Response to Comments from the Blogosphere

Blake Temple, October 2, 2011

When our paper *Self-similar solutions of the Einstein equations that perturb the Standard Model of Cosmology*[1] first came out in PNAS, there was a great deal of media attention because we argued that such a family of waves could in principle account for the anomalous acceleration of the galaxies without Dark Energy. There was a very intense debate on the internet. A number of criticisms have been posted on blogs, including references from Clara Moskowitz's article at Space.com

http://www.space.com/7145-big-wave-theory-offers-alternative-dark-energy.html

and the Discover Magazine article

http://blogs.discovermagazine.com/cosmicvariance/2009/08/28/dark-energy-still-a-puzzle/.

and Discover Fair:

http://content.usatoday.com/communities/sciencefair/post/2009/08/68497142/1

In particular, the following quote from Avi Loeb, Chair of Astronomy at Harvard, is often referred to: "*There are many observational tests of the standard cosmological model that the proposed model must pass, aside from the late phase of accelerated expansion," said Avi Loeb, director of the Institute for Theory and Computation at the Harvard-Smithsonian Center for Astrophysics. "These include big bang nucleosynthesis, the quantitative details of the microwave background anisotropies, the Lyman-alpha forest, and galaxy surveys. The authors do not discuss how their model compares to these tests, and whether the number of free parameters they require in order to fit these observational constraints is smaller than in the standard model. Until they do so, it is not clear why this alternative model should be regarded as advantageous.*"

My Reaction: Cosmologists should welcome this new knowledge about perturbations of the standard model backed up by rigorous mathematics, and welcome this new paradigm to test against the Dark Energy theory. They should not sell this mathematics short. It represents fundamental new information about the standard model.

My Response to the Bloggers:. First, we essentially agree with the quote from Avi Loeb. I think the word "correct" in place of "advantageous" would be more unequivocal, but we essentially agree. But be sure, the parameter in our family of self-similar waves is not to be treated on the level of the cosmological constant, a free parameter added to the equations to fit the data. That is the one indisputable advantage of the wave theory over the cosmological constant at this early stage. Let me respond to the other criticisms first, and come back to this one.

What the other critics have wholly overlooked, in my opinion, is the most interesting and important aspect of these new self-similar solutions of the Einstein equations: Namely, they represent *time-asymptotic wave forms* for the radiation epoch of Big Bang cosmology, the stage when the *non-*

[1]See also our longer paper *General Relativistic self-similar solutions of the Einstein equations that perturb the Standard Model of Cosmology*, which extends these results, to appear in Memoirs of AMS.

linearities which predict decay of complicated solutions to simple wave forms, are in place.[2] To our knowledge, these are the only known time-asymptotic waves that perturb the Friedmann solution during the radiation epoch of the standard model of cosmology. That makes them mathematically *very special* and *rare*, and any expert in conservation laws would immediately conjecture that these waves should be there on some scale by the time the radiation epoch ends. *Mathematicians* know this.

Here is how it works (the theory was worked out rigorously by Glimm and Lax for periodic solutions): In a general solution that is not self-similar, the strong nonlinearities in the equations (the technical term is *genuine nonlinearity* in the sense of Peter Lax) produce steep gradients, and dissipation acts most strongly on steep gradients, so the fluctuations around the gradients are dissipated away. This cannot be stopped even if you neglect dissipation mechanisms in the equations (which we do during the radiation phase) because nonlinearities steepen the gradients until shock waves form, and shock wave dissipation models the massive dissipating effect of the steepening. Only when you are in a self-similar solution is there a perfect balance so that this steepening effect is neutralized. Self-similar solutions are very special. [When I refer to "self-similar solutions", I include all time-asymptotic expansion waves that can be rescaled into one of these by the coordinate freedom of the Einstein equations.] The result is that dissipation due to nonlinearities should drive a general solution with fluctuations, back to a self-similar solution. Think of a nuclear explosion decaying in time to a simple spherical expansion wave behind a self-similar shock wave. This point of view is well established and not at all controversial any more, among the community of mathematicians working on shock wave theory.

Conclude: if the Einstein equations are correct, we should see these waves by the end of the radiation epoch.

Thus, when we find the first self-similar solutions that perturb the standard model during the radiation phase, we have found something important. Mathematics tells us they should be what is left after the dissipation is done. If they are not there in cosmology, they are still important, because this puts strong constraints on whatever underlying physical mechanism has ruled them out. (E.g., the $k \neq 0$ Friedmann spacetimes have not been observed in cosmology, but they are still fundamental to the physics. Indeed, one motivation for the theory of inflation was to explain why $k \neq 0$ Friedmann got ruled out. If you only knew the $k = 0$ solution existed, you couldn't even raise this issue.) And the Friedmann spacetimes are often taken as the local asymptotic model for early galaxy formation from local fluctuations on that scale, so why not these new self-similar waves? Thus I believe the mathematical interest of these waves transcends any one physical application. Because of this, together with the fact that the $k = 0$ Friedmann spacetime of the radiation phase is now *characterized* as the unique point of intersection of the one parameter family of Friedmann spacetimes and this new one-parameter family of self-similar spacetimes, I believe these new waves are of fundamental importance to cosmology.

Now it would be remarkable just to construct any new self-similar solution of the Einstein equations during the radiation phase because these are *real* things, they are what's left after everything settles down. But we have also shown that these new solutions have the additional remarkable property that they can approximate the standard model arbitrarily well, the standard model $k = 0$

[2] These nonlinearities are *not* in place after the pressure drops to zero, so self-similar solutions do not represent time asymptotic wave forms after the radiation phase has passed.

Friedmann universe is one of them, and moreover, every member of the family looks just like the $k = 0$ Friedmann universe until very far from the center, where the first divergence to appear is in redshift vs luminosity. Finally, we make contact with physics by showing that, since there is a one parameter family of these solutions, they could account for the second order correction to redshift vs luminosity observed at a later time in the supernova data, and the third order correction would be a prediction. That is, something *real* could account for the anomalous acceleration of the galaxies. Since our formulas only apply during the radiation phase and the solutions lose the property of self-similarity after the pressure drops to zero, numerics are required to get further information. That is our work in progress.

To respond then, my view is that the other criticisms, (eg in http://blogs.discovermagazine .com/cosmicvariance/2009/08/28/dark-energy-still-a-puzzle/) are off target: we are not fitting a solution of the Einstein equations to the data, like finding solutions of the constraint equations that fit the present expansion initially and evolving them backward to make a cosmological model. That is closer to the local under-density models of Kob, Marra, Matarrese, Clifton, Ferreira and Land, [referenced in the articles]. What we have done is to show that the discovery of a *very rare thing*, a family of *real* self-similar time asymptotic waves at an early time, could account for the formation of a local under-density at present time, and thereby account for the leading order correction to red-shift vs luminosity observed in the supernova data. Our theory supplies a mechanism and a prediction, indicating a new test for the Dark Energy theory. We are currently working on getting numbers out for the prediction.

Note also that all of the implications of precision cosmology were already debated in those under-density papers. Thus, this work is interesting in its own right, and also addresses a very current issue of serious debate in physics. (The Clifton-Ferreira paper was the cover article of Scientific American in 2009, see attached).

Let me return now to the comment of Avi Loeb.

There are three purported contacts of Dark Energy with experiment: (1) the corrections to the Hubble Law, and (2,3) the connections between the length scale calculated from the theory of Baryonic Acoustic Oscillations at the end of the radiation phase, and the corresponding largest length scales observed in (2) the clusters of galaxies and (3) microwave background radiation. (That is plain English for what he said.) Connecting with the other points is part of our program, and requires numerics. Note also that he did not dismiss our work, he concluded that "Until they do so, it is not clear why this alternative model should be regarded as advantageous." We agree! An important mathematical discovery with a surprising implication for cosmology, maybe, but certainly not advantageous *yet*. But at this stage, I believe what we have makes for the most plausible physical mechanism for the anomalous acceleration based on classical general relativity without the cosmological constant.

And regarding Laub's comment about the number of free parameters, keep in mind that the parameter in our family of self-similar waves is not to be treated on the level of the cosmological constant. Indeed, exactly which asymptotic wave in the family would emerge from a given local fluctuation would be highly dependent on the details of the fluctuations, (the shock wave emerging from a nuclear explosion depends on the energy of the explosion)—the fact that any of the waves could emerge and one of them did, is not to be compared with an adjustable constant added to the equations to fit the data.

(Aside: The cosmological constant adds one free parameter to the equations. One free parameter plus three experiments equals two points of contact with the data. If these latter two are correlated, then maybe only one. We have one point of contact with the data without adding an adjustable parameter to the equations. So by this count the score would be tied!)

And we agree, the wave model is "finely tuned". According to Clifton-Ferreira, Earth would have to lie within an under-density that extends out to about a quarter of the way across the visible universe, and the size of the "center" about the distance between clusters of galaxies.[3] But the theory of Dark Energy based on the cosmological constant is just as "finely tuned". You can't rule out the wave theory by reasons that apply equally to the Dark Energy theory.[4]

My Conclusion: We should not sell this mathematics short. It represents fundamental new information about the standard model.

[3] This is fully consistent with the time over which the dissipation acts during the radiation phase of the expansion.

[4] We would have to live in a wink of time when the energy density of matter, which decreases with the expansion, is on the order of the cosmological constant, which is constant. And the magnitude of Dark Energy is 50 orders of magnitude off from what *string theory* might predict. Dark Energy is controversial.