Name: ____________________

Student ID#: ____________________

Section: ____________________

Midterm Exam 1

Wednesday, Jan 29

MAT 125A, Temple, Winter 2014

Show your work on every problem. Correct answers with no supporting work will not receive full credit. Be organized and use notation appropriately. No calculators, notes, books, cellphones, etc. may be used on this exam. Please write legibly. Please have your student ID ready to be checked when you turn in your exam.

Problem	Your Score	Maximum Score
1		20
2		20
3		20
4		20
5		20
Total		100

Problem #1 (20pts): (a) State the $\epsilon - \delta$ condition for a function to be continuous at a point x_0, and state its negation. (You may assume that f is defined for all real numbers.)

(b) Use the negation to prove directly that if for every sequence $x_n \to x_0$ we have $f(x_n) \to f(x_0)$, then f is continuous at x_0 by the $\epsilon - \delta$ condition.

Problem #2 (20pts): (a) Recall that a continuous function on a closed interval takes on its max and min values m and M, respectively. Prove the case that it takes on its minimum value m.

(b) State the Intermediate Value Theorem.

(c) Using only parts (a) and (b), prove that if $f : [a, b] \to \mathcal{R}$ is continuous, then its range is exactly $[m, M]$.

Problem #3 (20pts): Assume that $f(x)$ is a function defined and continuous on the closed interval $[a, b]$. Let y be given, and define $x_0 = Inf\mathcal{S}$ where $\mathcal{S} = \{x : f(x) > y\}$. Assume that $a < x_0 < b$, and prove that $f(x_0) = y$.

Problem #4 (20pts): Let $f : \mathcal{R} \to \mathcal{R}$ be a uniformly continuous function, and let x_n denote a sequence of real numbers.

(a) State the Cauchy criterion for convergence of x_n.

(b) State the definition of uniform continuity of f.

(c) Use (a) and (b) to prove directly that if x_n is Cauchy, then so is $f(x_n)$.

Problem #5 (20pts): Let $f : \mathcal{R} \to \mathcal{R}$ and $g : \mathcal{R} \to \mathcal{R}$ be uniformly continuous functions. Prove that $f \circ g$ is uniformly continuous. (Recall, $(f \circ g)(x) = f(g(x))$.)