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Shock Wave Interactions in General Relativity

A Locally Inertial Glimm Scheme for Spherically Symmetric Spacetimes

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Shock Wave Interactions in General Relativity: A Locally Inertial Glimm Scheme for Spherically Symmetric Spacetimes

This monograph is a self contained mathematical treatment of the initial value problem for shock wave solutions of the Einstein equations in General Relativity. The first two chapters provide background for the introduction of a locally inertial Glimm Scheme, a non-dissipative numerical scheme for approximating shock wave solutions of the Einstein equations in spherically symmetric spacetimes. What follows is a careful analysis of this scheme providing a proof of the existence of (shock wave) solutions of the spherically symmetric Einstein equations for a perfect fluid, starting from initial density and velocity profiles that are only locally of bounded total variation. In particular, the result establishes the consistency of the Einstein equations at the level of shock waves in a class of curved spacetimes rich enough to incorporate the interaction of arbitrary numbers of shock waves of arbitrary strength. The book covers the initial value problem for Einstein's gravitational field equations with fluid sources and shock wave initial data. It has a clearly outlined goal: proving a certain local existence theorem. Concluding remarks are added and commentary is provided throughout. The book will be useful to graduate students and researchers in mathematics and physics.

Blake Temple, who organized these notes from research articles with the other two authors, is currently a Professor of Mathematics, and Chair of the Graduate Group in Applied Mathematics, at the University of California, Davis. Joel Smoller teaches Mathematics at the University of Michigan, and Jeffrey Groah is at the Department of Mathematics, Montgomery College, Conroe, Texas.

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1 Introduction

These notes present a self contained mathematical treatment of the initial value problem for shock wave solutions of the Einstein equations in General Relativity. The first two chapters provide background for the introduction of a locally inertial Glimm Scheme in Chapter 3, a non-dissipative numerical scheme for approximating shock wave solutions of the Einstein equations in spherically symmetric spacetimes. In Chapter 4 a careful analysis of this scheme provides a proof of the existence of (shock wave) solutions of the *spherically symmetric* Einstein equations for a perfect fluid, starting from initial density and velocity profiles that are only locally of bounded total variation. To keep the analysis as simple as possible, we assume throughout that the equation of state is of the form $p = \sigma^2\rho$, $\sigma = const.$ For these solutions, the components of the gravitational metric tensor are only *Lipschitz continuous* functions of the spacetime coordinates at shock waves, and so it follows that these solutions satisfy the Einstein equations, as well as the relativistic compressible Euler equations, only in the weak sense of the theory of distributions. The existence theory presented here establishes the consistency of the initial value problem for the Einstein equations at the weaker level of shock waves, for spherically symmetry spacetimes.

The material of Chapter 4 is taken from the work of Groah and Temple [13], and relies on the results of Chapters 2 and 3. The material of Chapter 3 is taken from the work of Groah and Temple, [12], and Chapter 2 is taken from the work of Smoller and Temple [27]. The introductory material in Sections 1.1 and 1.2 of Chapter 1 is taken mostly from [30, 31], while the material in Sections 1.3 and 1.4 is from [12, 13].