Name: ____________________

Student ID#: ____________________

Section: ____________________

Midterm Exam 2

Monday, March 3

MAT 125A, Temple, Winter 2014

Show your work on every problem. Correct answers with no supporting work will not receive full credit. Be organized and use notation appropriately. No calculators, notes, books, cellphones, etc. may be used on this exam. Please write legibly. Please have your student ID ready to be checked when you turn in your exam.

Problem	Your Score	Maximum Score
1		15
2		15
3		20
4		15
5		20
6		15
Total		100

Problem #1 (15pts):

(a) State the $\epsilon - \delta$ definition of a uniformly continuous function f defined on the Domain set $\mathcal{S}$. Then state the negation of this statement. That is, give the definition of *not* uniformly continuous.

(b) Let $f : \mathcal{R} \to \mathcal{R}$ and $g : \mathcal{R} \to \mathcal{R}$ be uniformly continuous functions. Prove that $f \circ g$ is uniformly continuous. (Recall, $(f \circ g)(x) = f(g(x))$.)

Problem #2 (15pts): Assume that f and g are differentiable at $x = x_0$, and assume that $g(x_0) \neq 0$. [*Corrected from* $g'(x_0) \neq 0$ *in exam.*] Give a careful proof that

$$\frac{d}{dx}\left(\frac{f}{g}\right)(x_0) = \frac{g(x_0)f'(x_0) - g'(x_0)f(x_0)}{g(x_0)^2}.$$

Problem #3 (20pts): Consider the geometric series starting at $k = 2$,

$$f(x) = \sum_{k=2}^{\infty} x^k.$$

(a) Derive a formula for the partial sum $S_n(x) = \sum_{k=2}^{n} x^k$.

(b) Define what it would mean for $S_n(x)$ to be a uniformly Cauchy sequence of functions.

(c) Prove that $S_n(x)$ is a uniformly Cauchy sequence of functions on Domain $|x| < r$ for any $r < 1$.

Problem #4 (15pts): Let f be a function continuous on the closed interval $[a, b]$, assume f is differentiable on the open interval (a, b), and assume $f(a) = f(b)$. Using only theorems about continuous functions that we proved before, give a careful proof of Rolle's Theorem: There exists a point $x^* \in (a, b)$ at which $f'(x^*) = 0$.

Problem #5 (20pts): Assume that $f_n(x) \to f(x)$ for each $x \in [0,1]$, and assume that each f_n is continuous on $[0,1]$.

(a) Give an example of a sequence f_n such that the limit f is discontinuous.

(b) Define what it means for the sequence of functions f_n to converge *uniformly* to f.

(c) Give a careful proof that if $f_n \to f$ uniformly, then f is continuous.

Problem #6 (15pts): Assume $f(x) = \sum_{n=0}^{\infty} a_n x^n$ has radius of convergence $R = \infty$. Use our theorems about power series to derive a formula for the coefficients a_n in terms of derivatives of f at $x = 0$. Then marvel at the idea that the entire function is determined by what is happening at $x = 0$!.