1. (pts) Prove that $f(x) = 5x^2 - 7$ is continuous on the interval $(1, \infty)$ by verifying the $\epsilon\delta$-property.
2. (pts) Prove that \( f(x) = 5x^2 - 7 \) is not uniformly continuous on the interval \((1, \infty)\) by definition.
3. (pts) Let \( f(x) = \lfloor x \rfloor \) be the floor function (i.e. \( f(x) \) is the largest integer less than \( x \), which can be defined as \( \lfloor x \rfloor := \max\{p \in \mathbb{Z} : p \leq x\} \)). Define the function \( g(x) = x - \lfloor x \rfloor \) to be the fractional part of \( x \).

(a) Sketch both functions \( f(x) \) and \( g(x) \) over the interval \([-4, 4]\), and determine where each function is discontinuous on \( \mathbb{R} \).

(b) Prove \( g(x) \) is discontinuous at \( x_0 = 0 \) using the definition.
4. (pts) Prove that \( \ln(x + 1) = 1 - x \) is solvable.

5. (pts) Give an example of a continuous function \( f(x) \) bounded on \([0, \infty)\) that does not obtain it’s maximum value (i.e. \( \not\exists x^* \in [0, \infty) \) such that \( f(x) \leq f(x^*) \ \forall x \in [0, \infty) \)).
6. (pts) Which are the following functions on the indicated domain are continuous and/or uniformly continuous or neither? Briefly justify your answer, using any theorem (in the book) you wish.

(a) \( f(x) = 2^x \) on \([-7, 5]\).

(b) \( g(x) = \frac{1}{x^3} \) on \((0, 1)\).

(c) \( h(x) = \begin{cases} -1 & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases} \) on \((-2, 2)\).
7. (pts) Let $f$ be a continuous function with domain $(a, b)$. Prove that if $f(r) = 0$ for each rational number $r$ in $(a, b)$, then $f(x) = 0$ for all $x \in (a, b)$. 
The following extra credit problems are OPTIONAL and you are advised to finish the rest of the test before trying these problems.

1. (pts) Prove that for all \( x_0 \in \mathbb{R} \setminus \mathbb{Q} \) (i.e. the irrationals) there exists a sequence \( x_n \subseteq \mathbb{Q} \) which converges to \( x_0 \).