**Math 280:** *Introduction to Shock Wave Theory*
Blake Temple, Winter Quarter 2013, UC-Davis

This class is a mathematical introduction to the theory of *Shock Waves* and the nonlinear equations that support them, *Conservation Laws*. Shock waves are steep fronts that form in nonlinear equations due to nonlinear focusing. Shock waves incorporate disparate physical phenomenon under a single theory: sonic booms, breaking waves on a beach, traffic jams, chromatography waves and discontinuities in spacetime curvature, are all understood as aspects of the same nonlinear shock wave phenomenon. Shock wave theory lies at the heart of modern partial differential equations (PDE) because shock wave discontinuities are typically the most interesting effect in a given scientific problem, they provide an important application of the theory of distributions to nonlinear equations, and they represent "what can go wrong", and hence what must be ruled out, whenever the existence of a smooth solution to a PDE is important.

Shock waves re-introduce dissipation back into equations whose derivation begins by neglecting dissipation. Based on this, Peter Lax extended the concept of *entropy* and loss of information to general systems of conservation laws. Thus the theory of shock waves is of great intellectual, as well as mathematical and scientific interest. Shock wave theory has directed the attention of many great mathematicians including Bernard Riemann, John Von Neumann, Kurt Friedrichs and more recently Peter Lax (2005 Abel Prize) and James Glimm (2002 Medal of Science, President of AMS 2007-9).

In this class, which is self-contained, we begin by deriving the compressible Euler equations of gas dynamics, Newton's laws for a continuous media, and the prototype of a system of conservation laws. From there, topics of study will include: the second law of thermodynamics, entropy, the Riemann problem for the $p$-system and the general solution due to Lax, Lax's generalization of entropy, shock profiles, numerical methods by the Lax Wendroff Theorem, and existence of weak solutions by the genius of the Glimm Scheme.

At the end, instructor will give a seminar level introduction to the research problems he and collaborators are engaged in. These include a shock wave cosmological model of the Big Bang; a wave theory for Dark Energy; *Regularity Singularities* created by shock wave collision; a causal theory of relativistic viscosity; and a theory of nonlinear dissipation-free sound wave transmission. Lecture notes and references to recent research are posted on instructors webpage: http://www.math.ucdavis.edu/ temple/.