728–1777), and Adrien Marie Legendre (1752–1833). Without exception, their efforts only succeeded in replacing the fifth postulate with some other equivalent postulate, which might or might not seem more self-evident, but which in any case could not be proved from Euclid's other postulates either. Thus, the Athenian neo-Platonist Proclos offered the substitute postulate: "If a straight line intersects one of two parallels, it will intersect the other also." (That is, if we define parallel lines as straight lines that do *not* intersect however far extended, then there can be at most one line that passes through any given point and is parallel to a given line.) John Wallis, Savillian Professor at Oxford, showed that Euclid's fifth postulate could be replaced with the equivalent statement "Given any figure there exists a figure, similar to it, of any size." And Legendre proved the equivalence of the fifth postulate with the statement "There is a triangle in which the sum of the three angles is equal to two right angles."[3]

The attempt to dispense with Euclid's fifth postulate began to take a different direction in the eighteenth century. In 1733 the Jesuit Geralamo Saccheri published a detailed study of what geometry would be like if the fifth postulate were false. He particularly examined the consequences of what he called the "hypothesis of the acute angle," that is, that "a straight line being given, there can be drawn a perpendicular to it and a line cutting it at an acute angle, which do not intersect each other."[3] However, Saccheri did not really think that this is possible; he still believed in the logical necessity of the fifth postulate, and explored non-Euclidean geometry only in the hope of eventually turning up a logical contradiction. Similar tentative explorations of non-Euclidean geometry were begun by Lambert and Legendre.

It seems to have been Carl Friedrich Gauss (1777–1855) who first had the courage to accept non-Euclidean geometry as a logical possibility. His gradual enlightenment is recorded in a series of letters[4] to W. Bólyai, Olbers, Schumacher, Gerling, Taurinus, and Bessel, extending from 1799 to 1844. In a letter dated 1824 he begged Taurinus to keep silent about the "heretical opinions" he had revealed. Gauss even went to the extent of surveying a triangle[40] in the Harz mountains formed by Inselberg, Brocken, and Hoher Hagen to see if the sum of its interior angles was 180°! (It was.) Then, in 1832, Gauss received a letter from his friend Wolfgang Bólyai, describing the non-Euclidean geometry developed by his son, Janos Bólyai (1802–1860), an Austrian army officer. He subsequently also learned that a professor in the Kazan, Nikolai Ivanovich Lobachevski (1793–1856), had obtained similar results in 1826.

Gauss, Bólyai, and Lobachevski had independently discovered what in modern terms is called the *two-dimensional space of constant negative curvature*. Such spaces are still very interesting; we shall see in the chapter on cosmography that the space in which we actually live may be a three-dimensional space of constant curvature. But to its discoverers the important thing about their new geometry was that it describes an infinite two-dimensional space in which all of Euclid's assumptions are satisfied—except the fifth postulate! In this it is unique, which perhaps explains why it was discovered more or less independently in Germany, Austria, and Russia. (The surface of a sphere also satisfies Euclidean geometry without the fifth postulate, but being finite it does not have room for parallel lines.) We shall see in Chapter 13, on symmetric spaces, that the two-dimensional space of constant negative curvature cannot be realized as a surface in ordinary three-dimensional Euclidean space, which is doubtless why it took two millennia to find it. And of course it also violates the alternative "common-sense" versions of Euclid's fifth postulate given by Proclos, Wallis, and Legendre—through a given point there can be drawn *infinitely* many lines parallel to any given line; *no* figures of different size are similar; and the sum of the angles of any triangle is *less* than 180°.

However, it still remained an open possibility that Euclid's fifth postulate [illegible] derived from the others, for it was not at all obvious that the geometry of

Gauss, Bólyai, and Lobachevski did not contain a logical inconsistency. The usual way to "prove" that a system of mathematical postulates is self-consistent is to construct a model that satisfies the postulates out of some other system whose consistency is (for the moment) unquestioned. For both Euclidean and non-Euclidean geometry the "model" is provided by the theory of real numbers. Descartes' analytic geometry shows that if a point is identified with a pair of real numbers (x_1, x_2) and the distance between two points (x_1, x_2) and (X_1, X_2) is identified as $[(x_1 - X_1)^2 + (x_2 - X_2)^2]^{1/2}$, then all of Euclid's postulates can be proved as theorems about real numbers. In 1870 a similar analytic geometry[5] was constructed by Felix Klein (1849–1925) for the geometry of Gauss, Bólyai, and Lobachevski—a "point" is represented as a pair of real numbers x_1, x_2 with

$$x_1{}^2 + x_2{}^2 < 1 \tag{1.1.1}$$

and the distance $d(x, X)$ between two points x, X is defined by

$$\cosh\left[\frac{d(x, X)}{a}\right] = \frac{1 - x_1X_1 - x_2X_2}{(1 - x_1{}^2 - x_2{}^2)^{1/2}(1 - X_1{}^2 - X_2{}^2)^{1/2}} \tag{1.1.2}$$

where a is a fundamental length which sets the scale of the geometry. Note that this space *is* infinite, because $d(x, X) \to \infty$ as $X_1{}^2 + X_2{}^2$ approaches unity. With this definition of "point" and "distance" one can verify that this model satisfies all of Euclid's postulates except the fifth, and in fact obeys the geometry discovered by Gauss, Bólyai, and Lobachevski. Thus after two millennia the logical independence of Euclid's fifth postulate was at last established.

This was just the beginning of the development of non-Euclidean geometry. We saw that in order to discover the geometry of Gauss, Bólyai, and Lobachevski it was necessary to give up the idea that a curved surface could only be described in terms of its embedding in ordinary three-dimensional spaces. How then *can* we describe and classify curved spaces? To pick up our story we must go back to 1827 when Gauss published his *Disquisitiones generales circa superficies curvas*. Gauss for the first time distinguished the *inner* properties of a surface, that is, the geometry experienced by small flat bugs living in the surface, from its *outer* properties, that is, its embedding in a higher-dimensional space, and he realized that it is the inner properties of surfaces that are "most worthy of being diligently explored by geometers."

Gauss also realized that the essential inner property of any surface is the metric function $d(x, X)$, which gives the distance between x and X along the shortest path between them on the surface. For instance, a cone or a cylinder has the same local inner properties as a plane, since a plane can be rolled without stretching or tearing (i.e., without distorting metric relations) into a cone or a cylinder. On the other hand, all cartographers know that a sphere cannot be unrolled onto a plane surface without distortion, and thus its local inner properties are not the same as the plane's.

There is a simple example that has been used by Einstein, Wheeler, and others to illustrate how the inner properties of a surface can be discovered by exploring its metric. (See Figure 1.1.) Consider N points in a plane. We can use one point as

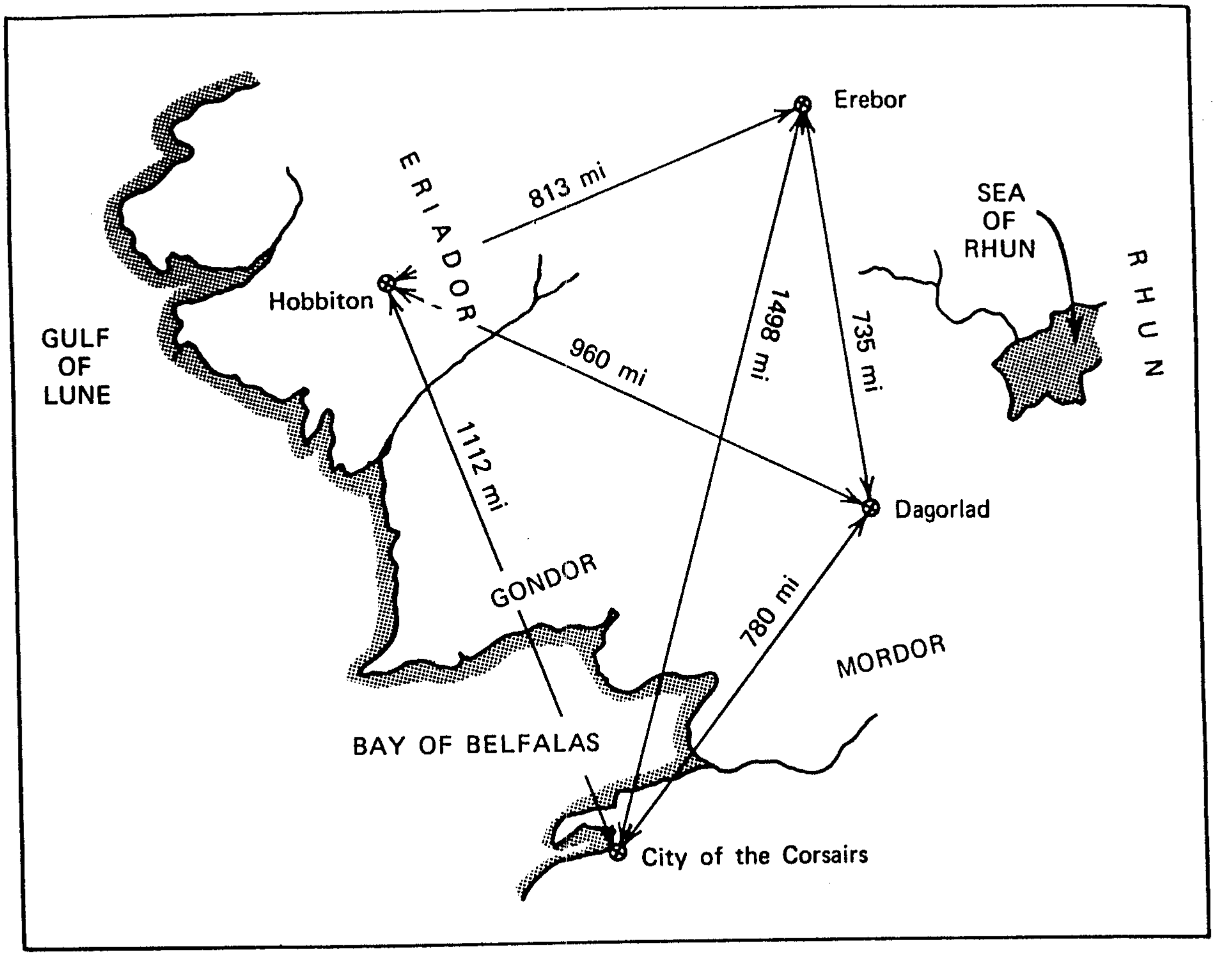

Figure 1.1 Problem: Is Middle Earth flat?

distances between the various points are described in terms of $(2N - 3)$ coordinates, that is, the x-coordinate of the second point and the x- and y-coordinates of the remaining $(N - 2)$ points. But there are $N(N - 1)/2$ different distances between the N points, and thus for large enough N these distances must be subject to M algebraic relations, where

$$M = \frac{N(N-1)}{2} - (2N - 3) = \frac{(N-2)(N-3)}{2} \tag{1.1.3}$$

For instance, in the simplest interesting case, $N = 4$, we can easily show that the distances d_{mn} between points m and n satisfy the single relation

$$\begin{aligned} 0 = {} & {d_{12}}^4 {d_{34}}^2 + {d_{13}}^4 {d_{24}}^2 + {d_{14}}^4 {d_{23}}^2 + {d_{23}}^4 {d_{14}}^2 + {d_{24}}^4 {d_{13}}^2 + {d_{34}}^4 {d_{12}}^2 \\ & + {d_{12}}^2 {d_{23}}^2 {d_{31}}^2 + {d_{12}}^2 {d_{24}}^2 {d_{41}}^2 + {d_{13}}^2 {d_{34}}^2 {d_{41}}^2 + {d_{23}}^2 {d_{34}}^2 {d_{42}}^2 \\ & - {d_{12}}^2 {d_{23}}^2 {d_{34}}^2 - {d_{13}}^2 {d_{32}}^2 {d_{24}}^2 - {d_{12}}^2 {d_{24}}^2 {d_{43}}^2 - {d_{14}}^2 {d_{42}}^2 {d_{23}}^2 \\ & - {d_{13}}^2 {d_{34}}^2 {d_{42}}^2 - {d_{14}}^2 {d_{43}}^2 {d_{32}}^2 - {d_{23}}^2 {d_{31}}^2 {d_{14}}^2 - {d_{21}}^2 {d_{13}}^2 {d_{34}}^2 \\ & - {d_{24}}^2 {d_{41}}^2 {d_{13}}^2 - {d_{21}}^2 {d_{14}}^2 {d_{43}}^2 - {d_{31}}^2 {d_{12}}^2 {d_{24}}^2 - {d_{32}}^2 {d_{21}}^2 {d_{14}}^2 \end{aligned}$$

This relation will be satisfied on any simply connected patch of a cylinder or a cone, which share the same inner properties as the plane, but it will *not* be satisfied by a table of airline distances among any four cities, because the earth's surface has different inner properties. There is a different relation appropriate to spherical surfaces, which *is* satisfied by airline mileage tables, and can be used to measure the radius of the earth. Of course, this is not the most convenient method and it is not the method used by Eratosthenes, but the important point here is that the curvature of the earth's surface can be determined from its local inner properties.

Were our imaginations given free rein, we could conceive of a great variety of peculiar metric functions $d(x, X)$. It was Gauss's great contribution to pick out one particular class of metric spaces, which was broad enough to include the space of Gauss, Bólyai, and Lobachevski as well as that of ordinary curved surfaces, but narrow enough to deserve the name of geometry. Gauss assumed that in any sufficiently small region of the space it would be possible to find a locally Euclidean coordinate system (ξ_1, ξ_2) so that the distance between two points with coordinates (ξ_1, ξ_2) and $(\xi_1 + d\xi_1, \xi_2 + d\xi_2)$ satisfies the law of Pythagoras,

$$ds^2 = d\xi_1^{\,2} + d\xi_2^{\,2} \tag{1.1.5}$$

For instance, we can set up such a locally Euclidean coordinate system at any point in an ordinary smooth curved surface by using the Cartesian coordinates of a plane tangent to the surface at the given point. However, this should not make us suppose that Gauss's assumption has anything to do with outer properties; it deals only with inner metric relations for infinitesimal neighborhoods.

If a surface is not Euclidean, it will not be possible to cover any *finite* part of it with a Euclidean coordinate system (ξ_1, ξ_2) satisfying the law of Pythagoras. Suppose that we use some other coordinate system (x_1, x_2) that *does* cover the space, and ask what form Gauss's assumption takes in these coordinates. It is easy to calculate that the distance ds between points (x_1, x_2) and $(x_1 + dx_1, x_2 + dx_2)$ is given by

$$ds^2 = g_{11}(x_1, x_2)\, dx_1^{\,2} + 2g_{12}(x_1, x_2)\, dx_1\, dx_2 + g_{22}(x_1, x_2)\, dx_2^{\,2} \tag{1.1.6}$$

where

$$\begin{aligned} g_{11} &= \left(\frac{\partial\xi_1}{\partial x_1}\right)^2 + \left(\frac{\partial\xi_2}{\partial x_1}\right)^2 \\ g_{12} &= \left(\frac{\partial\xi_1}{\partial x_1}\right)\left(\frac{\partial\xi_1}{\partial x_2}\right) + \left(\frac{\partial\xi_2}{\partial x_1}\right)\left(\frac{\partial\xi_2}{\partial x_2}\right) \\ g_{22} &= \left(\frac{\partial\xi_1}{\partial x_2}\right)^2 + \left(\frac{\partial\xi_2}{\partial x_2}\right)^2 \end{aligned} \tag{1.1.7}$$

This form for ds^2 is the hallmark of a *metric space*. [We shall see in Chapter 3 that this derivation can be reversed; given any space with ds given by (1.1.6), we can at any point choose *locally* Euclidean coordinates ξ_1, ξ_2 satisfying (1.1.5).] For the case of a sphere of radius a we can use spherical polar coordinates θ, φ, and the metric is

$$g_{\theta\theta} = a^2 \qquad g_{\theta\varphi} = 0 \qquad g_{\varphi\varphi} = a^2 \sin^2\theta \tag{1.1.8}$$

It is the factor $\sin^2 \theta$ in $g_{\varphi\varphi}$ that gives a sphere different inner properties from a plane. In the geometry of Gauss, Bólyai, and Lobachevski, we can use the coordinates x_1, x_2 of Klein's model, and find from the posited formula for $d(x, X)$ that

$$g_{11} = \frac{a^2(1 - x_2{}^2)}{(1 - x_1{}^2 - x_2{}^2)^2} \qquad g_{12} = \frac{a^2 x_1 x_2}{(1 - x_1{}^2 - x_2{}^2)^2} \qquad g_{22} = \frac{a^2(1 - x_1{}^2)}{(1 - x_1{}^2 - x_2{}^2)^2} \tag{1.1.9}$$

The length of any path can be determined by integrating ds along the path.

The metric functions g_{ij} determine all inner properties of a metric space, but they also depend on how we choose the coordinate mesh. For instance, we can use polar coordinates r, θ to describe a plane surface, and find that the metric functions are

$$g_{rr} = 1 \qquad g_{r\theta} = 0 \qquad g_{\theta\theta} = r^2 \tag{1.1.10}$$

This does not *look* like a Euclidean space, but of course it is, as we can show formally by transforming to Cartesian coordinates $x = r \cos \theta$, $y = r \sin \theta$. More generally, a change of coordinates from (x_1, x_2) to (x_1', x_2') will change the metric functions g_{ij} to g_{ij}', where, for instance,

$$\begin{aligned} g_{11}' &= \left(\frac{\partial \xi_1}{\partial x_1'}\right)^2 + \left(\frac{\partial \xi_2}{\partial x_1'}\right)^2 \\ &= \left(\frac{\partial \xi_1}{\partial x_1}\frac{\partial x_1}{\partial x_1'} + \frac{\partial \xi_1}{\partial x_2}\frac{\partial x_2}{\partial x_1'}\right)^2 + \left(\frac{\partial \xi_2}{\partial x_1}\frac{\partial x_1}{\partial x_1'} + \frac{\partial \xi_2}{\partial x_2}\frac{\partial x_2}{\partial x_1'}\right)^2 \\ &= g_{11}\left(\frac{\partial x_1}{\partial x_1'}\right)^2 + 2g_{12}\frac{\partial x_1}{\partial x_1'}\frac{\partial x_2}{\partial x_1'} + g_{22}\left(\frac{\partial x_2}{\partial x_1'}\right)^2 \end{aligned} \tag{1.1.11}$$

How then can we tell the inner properties of a space by looking at its metric coefficients? What we need is some function of the g_{ij} and their derivatives that depends only on the inner properties of the space and not, like the g_{ij}, also on the particular coordinate system chosen to describe the space.

Gauss found this function, and found it to be essentially unique; it is the so-called Gaussian curvature:

$$\begin{aligned} K(x_1, x_2) = {} & \frac{1}{2g}\left[2\frac{\partial^2 g_{12}}{\partial x_1 \partial x_2} - \frac{\partial^2 g_{11}}{\partial x_2{}^2} - \frac{\partial^2 g_{22}}{\partial x_1{}^2}\right] \\ & - \frac{g_{22}}{4g^2}\left[\left(\frac{\partial g_{11}}{\partial x_1}\right)\left(2\frac{\partial g_{12}}{\partial x_2} - \frac{\partial g_{22}}{\partial x_1}\right) - \left(\frac{\partial g_{11}}{\partial x_2}\right)^2\right] \\ & + \frac{g_{12}}{4g^2}\left[\left(\frac{\partial g_{11}}{\partial x_1}\right)\left(\frac{\partial g_{22}}{\partial x_2}\right) - 2\left(\frac{\partial g_{11}}{\partial x_2}\right)\left(\frac{\partial g_{22}}{\partial x_1}\right)\right. \\ & \left. + \left(2\frac{\partial g_{12}}{\partial x_1} - \frac{\partial g_{11}}{\partial x_2}\right)\left(2\frac{\partial g_{12}}{\partial x_2} - \frac{\partial g_{22}}{\partial x_1}\right)\right] \\ & - \frac{g_{11}}{4g^2}\left[\left(\frac{\partial g_{22}}{\partial x_2}\right)\left(2\frac{\partial g_{12}}{\partial x_1} - \frac{\partial g_{11}}{\partial x_2}\right) - \left(\frac{\partial g_{22}}{\partial x_1}\right)^2\right] \end{aligned} \tag{1.1.12}$$

where g is the determinant

$$g(x_1, x_2) \equiv g_{11}g_{22} - g_{12}^2$$

(The reader should not quail at the awful appearance of this formula. After introducing a certain amount of mathematical formalism, we shall be able to derive and discuss the curvature in a far more compact and elegant notation, in Chapter 6.) By applying Eq. (1.1.12) to the metric functions (1.1.8) and (1.1.9), we find that the surface of a sphere is a space of constant positive curvature

$$K = \frac{1}{a^2} \qquad \text{(sphere)} \tag{1.1.13}$$

whereas the space of Gauss, Bólyai, and Lobachevski has constant negative curvature

$$K = -\frac{1}{a^2} \qquad \text{(G–B–L)} \tag{1.1.14}$$

(Incidentally, there is nothing very exotic about negative curvature; an ordinary saddle is negatively curved. It is the *constancy* of K that makes the geometry of Gauss, Bólyai, and Lobachevski unrealizable for ordinary curved surfaces. It is also obvious that only with K constant could the other postulates of Euclid be satisfied, because these other postulates describe an intrinsically homogeneous space, whereas if K varied from point to point then the inner properties of the space would vary with it.) Finally, if we apply our formula for K to the metric (1.1.10) that describes a plane in polar coordinates, then we find

$$K = 0 \qquad \text{(plane)} \tag{1.1.15}$$

as of course we must. Thus, however perverse we are in our choice of coordinate system, the inner properties of a space can still be revealed by the straightforward procedure of calculating K.

Having come so far, it was not long before mathematicians turned to the problem of describing the inner properties of curved spaces having three or more dimensions. It was not a trivial matter to expand the work of Gauss to more than two dimensions, because the inner properties of such spaces cannot be described by a single curvature function K. In D dimensions there will be $D(D + 1)/2$ independent metric functions g_{ij}, and our freedom to choose the D coordinates at will allows us to impose D arbitrary functional relations on the g_{ij}, leaving C functions that truly express the inner properties of the space, where

$$C = \frac{D(D + 1)}{2} - D = \frac{D(D - 1)}{2}$$

For $D = 2$, $C = 1$, as found by Gauss. For $D > 2$, $C > 1$, and the description of the geometry becomes much more complicated. This problem was completely solved in 1854 by Georg Friedrich Bernhard Riemann (1826–1866), who presented what we now call Riemannian geometry in his Göttingen inaugural lecture, *Über die Hypothesen, welche der Geometrie zu Grunde liegen*. Subsequent work by Christoffel, Ricci, Levi-Civita, Beltrami, and others developed Riemann's ideas into the beautiful mathematical structure described in our chapters on tensor analysis and curvature. However, it remained for Einstein to see the use physics could make of non-Euclidean geometry.

2 History of the Theory of Gravitation

At the end of the *Principia*, Isaac Newton (1642–1727) described gravitation as a cause that operates on the sun and planets "according to the quantity of solid matter which they contain and propagates on all sides to immense distances, decreasing always as the inverse square of the distances."[6] There are two parts to Newton's law, which were discovered in different ways, and which played different roles in the development of mechanics from Newton to Einstein.

It was of course Galileo Galilei (1564–1642) who discovered that bodies fall at a rate independent of their mass. His tools were an inclined plane to slow the fall, a water clock to measure its duration, and also a pendulum, to avoid rolling friction. These observations were later improved by Christaan Huygens (1629–1695). Newton could thus use his second law to conclude that the force exerted by gravitation is proportional to the mass of the body on which it acts; the third law then ensures that the force is also proportional to the mass of its source.

Newton was well aware that these conclusions might be only approximately true, and that the "inertial mass" entering in his second law might not be precisely the same as the "gravitational mass" appearing in the law of gravitation. If this were the case, we would have to write Newton's second law as

$$\mathbf{F} = m_i \mathbf{a} \tag{1.2.1}$$

and write the law of gravitation as

$$\mathbf{F} = m_g \mathbf{g} \tag{1.2.2}$$

where $\mathbf{g}$ is a field depending on position and other masses. The acceleration at a given point would be

$$\mathbf{a} = \left(\frac{m_g}{m_i}\right) \mathbf{g} \tag{1.2.3}$$

and would be different for bodies with different values for the ratio m_g/m_i; in particular pendulums of equal length would have periods proportional to $(m_i/m_g)^{1/2}$. Newton tested this possibility by experiments with pendulums of equal length but different composition, and found no difference in their periods. This result was later verified more accurately by Friedrich Wilhelm Bessel (1784–1846) in 1830. Then, in 1889, Roland von Eötvös[7] succeeded by a different method in showing that the ratio m_g/m_i does not differ from one substance to another by more than one part in 10^9. (See Figure 1.2.) Eötvös hung two weights A and B from the ends of a 40-cm beam suspended on a fine wire at its center. At equilibrium the beam would sag in such a way that

$$l_A(m_{gA}g - m_{iA}g_z') = l_B(m_{gB}g - m_{iB}g_z') \qquad (1.2.4)$$

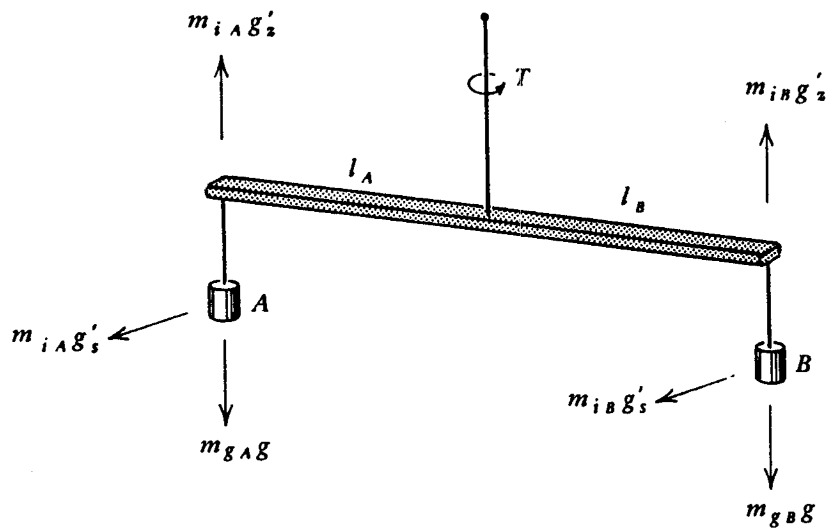

Figure 1.2 Schematic view of the Eötvös experiment.

where g is the earth's gravitational field, g_z' is the vertical component of the centripetal acceleration due to the earth's rotation, and l_A and l_B are the effective lever arms for the two weights. [Of course Eötvös chose weights and lever arms to be nearly equal, but the point of his method is that even if A is a little bigger than B, the beam will still sag just so as to make (1.2.4) correct.] At the latitude of Budapest the centripetal acceleration due to the earth's rotation also has an appreciable horizontal component g_s', giving to the balance a torque around the vertical axis equal to

$$T = l_A m_{iA} g_s' - l_B m_{iB} g_s'$$

Using the equilibrium condition to determine l_B, we have then

$$T = l_A m_{iA} g_s' \left[1 - \left(\frac{m_{gA}}{m_{iA}}g - g_z'\right)\left(\frac{m_{gB}}{m_{iB}}g - g_z'\right)^{-1}\right]$$

or, since g_z' is much less than g,

$$T = l_A g_s' m_{gA}\left[\frac{m_{iA}}{m_{gA}} - \frac{m_{iB}}{m_{gB}}\right]$$

Any inequality in the ratios m_i/m_g for the two weights would thus tend to twist the wire from which the balance was suspended. No twist was detected, and

Eötvös concluded from this that the difference of m_i/m_g for wood and platinum was less than 10^{-9}.

Einstein was very impressed with the observed equality of gravitational and inertial mass[8], and as we shall see, it served him as a signpost toward the Principle of Equivalence. (It also sets very stringent limits on any possible nongravitational forces that might exist. For instance, any new kind of electrostatic force in which the number of nucleons plays the role of charge would have to be much weaker than gravitation.[9]) In recent years a group under R. H. Dicke[10] at Princeton has improved on Eötvös' method, by using the gravitational field of the sun and the earth's centripetal acceleration toward the sun, rather than the rotation of the earth, to produce the torque on the balance. The advantage is that the angle between the direction of the sun and the balance arm changed with a 24-hr period, and so Dicke could filter out of his data any noise not at the diurnal frequency. In this way he concluded that "aluminum and gold fall toward the sun with the same acceleration, the accelerations differing from each other by at most one part in $10.^{11}$" It has also been shown (with very much less precision) that neutrons fall with the same acceleration as ordinary matter,[11] and that the gravitational force on electrons in copper is the same as on free electrons.[12]

We now move on to the second part of Newton's law of gravitation, which says that the force decreases as the inverse square of the distance. This idea was not entirely original with Newton. Johannus Scotus Erigena (c. 800–c. 877) had guessed that heaviness and lightness vary with distance from the earth. This theory was taken up by Adelard of Bath (twelfth century), who realized that a stone dropped into a very deep well could fall no farther than the center of the earth. (Incidentally, Adelard also translated Euclid from Arabic into Latin, thus making it available to medieval Europe.) The first suggestion of an inverse-square law may have been made around 1640 by Ismael Bullialdus (1605–1694). However, it was certainly Newton who in 1665 or 1666 first deduced the inverse-square law from observations. He knew that the moon falls toward the earth a distance 0.0045 ft. each second, and he knew that the moon is 60 earth radii away from the center of the earth. Hence, if the gravitational force obeys an inverse-square law, then an apple in Lincolnshire (which is 1 earth radius away from the center of the earth) should fall in the first second 3600 times 0.0045 ft, or about 16 ft, in good agreement with the measured value. However, Newton did not publish this calculation for twenty years, because he did not know how to justify the fact that he had treated the earth as if its whole mass were concentrated at its center. Meanwhile, it became known to several members of the Royal Society, including Edmund Halley (1656–1742), Christopher Wren (1632–1723), and Robert Hooke (1635–1703), that Kepler's third law would imply an inverse-square law of force if the orbits of planets were circular. That is, if the squares of the periods, r^2/v^2, are proportional to the cubes of the radii r^3, then the centripetal acceleration v^2/r is proportional to $1/r^2$. However, the planets actually move on ellipses, not circles, and no one knew how to calculate their centripetal acceleration. Under Halley's instigation, Newton in 1684 proved that planets moving under the influence of an inverse-square-law force would indeed obey all the empirical laws of Johannes Kepler (1571–1630); that is, they would move on ellipses with the sun at a focus, they would sweep out equal areas in equal times, and the square of their periods would be proportional to the cube of their major axes. Finally, in 1685, Newton was able to complete his lunar calculation of 1665. These stupendous accomplishments were published on July 5, 1686, under the title *Philosophiae Naturalis Principia Mathematica.*[13]

In the following centuries Newton's law of gravitation met with a brilliant series of successes in explaining the motion of the moon and planets. Some irregularities in the orbit of Uranus remained unexplained until, in 1846, they were independently used by John Couch Adams (1819–1892) in England and Urbain Jean Joseph LeVerrier (1811–1877) in France to predict the existence and position of Neptune. The discovery of Neptune shortly thereafter was perhaps the most splendid verification of Newton's theory. The motion of the moon and Encke's comet (and, later, Halley's comet) still showed departures from Newtonian theory, but it was clear that nongravitational forces could be at work.

One problem remained. A year before his prediction of Neptune, LeVerrier had calculated that the observed precession of the perihelia of Mercury was 35″/century faster than what would be expected according to Newton's theory from the known perturbing fields of the other planets. This discrepancy was confirmed in 1882 by Simon Newcomb (1835–1909), who gave a value of 43″ for the excess centennial precession.[14] LeVerrier had thought that this excess was probably due to a group of small planets between Mercury and the sun, but after a careful search none were discovered. Newcomb then suggested that perhaps the matter responsible for the faint "zodiacal light" seen in the plane of the ecliptic of the solar system was also responsible for the excess precession of Mercury. However, his calculations showed that the amount of matter needed to account for the precession of Mercury would, if placed in the plane of the ecliptic, produce a rotation of the plane of the orbits (that is, a precession of the nodes) of both Mercury and Venus different from what had been observed. For this reason, Newcomb was led by 1895 "to drop these explorations as unsatisfactory, and to prefer provisionally the hypothesis that the Sun's gravitation is not exactly as the inverse square."[15]

Unfortunately this was not the last word. In 1896 H. H. Seeliger constructed an elaborate model of the zodiacal light, placing the matter responsible on ellipsoids close to the sun, which could account for the excess precession of Mercury without upsetting the agreement between theory and experiment for the rotation of the planes of the inner planets' orbits. Today we know that this model is totally wrong, and that there simply is not enough interplanetary matter to account for the observed excess precession of Mercury. However, Seeliger's hypothesis, together with the continued success of Newtonian theory elsewhere, convinced Newcomb that there was no need to alter the law of gravitation.[15]

I do not know whether Einstein was very much influenced, in creating general relativity, by the problem of the precession of Mercury's perihelia. However, there is no doubt that the first confirmation of his theory was that it predicted an excess precession of precisely 43″/century.

3 History of the Principle of Relativity

Newtonian mechanics defined a family of reference frames, the so-called *inertial frames*, within which the laws of nature take the form given in the *Principia*. For instance, the equations for a system of point particles interacting gravitationally are

$$m_N \frac{d^2\mathbf{x}_N}{dt^2} = G \sum_M \frac{m_N m_M(\mathbf{x}_M - \mathbf{x}_N)}{|\mathbf{x}_M - \mathbf{x}_N|^3} \tag{1.3.1}$$

where m_N is the mass of the Nth particle and $\mathbf{x}_N$ is its Cartesian position vector at time t. It is a simple matter to check that these equations take the same form when written in terms of a new set of space-time coordinates:

$$\begin{aligned} \mathbf{x}' &= R\mathbf{x} + \mathbf{v}t + \mathbf{d} \\ t' &= t + \tau \end{aligned} \tag{1.3.2}$$

where $\mathbf{v}$, $\mathbf{d}$, and τ are any real constants, and R is any real orthogonal matrix. (If O and O′ use the unprimed and primed coordinate system, respectively, then O′ sees the O coordinate axes rotated by R, moving with velocity $\mathbf{v}$, displaced at $t = 0$ by $\mathbf{d}$, and O′ sees the O clock running behind his own by a time τ.) The transformations (1.3.2) form a 10-parameter group (three Euler angles in R, plus three components each for $\mathbf{v}$ and $\mathbf{d}$, plus one τ) today called the *Galileo group*, and the invariance of the laws of motion under such transformations is today called Galilean invariance, or the *Principle of Galilean Relativity*.

What really impressed Newton about all this was that there are a great many more transformations that do *not* leave the equations of motion invariant. For instance, (1.3.1) does not retain its form if we transform into an accelerating or a rotating coordinate system, that is, if we let $\mathbf{v}$ or R depend on t. The equations of motion can hold in their usual form in only a limited class of coordinate systems, called *inertial frames*. What then determines which reference frames are inertial frames? Newton answered that there must exist an absolute space, and that the inertial frames were those at rest in absolute space, or in a state of uniform motion with respect to absolute space. In his words[16]:

> "Absolute space, in its own nature and with regard to anything external, always remains similar and unmovable. Relative space is some movable dimension or measure of absolute space, which our senses determine by its position with respect to other bodies, and is commonly taken for absolute space."

Newton also described several experiments that demonstrated what he interpreted as the effects of rotation with respect to absolute space. The most famous is the rotating bucket[17]:

> "If a bucket, suspended by a long cord, is so often turned about that finally the cord is strongly twisted, then is filled with water, and held at rest together with the water; and afterwards by the action of a second force, it is suddenly set whirling about the contrary way, and continues, while the cord is untwisting itself, for some time in this motion; the surface of the water will at first be level, just as it was before the vessel began to move; but subsequently the vessel, by gradually communicating its motion to the water, will make it begin sensibly to rotate, and the water will recede little by little from the middle and rise up at the sides of the vessel; its surface assuming a concave form. (This experiment I have made myself.) . . . At first, when the *relative* motion of the water in the vessel was greatest, that motion produced no tendency whatever of recession from the axis, the water made no endeavor to move upwards towards the circumference, by rising at the sides of the vessel, but remained level, and for that reason its true circular motion had not yet begun. But afterwards, when the relative motion of the water had decreased, the rising of the water at the sides of the vessel indicated an endeavor to recede from the axis; and this endeavor reveals the real circular motion of the water, continually increasing till it had reached its greatest point, when relatively the water was at rest in the vessel. . . ."

Newton's conception of absolute space was rejected by his great opponent Gottfried Wilhelm von Leibniz (1646–1716), who argued that there is no philosophical need for any conception of space apart from the relations of material objects. The issue was debated in a famous series of letters[18] (1715–1716) between Leibniz and Newton's supporter, Samuel Clarke (1675–1729), and philosophers continued the argument, with Newton's position defended by Leonhard Euler (1707–1783) and Immanuel Kant (1724–1804) and attacked by Bishop George Berkeley (1685–1753) in his *Principles of Human Knowledge* (1710) and *Analyst* (1734). Of course none of this high-minded metaphysics led to any idea about how to develop a dynamical theory that might replace Newton's.

The first constructive attack on Newtonian absolute space was launched in the 1880's by the Austrian philosopher Ernst Mach (1836–1916). In his book *Die Mechanik in ihrer Entwicklung*[19] he remarks that

> "Newton's experiment with the rotating vessel of water simply informs us, that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces *are* produced by its relative motion with respect to the mass of the Earth and the other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass until they were several leagues thick."

The hypothesis, that there is some influence of "the mass of the Earth and the other celestial bodies" which determines the inertial frames, is called *Mach's principle.*

There is a simple experiment that anyone can perform on a starry night, to clarify the issues raised by Mach's principle. First stand still, and let your arms hang loose at your sides. Observe that the stars are more or less unmoving, and that your arms hang more or less straight down. Then pirouette. The stars will seem to rotate around the zenith, and at the same time your arms will be drawn upward by centrifugal force. It would surely be a remarkable coincidence if the inertial frame, in which your arms hung freely, just happened to be the reference frame in which typical stars are at rest, unless there were some interaction between the stars and you that determined your inertial frame.

This argument can be made more precise. The surface of the earth is not exactly an inertial frame, and of course the rotation and revolution of the earth give the stars an apparent motion, but these effects can be eliminated by using the inertial frame defined by the solar system as a whole. In this inertial frame of reference the average observed rotation of the galaxies with respect to any axis through the sun is less than about 1 arc-sec/century![20]

We seem to be faced with an unavoidable choice: Either we admit that there is a Newtonian absolute space-time, which defines the inertial frames and with respect to which typical galaxies happen to be at rest, or we must believe with Mach that inertia is due to an interaction with the average mass of the universe. And if Mach is right, then the acceleration given a particle by a given force ought to depend not only on the presence of the fixed stars but also, very slightly, on the distribution of matter in the immediate vicinity of the particle. We shall see in Chapter 3 that Einstein's equivalence principle gives an answer to the problem of inertia that does not refer to a Newtonian absolute space and yet does not quite agree with the conclusions of Mach. The issue is not closed.

I have not yet mentioned special relativity because, despite its name, it really does not affect the antinomy between absolute and relative space. However, we shall have to formulate the equivalence principle in special-relativistic terms, so a detailed review of special relativity is presented in the next chapter; for the moment we only take a glance at its history.

The theory of electrodynamics presented in 1864 by James Clark Maxwell (1831–1879) clearly did not satisfy the principle of Galilean relativity. For one thing, Maxwell's equations predict that the speed of light in vacuum is a universal constant c, but if this is true in one coordinate system x^i, t, then it will not be true in the "moving" coordinate system x'^i, t' defined by the Galilean transformation (1.3.2). Maxwell himself thought that electromagnetic waves were carried by a medium,[21] the luminiferous ether, so that his equations would hold in only a limited class of Galilean inertial frames, that is, in those coordinate frames at rest with respect to the ether.

However, all attempts to measure the velocity of the earth with respect to the ether failed,[22] even though the earth has a velocity of 30 km/sec relative to the sun, and about 200 km/sec relative to the center of our galaxy. The most important experiment was that of Albert Abraham Michelson (1852–1931) and E. W. Morley,[23] which showed in 1887 that the velocity of light is the same, within 5 km/sec, for light traveling along the direction of the earth's orbital motion and transverse to it. The accuracy of this result has been recently improved to about 1 km/sec.[24]

The persistent failure of experimentalists to discover effects of the earth's motion through the ether led theorists, including George Francis Fitzgerald[25] (1851–1901), Hendrik Antoon Lorentz[26] (1853–1928), and Jules Henri Poincaré[27] (1854–1912) to suggest reasons why such "ether drift" effects should be in principle unobservable. (See Figure 1.3.) Poincaré in particular seems to have glimpsed the revolutionary implications that this would have for mechanics, and Whittaker[28] gives the credit for special relativity to Poincaré and Lorentz. Without entering this controversy,[29] it is safe to say that a comprehensive solution to the problems

Figure 1.3 Founders of the Special Theory of Relativity, at the First Solvay Conference in 1911.

of relativity in electrodynamics and mechanics was first set out in detail in 1905 by Albert Einstein[30] (1879–1955).

Einstein proposed that the Galilean transformation (1.3.2) should be replaced with a different 10-parameter space-time transformation, called a *Lorentz transformation*, that does leave Maxwell's equations and the speed of light invariant. (It is not clear that Einstein was directly influenced by the Michelson-Morley experiment itself,[31] but he specifically refers to "the unsuccessful attempts to discover any motion of the earth relative to the 'light medium'" in his 1905 paper.[32]) The equations of Newtonian mechanics, such as Eq. (1.3.1), are not invariant under Lorentz transformations; therefore Einstein was led to modify the laws of motion so that they would be Lorentz-invariant. The new physics, consisting of Maxwell's electrodynamics and Einstein's mechanics, then satisfied a new principle of relativity, the Principle of Special Relativity, which says that all physical equations must be invariant under Lorentz transformations. These developments are discussed in detail in the next chapter.

The Lorentz group of transformations is not in any way larger than the Galileo group, and therefore the principle of relativity was not originated by the special theory of relativity, but rather *restored* by it. Before Maxwell, it might have been supposed that all of physics is invariant under the Galileo group. Maxwell's equations were not invariant under this group, and for half a century it appeared that only mechanics, not electrodynamics, obeys the principle of relativity. After Einstein, it was clear that the equations of both mechanics and electrodynamics are invariant, but with respect to Lorentz transformations, not Galileo transformations. The laws of physics in the form given them by Maxwell and Einstein could still only be true in a limited class of inertial reference frames, and the question of what determines these inertial frames was as mysterious after 1905 as in 1686.

It remained to construct a relativistic theory of gravitation. A crucial step toward this goal was taken in 1907, when Einstein introduced the Principle of Equivalence of Gravitation and Inertia,[33] and used it to calculate the red shift of light in a gravitational field. As we shall see in Chapter 3, this principle determines the effects of gravitation on arbitrary physical systems, but it does not determine the field equations for gravitation itself. Einstein tried to use the equivalence principle in 1911 to calculate the deflection of light in the sun's gravitational field,[34] but the structure of the field was not then correctly understood, and Einstein's answer was one-half the "correct" general-relativistic result, derived here in Chapter 8. A number of attempts were made in 1911–1912 by Einstein,[35] Abraham,[36] and Nordström[37] to construct relativistic field equations for a single scalar gravitational field, but Einstein soon became dissatisfied with all such theories, largely on aesthetic grounds. (The gravitational deflection of light by the sun had not yet been measured.) A collaboration with the mathematician Marcel Grossman led Einstein by 1913 to the view[38] that the gravitational field must be identified with the 10 components of the metric tensor of Riemannian space-time geometry. As discussed in Chapters 4 and 5, the Principle of Equivalence is incorporated into this formalism through the requirement that the physical equations be invariant under *general* coordinate transformations, not just Lorentz transformations, though I do not know to what extent this "General Principle of Relativity" took on in Einstein's mind a life of its own, apart from the Principle of Equivalence. During the next two years, Einstein presented to the Prussian Academy of Sciences a series of papers[39] in which he worked out the field equations for the metric tensor and calculated the gravitational deflection of light and the precession of the perihelia of Mercury. These magnificent achievements were finally summarized by Einstein in his 1916 paper,[1] titled "The Foundation of the General Theory of Relativity."

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