RESEARCH CONTRIBUTIONS

Professor Blake Temple's primary fields of research are the mathematical theory of shock waves and general relativity. Putting these two theories together, he and collaborators have formulated a theory of shock wave propagation in the Einstein equations of general relativity. Shock waves are steep fronts that form and propagate in fluids because of the nonlinear effects of compression. A sonic boom off the wing of a plane, grid-lock in traffic, waves breaking on the shoreline, flame fronts in combustion, the leading edge of a nuclear explosion, tidal waves, the bore of water let loose when a dam breaks, saturation waves in an oil reservoir, adsorption lines in chromatography, the water-hammer problem in pipelines, all are examples of the same phenomenon as understood by the modern mathematical theory of shock-waves. The mathematical theory of shock-waves has provided a unified conceptual picture capable of describing all of the above applications in terms of a single theory. Most interestingly, the modern theory has explained how *entropy* and *time-irreversibility*, (concepts that originally were understood only in the context of ideal gases), could be given meaning in an arbitrary conservative, first order system of nonlinear partial differential equations, much more general than gas dynamics. The conclusion: Shock-waves introduce dissipation and increase of entropy into the dynamics of solutions, and this provides a mechanism by which complicated solutions can settle down to orderly self-similar wave patterns, even when dissipative terms are neglected in the formulation of the equations. A rock thrown into a pond demonstrates how the mechanism can transform a chaotic "plunk" into a series of orderly outgoing self-similar waves moments later. Shock waves are also one of the celebrated applications of the theory of distributions to nonlinear equations.

In Temple's current research [80,81,82], he and collaborator Joel Smoller have discovered a new one parameter family of general relativistic self-similar expansion waves containing the Standard Model of Cosmology during the radiation phase of the expansion. They have shown that these expansion waves look remarkably similar to the Friedmann spacetimes of the Standard Model, and can account for the leading order quadratic correction to redshift vs luminosity observed in the super nova data, without the need for dark energy or the cosmological constant. They are now investigating whether these self-similar perturbations are consistent with other known observational data in astrophysics. Reference [81] is a nice non-technical discussion of the issues. The idea that the anomalous acceleration of the galaxies might be due to an expansion wave that formed during the radiation epoch was first proposed by Temple in the talk *Numerical Shock-wave Cosmology*, New Orleans, January 2007, which is the fourth entry under *Conference/Seminar Talks* on his webpage.

In [77], Temple together with collaborator (former student) Robin Young introduced a new wave pattern that is consistent with time periodic propagation of nonlinear sound waves, and in [76,77,78,79] have set out a program to prove that such waves solve the compressible Euler equations exactly. To put this into context, it was 1753 when Leonard Euler first introduced the Compressible Euler Equations and showed that in the limit of weak signals, the density of air satisfies the same wave equation as his colleague D'Alambert had shown describes oscillations of a vibrating string, thereby establishing the modern theory of music and sound. It has been an open problem since that time as to whether the fully nonlinear equations could support analogous time-periodic oscillations that do not form shock waves. (Time-periodic propagation is inconsistent with shock waves because entropy strictly increases when shock waves are present.) Temple and Young argue that they have found a simplest periodic wave structure consistent with time-periodic evolution, and by tailoring a new framework of analysis to this wave pattern, they are one step away from a complete mathematical proof that such periodic wave patterns can solve the Compressible Euler Equations exactly. Indeed, they believe the final step in the proof is at hand. (Temple believes they have it, as of June 2010!) Such waves, if they exist, represent shock-free, dissipation free, long distance transmission of sound waves, thought not possible since 1854 when Riemann first proved that shock waves form in solutions of Euler's equations. The idea of Temple and Young is to first discover, by formal reasoning, the wave structure of time-periodic waves, and then to tailor the functional analysis to the underlying structure. Combinatoric reasoning based on a new characterization of Compression and Rarefaction in the scattering of nonlinear waves by an entropy jump, led to their discovery in [77] of a simplest possible time-periodic wave structure consistent with the condition that Compression and Rarefaction be in balance (in a formal sense) along every characteristic (sound wave). They then defined an operator consisting of two evolutions, two jumps and a half period shift–the composition of five non-commuting, non-symmetric operators whose kernel (eigenfunctions) represent time periodic solutions of compressible Euler exhibiting the identified simplest periodic wave structure. Linearizing about the constant state values at each entropy level, they show that the linearized operator has a 1-mode kernel (time-periodic solution) exhibiting the identified wave structure, and for almost every period, the operator is bounded and invertible on the complement of this 1-mode kernel. But although the linearized operator is invertible, (a marvelous simplification indicating the framework is correct!), the *inverse* of the linearized operator is not bounded on the complement of the kernel due to the presence of *small divisors*, (the obstacle KAM theory was invented to address). By demonstrating the validity of the Liapunov-Schmidt decomposition in [79], they have proven that the small divisors are the only obstacle to a complete proof of the existence of time-periodic solutions of compressible Euler. They currently believe they have a proof that the leading order corrections to the linearized operator, (which encode the effect of genuine nonlinearity), will kill the small divisors, and hope to have the complete proof written up very soon.

This study of time-periodic solutions has introduced fascinating new techniques for conservation laws, (KAM theory, bifurcation theory, combinatorics, etc), it could change our point of view on how sound waves actually propagation in compressible Euler, and Temple believes the program could well open the door to the discovery of fascinating new wave phenomena, including chaotic and quasi-periodic propagation, and the possible stabilizing effect of random entropy fields on the dynamics, (more realistic for actual sound waves).

In earlier joint work [46], Temple and Young gave the first existence theorem for the compressible Euler equations allowing for general initial data of large total variation, (strong shock waves). This was a deep analysis that used the method of re-orderings (developed by Young in his dissertation) together with a new method of analysis that estimated wave growth relative to what they called a degenerate quadratic model, an exactly solvable wave interaction model that played the role of the constant state in Glimm's (celebrated) small total variation analysis. This result has stood as state of the art since 1996.

In joint work with (former student) Jeff Groah [59,64], Temple introduced a locally inertial formulation of the Einstein equations for spherically symmetric spacetimes, and the two went on to prove the first general local existence theory for shock-wave propagation in general relativity. The theory allows for arbitrary numbers of interacting shock waves of arbitrary strength. Temple's recent student Zeke Vogler, (doctoral thesis, March 2010, fourth entry under *Recent Research Articles/Books*, on Temple's webpage), developed this into what they term a *locally inertial Godunov method with dynamic time-dilation*, and demonstrated convergence of the method on a one parameter family of initial data that match an exploding Friedmann spacetime inside the general relativistic version of a static singular isothermal sphere, a simple model for a GR explosion. The numerics demonstrate the creation of two shock waves, (one imploding, one exploding), in forward time, and black hole formation in the time-reversed problem. In particular, this is a natural starting point for a rigorous proof of shock wave or black hole formation in GR, both open problems for perfect fluids. In the locally inertial formulation of Groah and Temple, the gravitational metric is only Lipschitz continuous ($C^{0,1}$) at shock waves, and it is currently an open problem whether the gravitational metric can be smoothed to $C^{1,1}$ at points of shock wave interaction like the initial fluid discontinuity in Vogler's data. (Single shock surfaces can always be smoothed, and $C^{1,1}$ is the level of smoothness usually assumed, see e.g., [47].) Temple's student Moritz Rientes is currently investigating the smoothness of the gravitational metric tensor at points of shock wave interaction.

Professor Temple has spent three decades contributing to the mathematical theory of shock waves. His early work was on geometrical aspects of shock wave interactions, (see e.g., [1,3,4,5,6,10,11,17-19,22,32,38-40]. In 1965, building on a framework for the subject set out by Peter Lax, James Glimm introduced the idea that the study of weak solutions with shock waves could be reduced to a study of wave interactions. Glimm's paper revolutionized the subject, and his ideas and methods of analysis remain the foundation of the mathematical theory to this day. Temple had a productive collaboration with Eli Isaacson on resonant nonlinear conservation laws [17-19,22,32], culminating in [39] which introduced a canonical wave structure associated with the resonant interaction of a nonlinear wave with a stationary source. This work was picked up on by Temple's student John Hong, and Temple has continued in collaboration with Hong on extensions of this theory, [63,66]. As an outgrowth of his thesis, Hong showed that a general system of conservation laws with source terms of the form introduced in [39], can be handled exactly as the *source free* case originally done by Glimm, but the residual converges only *weakly* in L^1, not strongly as in Glimm's original first order method. (This is quite unusual, and there were experts in the field who actually thought the method did not converge at all!) In his thesis [1], Temple introduced a way to incorporate entropy variations into the exactly solvable 2×2 system that has come to be known as the Nishida system. In [33], Temple and Smoller showed that there is a relativistic version of the Nishida system, (the case when the pressure is proportional to the energy density, of fundamental importance in general relativity), and in his doctoral thesis Brian Wissman, picking up on Temple's thesis, showed that the relativistic Nishida system has two natural entropy extensions, (not just one like the non-relativistic case!), one corresponding to the Stefan-Boltzmann radiation law, and a different one corresponding to the extreme relativistic limit of free particles. No such interesting physical interpretation exists for the non-relativistic case!

In [5,6] Temple introduced a family of nonlinear conservation laws that have come to be known as systems *in the Temple's class*. Such systems have milder nonlinearities than general systems of conservation laws, and have turned up in many applications, including chromatography, multiphase flow, and even in a system identified by Ed Witten in mathematical physics. In particular, since these systems isolate certain nonlinear phenomenon from others, Temple proposed them as a useful setting in which to begin a difficult analysis of conservation laws, and in particular, the technical uniqueness and continuous dependence results for the Glimm method began with the study of these systems.

In Einstein's theory of general relativity, gravitational *forces* turn out to be just anomalies of spacetime *curvature*, and the propagation of curvature through spacetime is governed by the *Einstein equations*. In [41], Temple and his co-worker Smoller were the first to construct an exact shock-wave solution of the Einstein equations that models an explosion into a static, singular isothermal sphere, a simple model for star formation. The example emerged from a general theory set out by Temple and Smoller in [47,52,55,61]. Temple and Smoller did a number of studies related to shock wave propagation in general relativity [50,51,53,56,57], culminating in [62,65] (with implications studied in [67,69,71]), which introduced a physically plausible model for the expanding universe in which a shock wave provides a finite mass cut-off of the total mass of the Big Bang explosion, and is present at the leading edge of the expansion of the galaxies that we measure by the Hubble constant. In this model, the shock wave emerges from the center of an expanding spacetime at the instant of the Big Bang, but there is a shadow region near the center in which the universe appears no different from the Standard Model, until the shock wave comes into view from the far field. In [69], Smoller and Temple argue a natural scenario by which such waves might emerge from an inflationary spacetime, the wave arising from a natural transition of co-moving coordinate frames at the end of inflation. If such a shock wave were present in the actual universe, then we now lie in a time-reversed black hole from which we will eventually emerge, long after the shock wave comes into view. In fact, Temple and Smoller have argued that because of the increase of entropy and loss of information due to the shock wave, the present universe cannot be time reversed back to micro-seconds after the Big Bang, (at least in the continuum level). In the Standard Model of the Big Bang, the universe is *infinite* at every time after the Big Bang, and time-reversible all the way back to the beginning as well, so the Smoller-Temple model is quite controversial. But it stands as a physically plausible finite mass cutoff for the standard Friedmann universe of cosmology.

As Temple says: *The Einstein equations form a highly nonlinear system of wave equations that support the propagation of waves. So [regarding the current theory of cosmology], where are the waves? Our theory of shock waves and self-similar expansion waves identify a rich structure of general relativistic nonlinear waves, and I believe they will open up new doors for exploring the dynamics of solutions, and may well have important new implications for cosmology.*