1. (pts) Let \( f(x) = \sin \frac{1}{x} \). Use the definition of the limit to prove that \( \lim_{x \to 0} f(x) \) does not exist.

2. (pts) Find the interval of convergence for the following power series

\[
\sum \frac{2^n}{n5^{n+1}}x^n
\]
3. (pts) Let the sequence of functions \( \{f_n\} \) be \( f_n(x) = x - x^n \) for \( x \in [0, 1] \).

(a) Find \( f(x) \) such that \( \{f_n\} \to f \) on \([0, 1]\).

(b) Using the definition, prove \( \{f_n\} \) does not converge uniformly to \( f \) (found in part a) on \([0, 1]\).
4. (pts) Let the sequence of functions \( \{f_n\} \) be 
\[ f_n(x) = \frac{1}{1 + nx} \text{ for } x \in [2, \infty). \]
Let \( f(x) = 0 \) for \( x \in [2, \infty). \) Using the definition, prove \( \{f_n\} \) converges uniformly to \( f \) on \( x \in [2, \infty). \)
5. (pts) For \( x \in [0, 1] \), we have the following power series

\[ \sqrt{1 + x} = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{(1 - 2n)(n!)^2 (4^n)} x^n. \]

Use this fact to build a power series for \( \frac{1}{\sqrt{1 - x^2}} \).

6. (pts) Prove the following series converges uniformly on \( \mathbb{R} \) to a continuous function

\[ \sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx. \]
7. (pts) Use the definition of the derivative to prove the Quotient Rule.

8. (pts) Use the definition of the derivative to show \( f(x) = |x| + |x + 1| \) is not differentiable at \( x = -1 \).
9. (pts) Let the sequence of functions \( \{f_n\} \) be \( f_n(x) = \frac{nx}{1 + n^2x^2} \) for \( x \in [0, 1] \). Let \( f(x) = 0 \) for \( x \in [0, 1] \). Prove \( \{f_n\} \) does not converge uniformly to \( f \) on \( x \in [0, 1] \).
The following extra credit problem is OPTIONAL and you are advised to finish the rest of the test before trying this problem.

1. (pts) Prove that for all $x_0 \in \mathbb{R}$ there exists a sequence of rational numbers which converges to $x_0$. Also, there exists a sequence of irrational numbers which converges to $x_0$. 