Applied Mathematics

vs

Dark Energy

Blake Temple, UC-Davis

National Meeting of the Brazilian Math. Society
Penary Address
July 26, 2015

Collaborators: Joel Smoller and Zeke Vogler

GR Simple-Waves Trigger An Instability in SM which creates the Anomalous Acceleration without Dark Energy

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The 1999 observations of redshift vs luminosity for type 1A supernovae in nearby galaxies won the Nobel Prize because they discovered the

Anomalous Acceleration:

The universe is expanding faster than the Standard Model of Cosmology (SM), based on Einstein's original theory of General Relativity, allows.

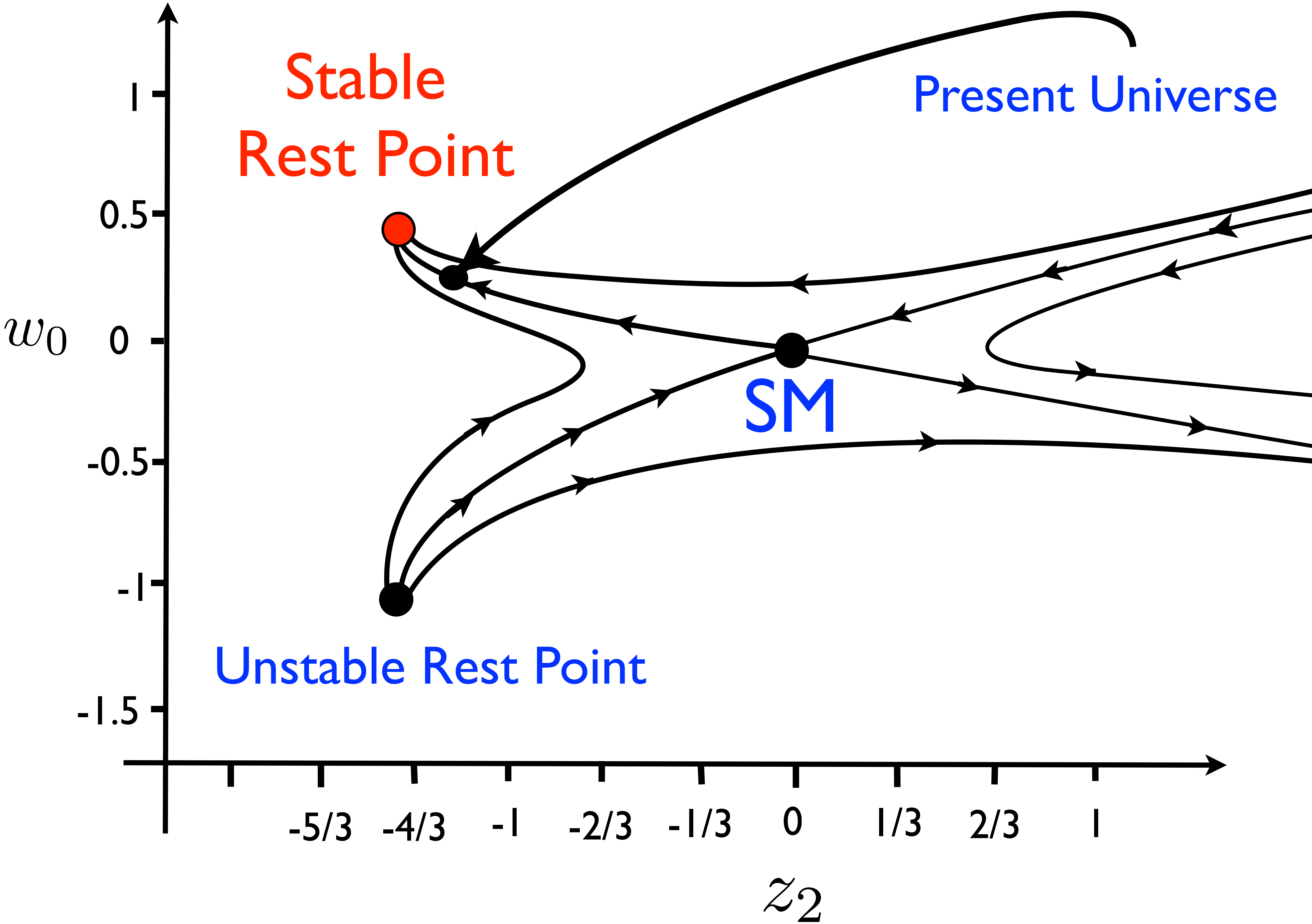
The only way to preserve the
Cosmological Principle-
that on the largest length scale the
universe is described by a
Friedmann Space-Time
which holds no special place-
is to add the
Cosmological Constant
to Einstein's equations as a source term.
Its interpretation is

Dark Energy.

A best fit among
Friedmann Space-Times
with
Dark Energy
leads to the conclusion that
the universe is a critical
k=0 Friedmann Space-Time
with
Seventy Percent Dark Energy

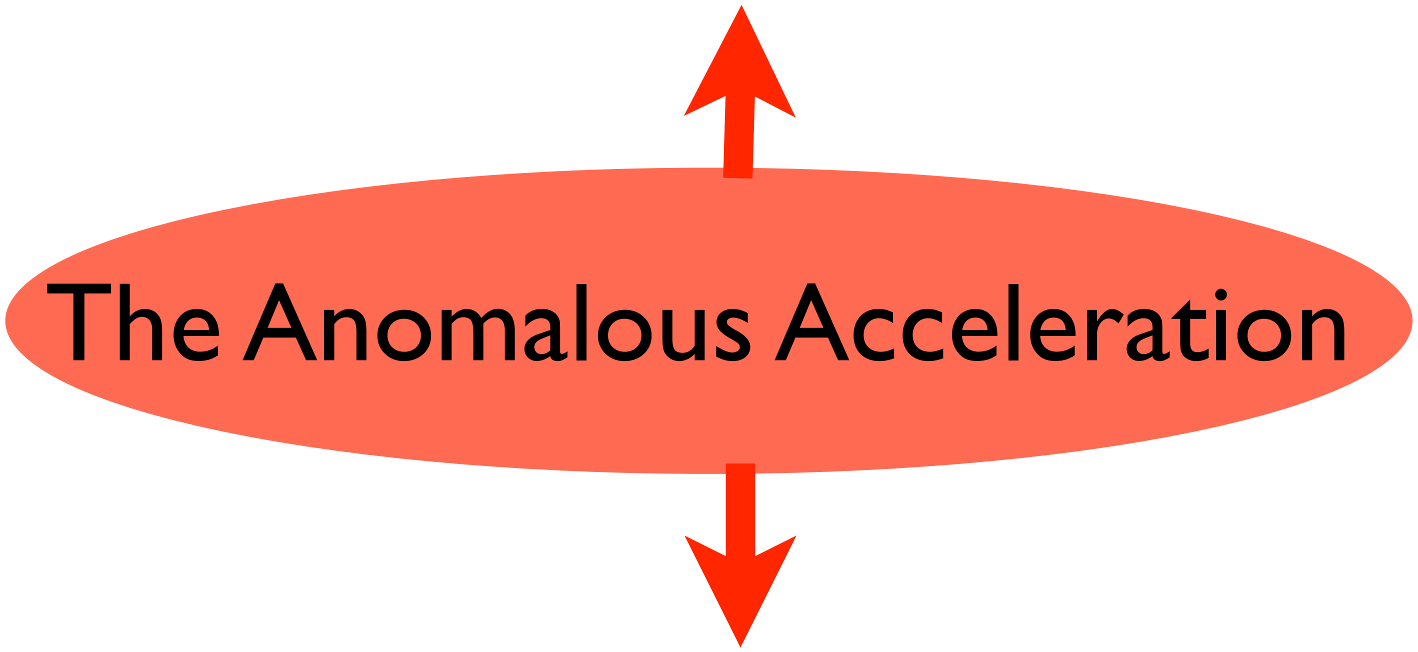
$$\Omega_\Lambda \approx .7$$

Our Wave Model:


Stable Rest Point
Present Universe
SM
Unstable Rest Point
w_0
1
0.5
0
-0.5
-1
-1.5
-5/3
-4/3
-1
-2/3
-1/3
0
1/3
2/3
1
z_2

$$H_0 d_\ell = z + .25\, z^2 - .125\, z^3$$

Standard Model



The Anomalous Acceleration

$$H_0 d_\ell = z + .425\, z^2 - .1804\, z^3$$

Dark Energy

$$H_0 d_\ell = z + .425 z^2 + .3591 z^3$$

Wave Model

2007 PI talk in Relativity Session at AMS National Meeting in New Orleans:

We proposed the idea that a
Simple Wave
from the
Radiation Epoch of the Big Bang
might account for the Anomalous
Acceleration of the Galaxies
Without Dark Energy

Our Motivation

The Radiation Epoch:

After Inflation

until about

30,000 years after the Big Bang

is evolution by

Relativistic Compressible Euler Equations

The p-system with $\boxed{p = \frac{c^2}{3}\rho}$

PURE RADIATION

Stefan-Bolzman Law: $$\rho = \mathrm{a}\, T^4$$

$$p = \frac{c^2}{3}\rho$$

(No Contact Discontinuities)

The p-system with:

- Enormous sound speed $\sigma \approx .57c$
- Enormous modulus of Genuine Nonlinearity

Every characteristic field contributes to Decay in the sense of Glimm and Lax

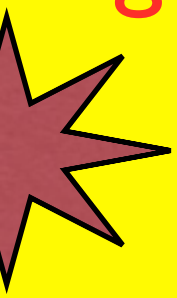
It is reasonable to expect
fluctuations
would decay to
simple wave patterns
by the
End of Radiation

This is our Starting Assumption

Stages of the Standard Model:

Inflation

Big



Bang

$10^{-35}s$
to
$10^{-30}s$

Pure Radiation

10^{-30} to 3×10^{5} yrs

$$p = \frac{c^2}{3}\rho$$

(Relativistic p-system)

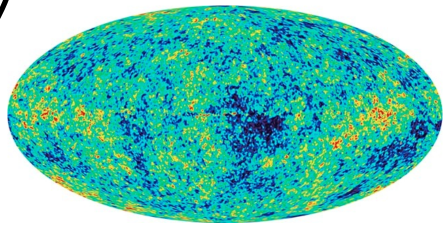
Uncoupling of
Matter and Radiation

$t \approx 3 \times 10^5$

(Neglect
Radiation
Pressure)

$p \approx 0$

Time of CMB
379,000 yr



It is reasonable to expect
near the center of a time asymptotic perturbation the simple wave should solve an ODE
by the
End of Radiation

This is our Starting Assumption

Pursuing this Idea…

…we discovered that there is only
ONE WAY
the Einstein equations can both
perturb the Friedmann spacetimes
and also
reduce to ODE's…

Pursuing this Idea…

…we identified a
1-parameter family of Self-Similar Waves
that perturb the Standard Model
during the Radiation Epoch-
And proposed that these might induce an
Anomalous Acceleration at a later time.

We set out our ideas in
PNAS in 2009
and
Memoirs of the AMS in 2011

Our interest is in the possible connection between these waves and the Anomalous Acceleration.

In Fact: This family of self-similar solutions was already known to exist

Cahill and Taub:

Commun Math Phys., 21, 1-40 (1971)

Extended by others, esp. Carr and Coley, Survey:

Physical Review D, 62, 044023-1-25 (1999)

Around 2007:

Other research groups began exploring the possibility that the anomalous acceleration might be due to the earth lying near the center of a large region of Under-Density

We first saw publication in 2009

The record is clear on one thing:

No one before us
proposed this family of waves
as a
mechanism
that could account for the
Anomalous Acceleration
without Dark Energy

We have now accomplished our goal of bringing the effects of these waves up to present time to compare with Dark Energy.

There are several surprises… in this talk I present what we have found…

Bullet Points to Discuss:

- We identify an instability in the SM based on a new (closed) asymptotic ansatz for local perturbations of the critical k=0 Friedmann Spacetime when p=0.
- The instability naturally creates a region of accelerated uniform expansion on the scale of the supernova data within Einstein's original theory, without Dark Energy.
- The region is one order of magnitude larger in extent than expected.

Bullet Points to Discuss:

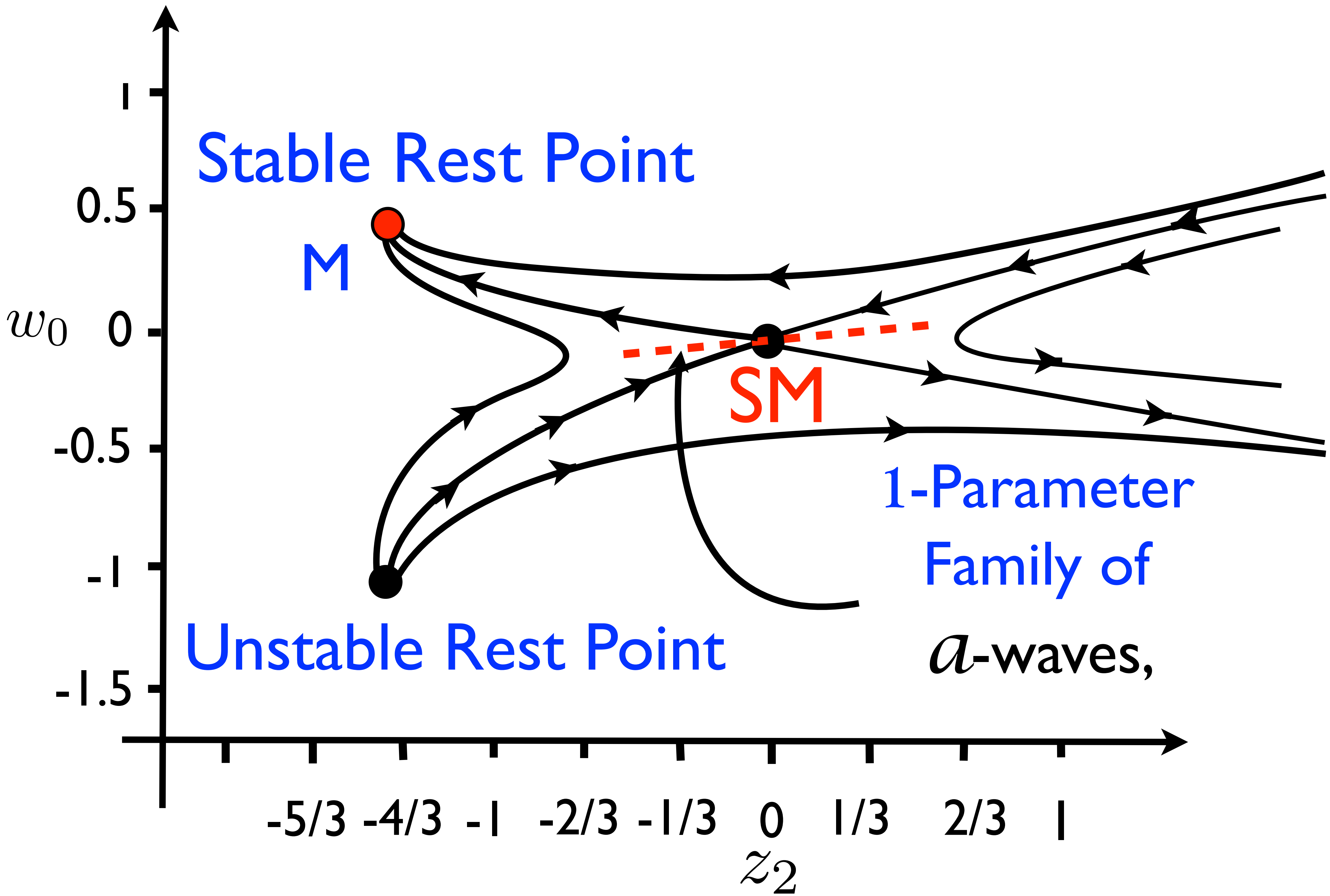
- The instability is triggered by our time asymptotic perturbations of SM from the

  Radiation Epoch when: $p = \frac{c^2}{3}\rho$

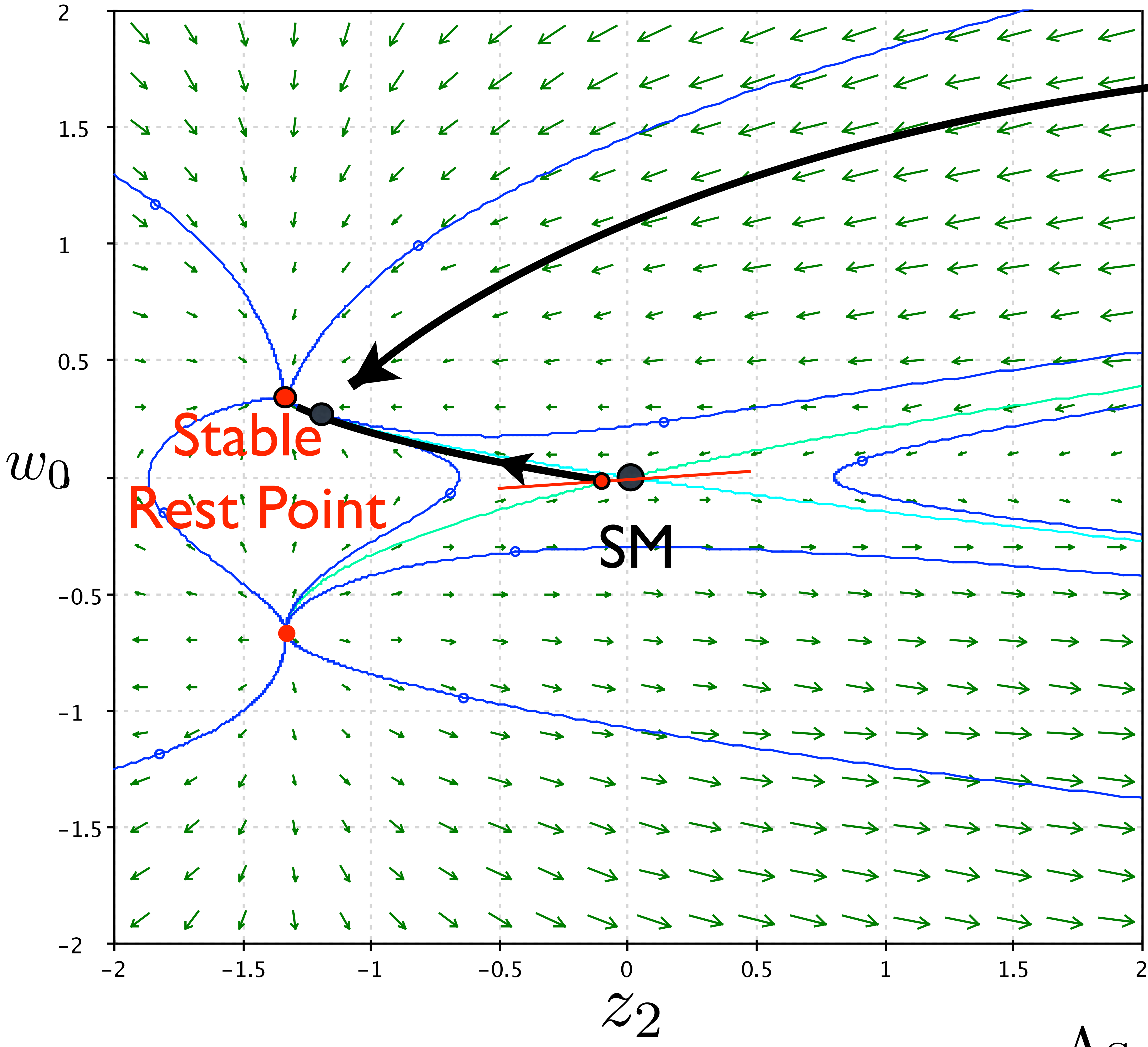
- Surprisingly—The perturbations at the end of radiation do not directly cause the Anomalous Acceleration as we originally conjectured in PNAS.

- Rather—It is the non-trivial phase portrait of the instability they trigger when p=0 that that creates the later accelerations.

- A phase portrait of the instability places the SM at a classic… **Unstable Saddle Rest Point**



1-Parameter Family of a-waves,



Present Universe
in the
Wave Theory

- Same Hubble Constant

- Same .425 Acceleration

As Dark Energy

Bullet Points to Discuss:

- The region of accelerated uniform expansion introduces precisely the same range of quadratic corrections to red-shift vs luminosity as does the cosmological constant in the theory of DE.

$$H_0 d_\ell = z + Qz^2 + O(z^3)$$

$$.25 \leq Q \leq .425 \leq .5$$

Bullet Points to Discuss:

- The results lead naturally to a testable alternative to Dark Energy within Einstein's original theory…

  Without the Cosmological Constant.

- Our Proposal: The AA is due to a local under-dense perturbation of the SM on the scale of the supernova data, arising from time-asymptotic perturbations of SM from the Radiation Epoch that trigger an instability in the SM when the pressure drops to zero.

Bullet Points to Discuss:

- A calculation shows the cubic correction is of the same order, but of a different sign, than the cubic correction in DE theory…

$$H_0 d_\ell = z + .425\, z^2 - \boxed{.1804}\, z^3$$

Dark Energy

$$H_0 d_\ell = z + .425 z^2 + \boxed{.3591} z^3$$

Wave Theory

Bullet Points to Discuss:

- We address ONLY the anomalous acceleration… further assumptions regarding space-time far from the center would be required to connect the theory with other measurements…

INTRODUCTION
TO
COSMOLOGY

Edwin Hubble (1889-1953)

- Hubble’s Law (1929):

``The galaxies are receding from us at a velocity proportional to distance”

↓

Universe is Expanding

- Based on Redshift vs Luminosity

9 January 2014 Last updated at 02:16 ET

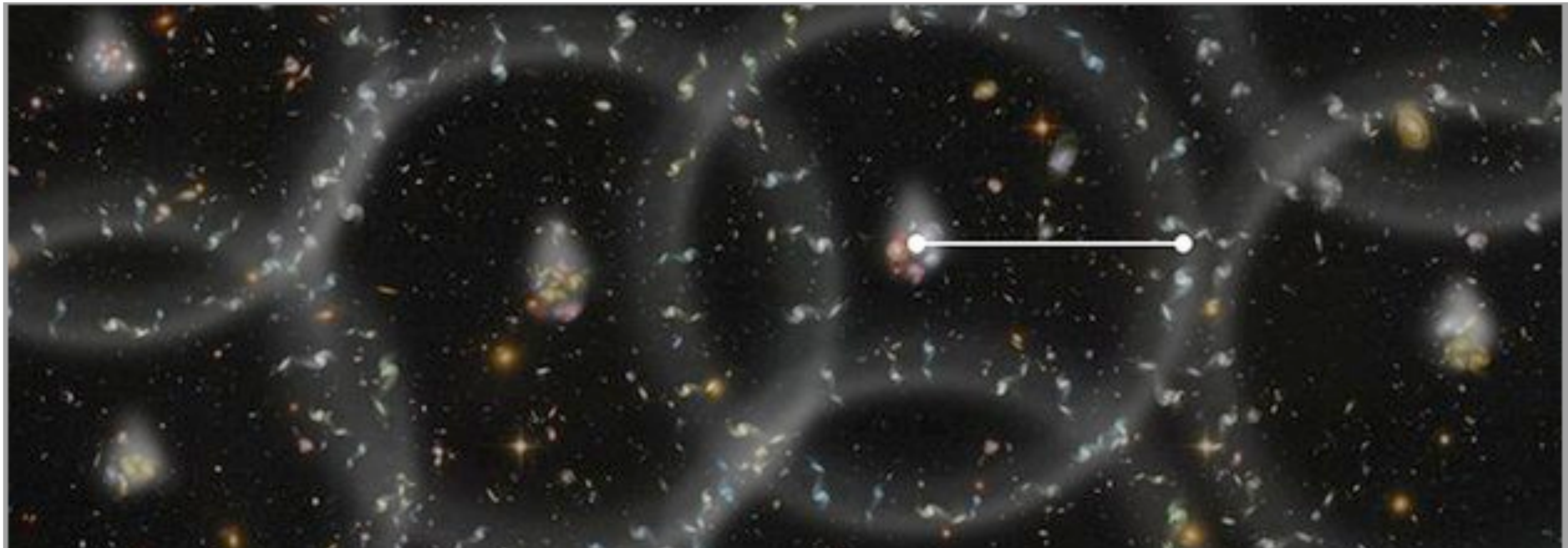
Universe measured to 1% accuracy

By James Morgan
Science reporter, BBC News, Washington DC

Astronomers have measured the distances between galaxies in the universe to an accuracy of just 1%.

This staggeringly precise survey - across six billion light-years - is key to mapping the cosmos and determining the nature of dark energy.

The new gold standard was set by BOSS (the Baryon Oscillation Spectroscopic Survey) using the Sloan Foundation Telescope in New Mexico, US.



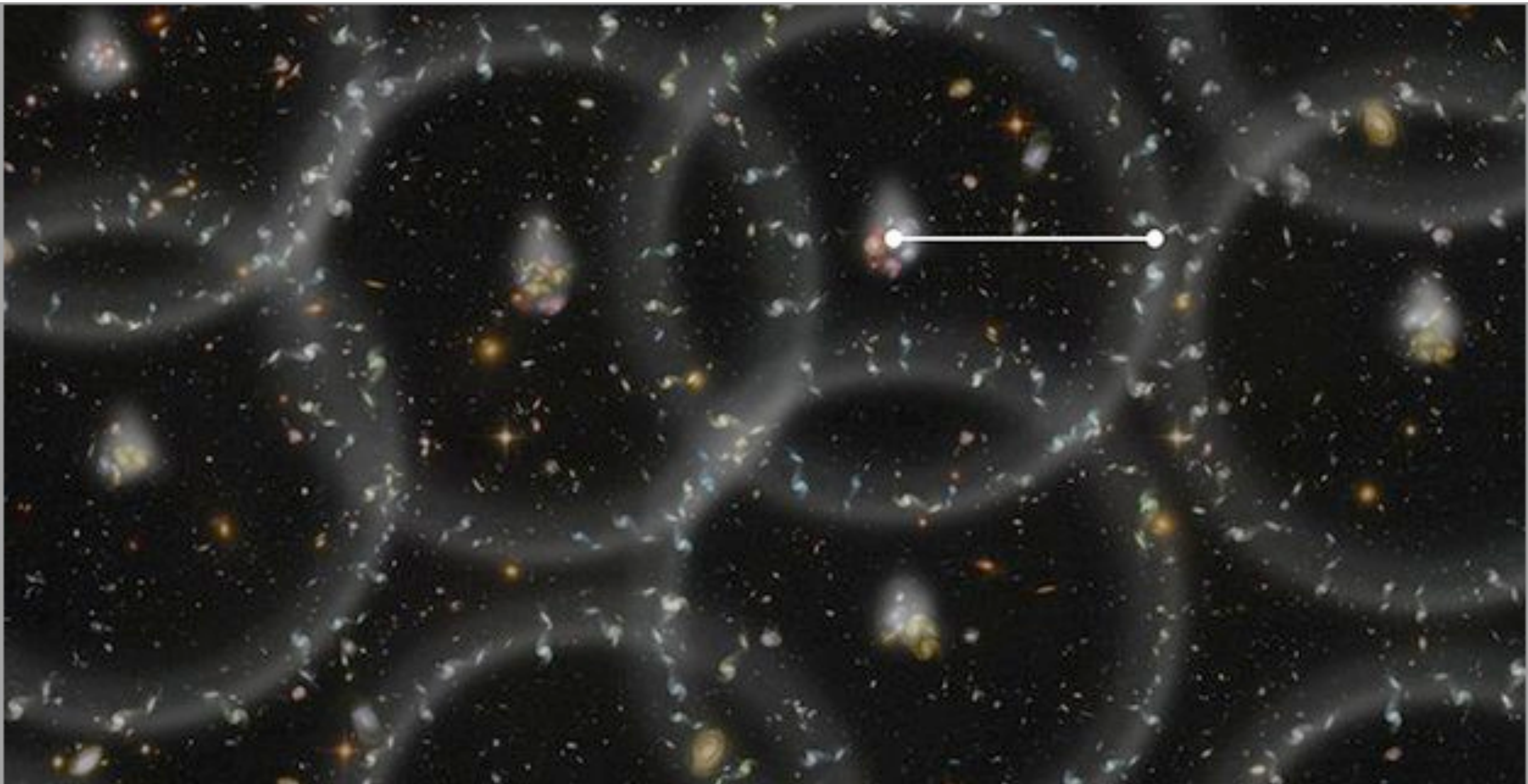
Frozen ripples

The BOSS team used baryon acoustic oscillations (BAOs) as a "standard ruler" to measure intergalactic distances.

BAOs are the "frozen" imprints of pressure waves that moved through the early universe - and help set the distribution of galaxies we see today.

"Nature has given us a beautiful ruler," said Ashley Ross, an astronomer from the University of Portsmouth.

"The ruler happens to be half a billion light years long, so we can use it to measure distances precisely, even from very far away."



Conclude: The universe appears (and is assumed) uniform on a scale of about 1/20th the distance across the visible universe

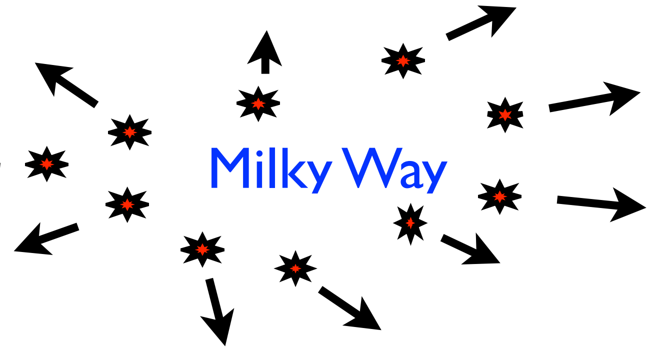
$$\xi = \frac{r}{ct} \approx \frac{.5\,\text{billion yr}}{13\,\text{billion yr}} \approx .4 \leq .5$$

Cosmic Length Scales

10 billion light-years ≈ Visible Universe

500 million light-years ≈ Uniform Density

- 50 million light-years ≈ Separation between clusters of galaxies



10 million light-years ≈ diameter of a cluster

- 1 million light-years ≈ separation between galaxies in a cluster

100 thousand light-years ≈ distance across Milky Way

- 28 thousand light-years ≈ distance to galactic center

Standard Model of Cosmology

- 1922 *Alexander Friedmann*:

  Derived FRW solutions of the Einstein equations: 3-space of constant curvature expanding in time:

$$ds^2 = -dt^2 + R(t)^2 \left\{ \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right\}$$

- The Big Bang theory based on the FRW metric was worked out by *George Lemaître* in the late 1920's leading to Hubble's comfirmation of redshift vs luminoscity consistent with an FRW spacetime

$$\boxed{\text{Hubble's Constant} \equiv H \equiv \frac{\dot{R}}{R}}$$

- In 1935: Howard Robertson and Arthur Walker derived Friedmann spacetime from the

Copernican Principle:
"Earth is not in a special place in the Universe"

- R-W: Friedmann uniquely determined by condition

Homogeneous and Isotropic about every point

↓

Any point can be taken as $r = 0$

↓

Each t=const surface is a 3-space of constant scalar curvature

Standard Model of Cosmology

Observations of the
micro-wave background
IMPLY

$$k = 0$$

“Critical expansion to within about 2-percent”

The Friedmann metric when k=0:

- $$ds^2 = -dt^2 + R(t)^2 \left\{ dr^2 + r^2 d\Omega^2 \right\}$$

The universe is infinite flat space $\mathbb{R}^3$ at each fixed time:

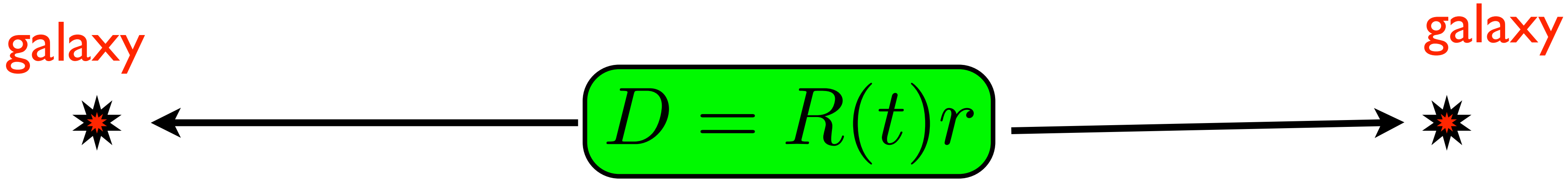
(Assumed to Apply on the Largest Length Scale)

Standard Model of Cosmology

- FRW metric, *k=0:*

$$ds^2 = -dt^2 + R(t)^2 \left\{dr^2 + r^2 d\Omega^2\right\}$$

- $\boxed{D = Rr}$ Measures distance between galaxies at each fixed t



- Conclude:

$$\dot{D} = \dot{R}r = \frac{\dot{R}}{R}Rr = HD$$

$$\boxed{\dot{D} = HD}$$

Hubble's Law

$$\boxed{\text{Hubble's Constant} \equiv H \equiv \frac{\dot{R}}{R}}$$

- Standard Model of Cosmology

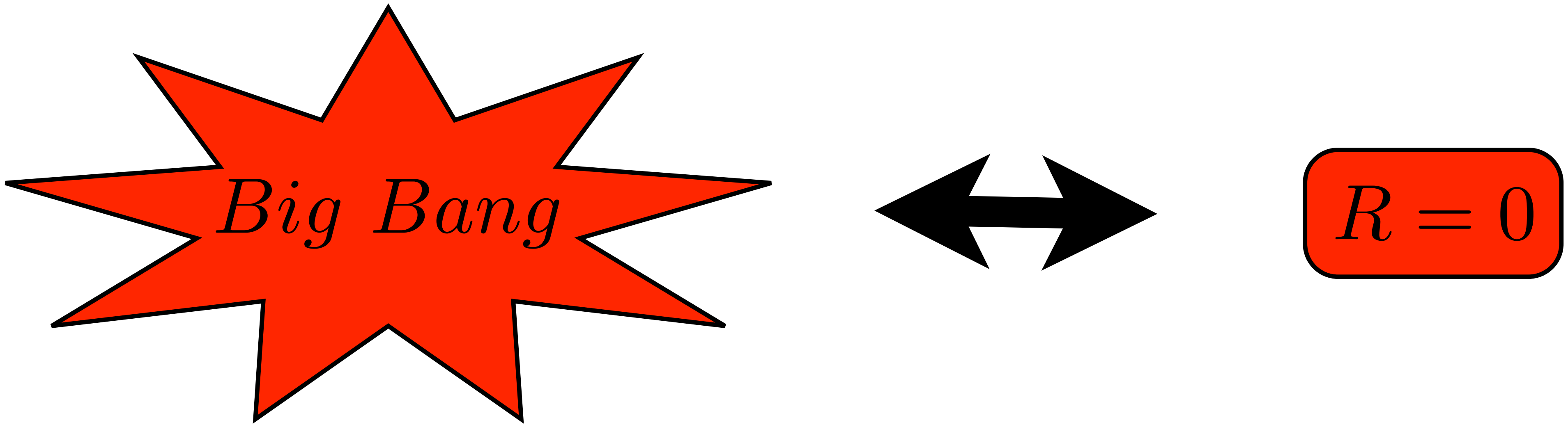
$$ds^2 = -dt^2 + R(t)^2 \left\{ dr^2 + r^2 d\Omega^2 \right\}$$

- Hubble's Law:

$$\dot{D} = HD$$

- Conclude--

``The universe is expanding like a balloon”



$Big\ Bang$

$R = 0$

The Hubble "Constant" at present time

- The inverse Hubble Constant estimates the Age of the Universe

$$\frac{1}{H_0} \approx 10^{10} \text{ years} \approx \text{age of universe}$$

- $\frac{c}{H_0}$ is the distance of light travel since the Big Bang, a measure of the size of the visible universe

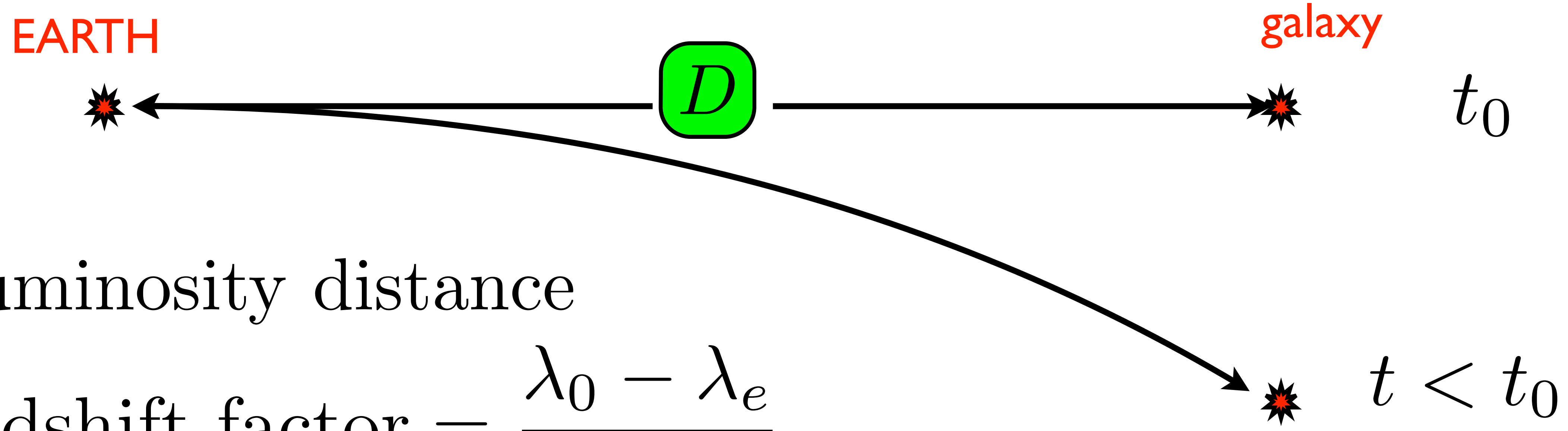
$$\frac{c}{H_0} = \text{Hubble Length} \approx 10^{10} \; lightyears$$

Measuring the Hubble Constant

$\boxed{D}$ Measures distance from Earth to distant galaxy at present time t_0

$$\boxed{H_0 D = \dot{D}}$$

Hubble's Law



$$D \approx d_\ell \equiv \text{luminosity distance}$$

$$\dot{D} \approx z \equiv \text{redshift factor} = \frac{\lambda_0 - \lambda_e}{\lambda_e}$$

$$\boxed{H_0 d_\ell = z + \frac{1}{4} z^2 - \frac{1}{8} z^3 + O(z^4)}$$

Friedmann

$k = 0$

Up until 1999, we could only measure the leading linear term:

$$H_0 d_\ell = z + \xcancel{\frac{1}{4}z^2} - \xcancel{\frac{1}{8}z^3} + \xcancel{O(z^4)}$$

Friedmann

$k = 0$

$z << 1$

$$H_0 \approx h_0\, 100 \frac{km}{s\, mpc} \qquad h_0 \approx .68$$

$$mpc \approx 3.2 \text{ million light years}$$

``A galaxy at a distance of one mega-parsec is receding at about 68 kilometers per second…”

The 1999 supernova data tested the dependence of the Hubble constant on time, and the results don’t fit standard model...

↓

“Anomalous Acceleration of Galaxies”

↓

Introduction of
“Cosmological Const” and “Dark Energy”

Dark energy is non-classical
Negative pressure → Anti-gravity effect

Recent supernova data have tested the dependence of the Hubble constant on time, and the results don't fit standard model...

$$\boxed{H_0 d_\ell = z + \frac{1}{4}z^2 - \xcancel{\frac{1}{8}z^3} + \xcancel{O(z^4)}}$$

Friedmann

$k = 0$

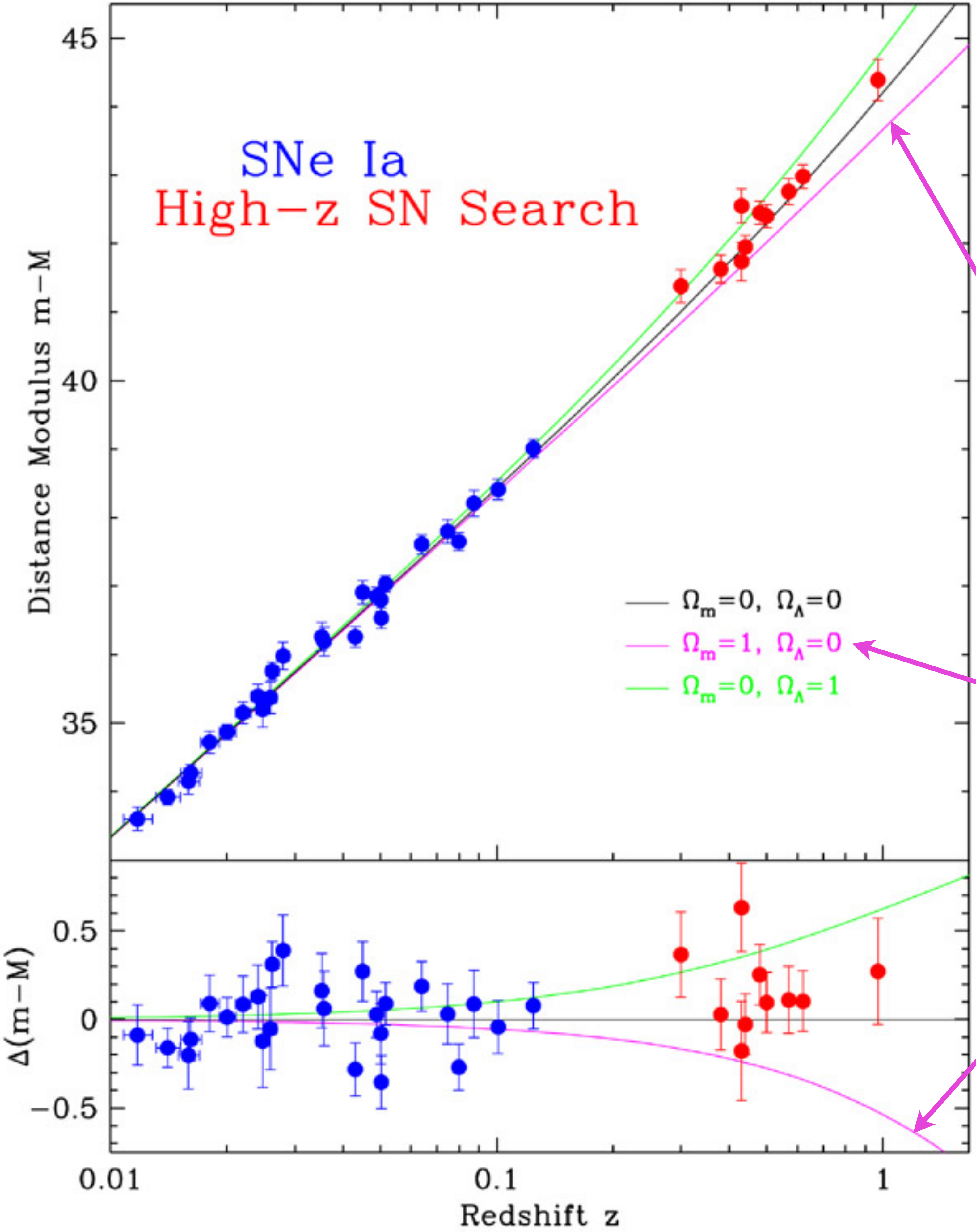
This is measured at about .425 not .25

Recent supernova data have tested the dependence of the Hubble constant on time, and the results don’t fit standard model...

This is usually interpreted in terms of a Best Fit to Friedmann Universes with the Cosmological Constant

$$(k, \Omega_\Lambda) \quad \rightarrow \quad k = 0,\ \ \Omega_\Lambda \approx .7$$

Supernova Data



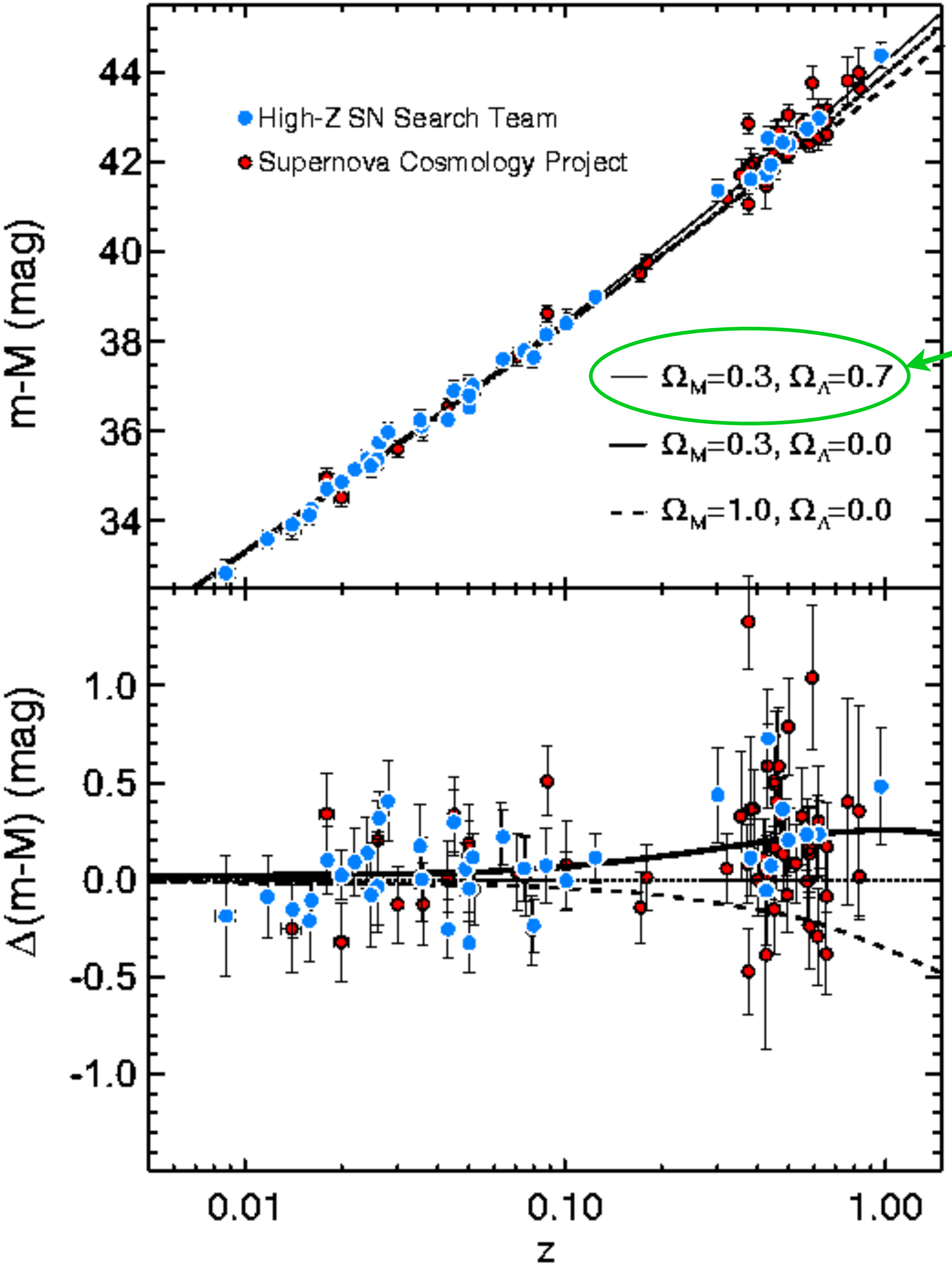
Standard Model
k=0 FRW

“Not a Good Fit”

Thanks to Philip Hughs
UM-Astronomy

That is: To preserve the
Copernican Principle,
that the Universe
on the Largest Length Scale
is evolving according to a
Uniform Friedmann Spacetime
with p=0, k=0
A Cosmological Constant
must be added
To Einstein's Equations

The Physical Interpretation is Dark Energy



✶ Best Fit: ———

70% Dark Energy

30% Classical Energy

Thanks to Philip Hughs
UM-Astronomy

Einstein Equations for Friedmann:

- Einstein Equations **(1915):** $G_{ij} = \kappa T_{ij}$

G_{ij}=Einstein Curvature Tensor

$T_{ij} = (\rho + p)u_i u_j + p g_{ij}$=Stress Energy Tensor (perfect fluid)

- Einstein Equations for k=0 Friedmann metric:

$$H^2 = \frac{\kappa}{3}\rho$$

$$\dot{\rho} = -3(\rho + p)H$$

✸ Solutions determined by equation of state: $p = p(\rho)$

Incorporating Dark Energy into Friedmann

- Assume Einstein equations with a cosmological constant:

$$\boxed{G_{ij} = 8\pi T_{ij} + \Lambda g_{ij}}$$

- Assume $k = 0$ FRW: $\boxed{ds^2 = -dt^2 + R(t)^2 \left\{dr^2 + r^2 d\Omega^2\right\}}$

- Leads to: $\boxed{H^2 = \frac{\kappa}{3}\rho + \frac{\kappa}{3}\Lambda}$

- Divide by $H^2 = \frac{\kappa}{3}\rho_{crit}$ $\boxed{1 = \Omega_M + \Omega_\Lambda}$

- Best data fit leads to $\boxed{\Omega_\Lambda \approx .7}$ and $\boxed{\Omega_M \approx .3}$

- Implies: The universe is 70 percent dark energy

Incorporating Dark Energy into Friedmann

More slowly…

Incorporating Dark Energy into Friedmann

$$H^2 = \frac{\kappa}{3}\rho + \frac{\kappa}{3}\Lambda$$

Incorporating Dark Energy into Friedmann

$$H^2 = \frac{\kappa}{3}\rho + \enclose{circle}{\frac{\kappa}{3}\Lambda}$$

Constant
in
time

Incorporating Dark Energy into Friedmann

$$H^2 = \frac{\kappa}{3}\rho + \frac{\kappa}{3}\Lambda$$

Decreases to zero as $t \to \infty$

Incorporating Dark Energy into Friedmann

$$\frac{H^2 = \frac{\kappa}{3}\rho + \frac{\kappa}{3}\Lambda}{H^2}$$

Incorporating Dark Energy into Friedmann

$$1 = \frac{\frac{\kappa}{3}\rho}{H^2} + \frac{\frac{\kappa}{3}\Lambda}{H^2}$$

Incorporating Dark Energy into Friedmann

$$1 = \frac{\frac{\kappa}{3}\rho}{H^2} + \underbrace{\frac{\frac{\kappa}{3}\Lambda}{H^2}}_{\Omega_\Lambda}$$

Incorporating Dark Energy into Friedmann

$$1 = \boxed{\frac{\frac{\kappa}{3}\rho}{H^2}} + \frac{\frac{\kappa}{3}\Lambda}{H^2}$$

$$\Omega_M$$

Incorporating Dark Energy into Friedmann

$$1 = \frac{\frac{\kappa}{3}\rho}{H^2} + \frac{\frac{\kappa}{3}\Lambda}{H^2}$$

$$1 = \Omega_M + \Omega_\Lambda$$

Incorporating Dark Energy into Friedmann

$$1 = \frac{\frac{\kappa}{3}\rho}{H^2} + \frac{\frac{\kappa}{3}\Lambda}{H^2}$$

$$1 = \Omega_M + \Omega_\Lambda$$

Conclude...

Incorporating Dark Energy into Friedmann

$$1 = \frac{\frac{\kappa}{3}\rho}{H^2} + \frac{\frac{\kappa}{3}\Lambda}{H^2}$$

$$1 = \Omega_M + \Omega_\Lambda$$

$\Omega_\Lambda \approx 0 \to 1$ as $t \approx t_{rad} \to \infty$

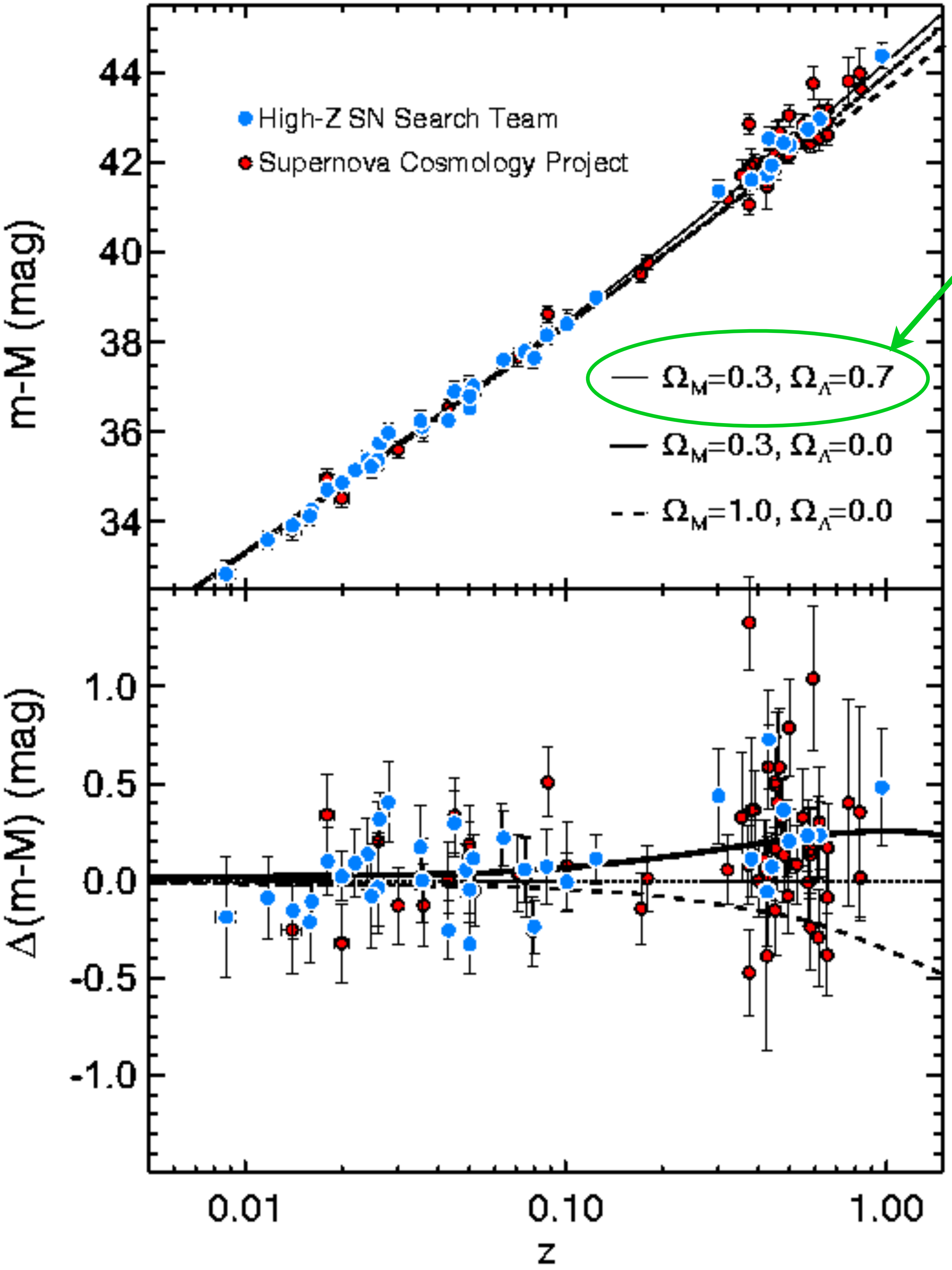
Incorporating Dark Energy into Friedmann

$$1 = \frac{\frac{\kappa}{3}\rho}{H^2} + \frac{\frac{\kappa}{3}\Lambda}{H^2}$$

$$1 = \Omega_M + \Omega_\Lambda$$

Best Fit...

$\Omega_\Lambda \approx .7$



✶ Best Fit:

70% Dark Energy

30% Classical Energy

- m - M = "Distance Modulus"

  M=absolute Magnitude

  m=apparent magnitude

- d=distance in parsecs:

  m - M = 5 log(d) - 5

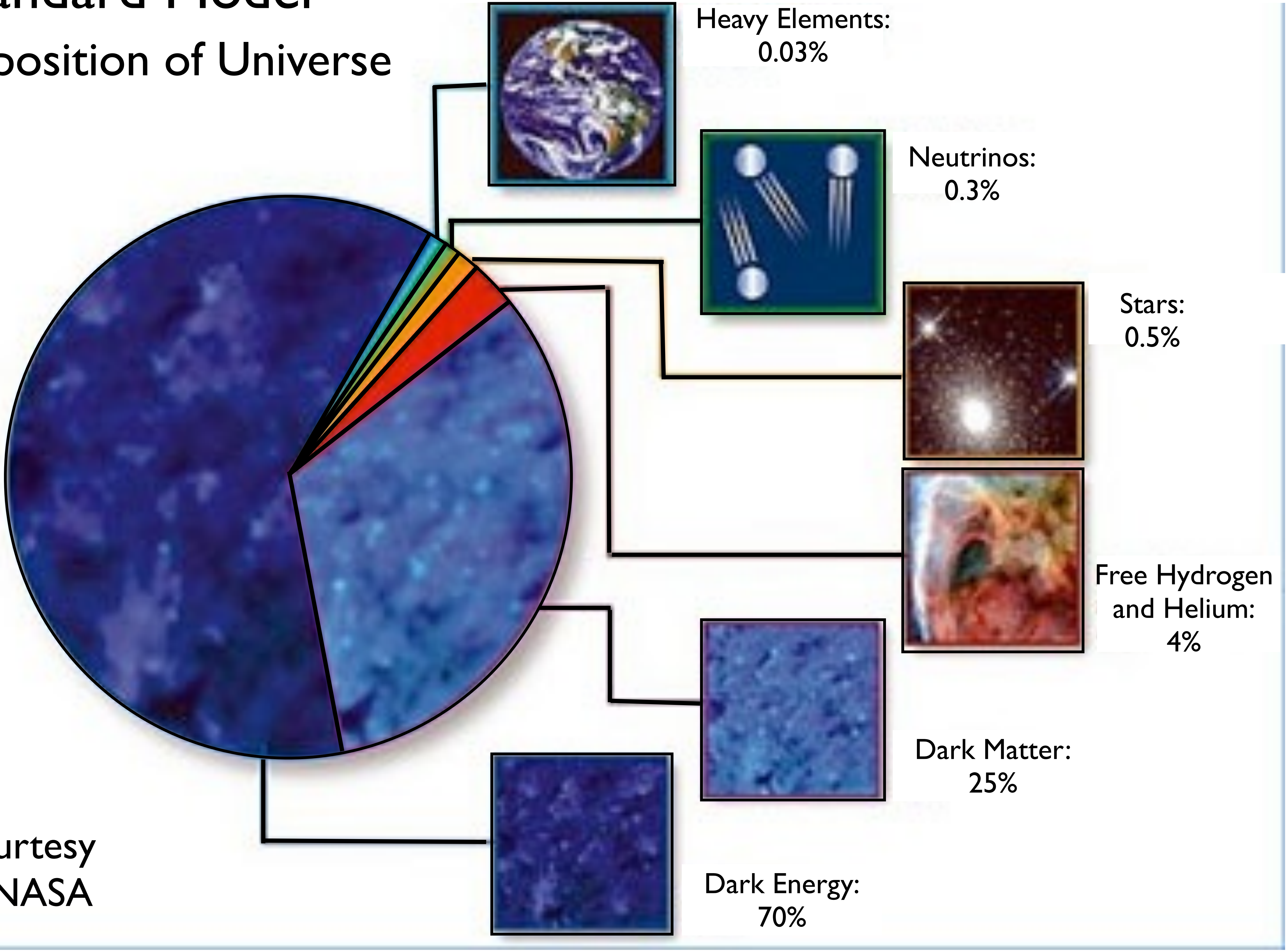
- z=redshift factor

  $1+z = \frac{\lambda_{emit}}{\lambda_{obs}}$

- $\Omega_m + \Omega_\Lambda = 1$ for a flat $(k = 0)$ universe.

Standard Model

Composition of Universe



Heavy Elements:
0.03%

Neutrinos:
0.3%

Stars:
0.5%

Free Hydrogen
and Helium:
4%

Dark Matter:
25%

Dark Energy:
70%

Courtesy
of NASA

The Question we Explore:

"Could the Anomalous Acceleration of the galaxies be due to the fact that we are looking outward into an expansion wave that formed during the Radiation Epoch of the Big Bang?"

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“Could the Anomalous Acceleration of the galaxies be due to the fact that we are looking outward into an expansion wave that formed during the Radiation Epoch of the Big Bang?”

- The Einstein equations have been confirmed without the cosmological constant in every setting except cosmology...

The Question we Explore:

"Could the Anomalous Acceleration of the galaxies be due to the fact that we are looking outward into an expansion wave that formed during the Radiation Epoch of the Big Bang?"

- ✸ The Einstein equations have been confirmed without the cosmological constant in every setting except cosmology...

Note: A general expansion wave has a center of expansion...

Summary of our results for the Wave Theory

Hubbles Law :

$$H_0 \, d_\ell = z$$

(1929)

Hubble's Constant

Luminosity Distance

Redshift Factor

Measured value:

$$H_0 = h_0 \, \frac{100 km}{s \, mpc}$$

$$h_0 \approx .68$$

The 1999 Supernova data was refined enough to measure the quadratic correction to Hubble's Relation:

$$H_0 d_\ell = z + \underset{\uparrow \; Q}{\boxed{??}} \, z^2$$

Einstein's Equations: $G = \kappa T + \Lambda g$

$$\Omega_M + \Omega_\Lambda = 1$$

Cosmological
Constant 1999

$$H_0 d_\ell = z + .25 z^2 + O(z^3)$$

Friedmann
$\Omega_\Lambda = 0$

Anomalous
Acceleration

$$H_0 d_\ell = z + .425 z^2 + O(z^3)$$

Friedmann
$\Omega_\Lambda = .7$

WE PROVE: The Friedmann Universe is UNSTABLE

A small wave perturbation at the end of radiation will expand to create a large region of accelerated uniform expansion at the Center of the Wave

WE PROVE: The Friedmann Universe is UNSTABLE

A small wave perturbation at the end of radiation will expand to create a large region of accelerated uniform expansion at the Center of the Wave

This induces exactly the same range of quadratic corrections to redshift vs luminosity as does Dark Energy

WE PROVE: The Friedmann Universe is UNSTABLE

The self-similar perturbations we identified
at the end of the radiation epoch
TRIGGER
this instability when p=0

This induces exactly the same range of quadratic corrections to redshift vs luminosity as does
Dark Energy

WE PROVE: The Friedmann Universe is UNSTABLE

The self-similar perturbations we identified at the end of the radiation epoch
TRIGGER
this instability when p=0

This induces exactly the same range of Q as does Dark Energy:

$$H_0 d_\ell = z + Qz^2 + O(z^3)$$

Dark Energy

$$H_0 d_\ell = z + \underbrace{.25\,(1+\Omega_\Lambda)}_{}\, z^2 - .125 \left(1 + \frac{2}{3}\Omega_\Lambda - \Omega_\Lambda^2\right) z^3 + O(z^4)$$

$$.25 \le Q \le .5$$

as

$$0 \le \Omega_\Lambda \le 1$$

$$\Omega_M + \Omega_\Lambda = 1$$

- In the case $\Omega_M = .3$, $\Omega_\Lambda = .7$ this gives

$$H_0 d_\ell = z + .425\, z^2 - .1804\, z^3 + O(z^4)$$

Our Wave Theory

$$H_0 d_\ell = z + \underbrace{Q(z_2, w_0)} z^2 + C(z_2, w_0, w_2) z^3 + O(z^4)$$

$$.25 \leq Q \leq .5$$

as

$$z_2' = -3w_0\left(\tfrac{4}{3} + z_2\right)$$

$$w_0' = -\left(\tfrac{1}{6} z_2 + \tfrac{1}{3} w_0 + w_0^2\right)$$

Orbit evolves to a NEW STABLE REST POINT

- A Wave with Underdensity: $\dfrac{\rho_{SM} - \rho_{ssw}}{\rho_{SM}} = 7.45 \times 10^{-6}$

$$H_0 d_\ell = z + .425 z^2 + .3591 z^3 + O(z^4)$$

Conclusion: The cubic correction is of the same order, but of a different sign, from Dark Energy…

…A Testable Prediction!

$$H_0 d_\ell = z + .425\, z^2 - .1804\, z^3$$

Dark Energy

$$H_0 d_\ell = z + .425 z^2 + .3591 z^3$$

Wave Theory

Self-Similar Solutions

The $k = 0$ Friedmann spacetimes admit self-similar expressions when $p = \sigma^2 \rho$

$$ds^2 = -B(\xi)dt^2 + \frac{1}{A(\xi)}dr^2 + r^2 d\Omega^2$$

$$\xi = \frac{r}{ct} \quad \text{“}Fractional\ Distance\ to\ Hubble\ Length''$$

$$\rho r^2 = z(\xi) \quad \text{“}Dimensionless\ Density''$$

$$\frac{v}{\xi} = w(\xi) \quad \text{“}Dimensionless\ Velocity''$$

$$\sigma = 0$$
$$p = 0$$

$$ds^2 = -B_F(\xi)d\bar{t}^2 + \frac{1}{A_F(\xi)}d\bar{r}^2 + \bar{r}^2 d\Omega^2$$

$$A_F(\xi) = 1 - \tfrac{4}{9}\xi^2 - \tfrac{8}{27}\xi^4 + O(\xi^6)$$
$$D_F(\xi) \equiv \sqrt{A_F B_F} = 1 - \frac{1}{9}\xi^2 + O(\xi^4).$$

$$z_F(\xi) = \frac{4}{3}\xi^2 + \frac{40}{27}\xi^4 + O(\xi^6)$$
$$w_F \equiv \frac{v}{\xi} = \frac{2}{3} + \frac{2}{9}\xi^2 + O(\xi^4)$$

The p=0 Friedmann Universe in Self-Similar Coordinates

$$p = \frac{c^2}{3}\rho$$

$$\sigma = \frac{1}{\sqrt{3}}$$

Self-similar coordinates for Friedmann with Pure Radiation

$$\bar{\xi} \neq \xi$$

$$z_{1/3} = \frac{3}{4}\bar{\xi}^2 + \frac{9}{16}\bar{\xi}^4 + O(\bar{\xi}^6)$$

$$v_{1/3} = \frac{1}{2}\bar{\xi} + \frac{1}{8}\bar{\xi}^3 + O(\bar{\xi}^5)$$

$$A_{1/3} = 1 - \frac{1}{4}\bar{\xi}^2 - \frac{1}{8}\bar{\xi}^4 + O(\bar{\xi}^6)$$

$$D_{1/3} = 1 + O(\bar{\xi}^4)$$

The $p = \frac{c^2}{3}\rho$ Friedmann Universe extends to 1-parameter family of Self-Similar spacetimes that perturb the Standard Model during the Radiation Epoch:

The $p = 0$ Friedmann Universe DOES NOT admit Self-Similar perturbations!

(Something has to give when p drops to zero!)

(The topic of our PNAS and MEMOIR)

A 1-parameter family of solutions depending on the Acceleration Parameter $0 < a < \infty$

$$z^a_{1/3} = \frac{3a^2}{4}\bar{\xi}^2 + \frac{3a^2(2+a^2)}{16}\bar{\xi}^4 + O(\bar{\xi}^6)$$

$$v^a_{1/3} = \frac{1}{2}\bar{\xi} + \frac{2-a^2}{8}\bar{\xi}^3 + O(\bar{\xi}^5)$$

$$A^a_{1/3} = 1 - \frac{a^2}{4}\bar{\xi}^2 + \frac{a^2(1-3a^2)}{16}\bar{\xi}^4 + O(\bar{\xi}^6)$$

$$D^a_{1/3} = 1 + O(\bar{\xi}^4)$$

The ANSATZ that triggers the instability when p=0:

The ANSATZ that triggers the instability:

$$\begin{aligned} z(t,\xi) &= \left(\frac{4}{3}+z_2(t)\right)\xi^2+\left\{\frac{40}{27}+z_4(t)\right\}\xi^4+O(\xi^6), \\ w(t,\xi) &= \left(\frac{2}{3}+w_0(t)\right)+\left\{\frac{2}{9}+w_2(t)\right\}\xi^2+O(\xi^4), \end{aligned}$$

The ANSATZ:

$$
\begin{aligned}
z(t,\xi) &= \left(\frac{4}{3} + z_2(t)\right)\xi^2 + \left\{\frac{40}{27} + z_4(t)\right\}\xi^4 + O(\xi^6), \\
w(t,\xi) &= \left(\frac{2}{3} + w_0(t)\right) + \left\{\frac{2}{9} + w_2(t)\right\}\xi^2 + O(\xi^4),
\end{aligned}
$$

$$\xi = \frac{r}{ct}$$

$$“Fractional\ Distance\ to\ Hubble\ Length''$$

The ANSATZ:

$$\begin{aligned} z(t,\xi) &= \left(\frac{4}{3}+z_2(t)\right)\xi^2+\left\{\frac{40}{27}+z_4(t)\right\}\xi^4+O(\xi^6), \\ w(t,\xi) &= \left(\frac{2}{3}+w_0(t)\right)+\left\{\frac{2}{9}+w_2(t)\right\}\xi^2+O(\xi^4), \end{aligned}$$

$$\xi = \frac{r}{ct} \quad \text{“}Fractional\ Distance\ to\ Hubble\ Length''$$

$$\boxed{z(t,\xi) = \rho r^2} \quad \text{“}Dimensionless\ Density''$$

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$$\xi=\frac{r}{ct} \qquad \text{“}\mathit{Fractional\ Distance\ to\ Hubble\ Length}''$$

$$z(t,\xi)=\rho r^2 \qquad \text{“}\mathit{Dimensionless\ Density}''$$

$$\boxed{w(t,\xi)=\frac{v}{\xi}} \qquad \text{“}\mathit{Dimensionless\ Velocity}''$$

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$$
\begin{aligned}
z(t,\xi) &= \left(\frac{4}{3} + z_2(t)\right)\xi^2 + \left\{\frac{40}{27} + z_4(t)\right\}\xi^4 + O(\xi^6), \\
w(t,\xi) &= \left(\frac{2}{3} + w_0(t)\right) + \left\{\frac{2}{9} + w_2(t)\right\}\xi^2 + O(\xi^4),
\end{aligned}
$$

Only EVEN powers of ξ...

The ANSATZ:

$$\boxed{z(t,\xi) \quad = \quad \left(\frac{4}{3} + z_2(t)\right)\xi^2 + \left\{\frac{40}{27} + z_4(t)\right\}\xi^4 + O(\xi^6),}$$

Uniform Density out to errors ξ^4

$$z(t,\xi) = \rho r^2$$

$$\boxed{\rho(t) \sim \frac{\left(\frac{4}{3} + z_2(t)\right)}{t^2} = \frac{f(t)}{t^2}}$$

THEOREM: The $p = 0$ waves take the asymptotic form

$$
\begin{aligned}
z(t,\xi) &= \left(\frac{4}{3} + z_2(t)\right)\xi^2 + \left\{\frac{40}{27} + z_4(t)\right\}\xi^4 + O(\xi^6),\\
w(t,\xi) &= \left(\frac{2}{3} + w_0(t)\right) + \left\{\frac{2}{9} + w_2(t)\right\}\xi^2 + O(\xi^4),
\end{aligned}
$$

where $z_2(t), z_4(t), w_0(t), w_2(t)$ evolve according to the equations

$$
\begin{aligned}
-t\dot{z}_2 &= 3w_0\left(\frac{4}{3} + z_2\right),\\
-t\dot{z}_4 &= -5\left\{\frac{2}{27}z_2 + \frac{4}{3}w_2 - \frac{1}{18}z_2^2 + z_2w_2\right\}\\
&\qquad -5w_0\left\{\frac{4}{3} - \frac{2}{9}z_2 + z_4 - \frac{1}{12}z_2^2\right\},\\
-t\dot{w}_0 &= \frac{1}{6}z_2 + \frac{1}{3}w_0 + w_0^2,\\
-t\dot{w}_2 &= \frac{1}{10}z_4 + \frac{4}{9}w_0 - \frac{1}{3}w_2 + \frac{1}{24}z_2^2 - \frac{1}{3}z_2w_0\\
&\qquad -\frac{1}{3}w_0^2 + 4w_0w_2 - \frac{1}{4}w_0^2z_2.
\end{aligned}
$$

Our Wave Theory

$$H_0 d_\ell = z + \underbrace{Q(z_2, w_0)}_{.25 \le Q \le .5} z^2 + C(z_2, w_0, w_2) z^3 + O(z^4)$$

as

$$z_2' = -3w_0 \left(\tfrac{4}{3} + z_2 \right)$$

$$w_0' = -\left(\tfrac{1}{6} z_2 + \tfrac{1}{3} w_0 + w_0^2 \right)$$

Orbit evolves to a NEW STABLE REST POINT

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Orbit evolves to a NEW STABLE REST POINT

$$Q(z_2, w_0) = \frac{1}{4} + \frac{24w_0 + 45w_0^2 + 3z_2}{4(2 + 3w_0)^2}$$

Our Wave Theory

$$H_0 d_\ell = z + \underbrace{Q(z_2, w_0)} z^2 + C(z_2, w_0, w_2) z^3 + O(z^4)$$

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as

$$z_2' = -3w_0 \left(\tfrac{4}{3} + z_2\right)$$

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$$\frac{1}{4} = Q(0,0) \le Q \le Q\,(M)$$

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$$\frac{1}{4} = Q(0,0) \le Q \le Q\left(M\right)$$

(Along the orbit $SM \to M$)

Our Wave Theory

$$H_0 d_\ell = z + \underbrace{Q(z_2, w_0)} z^2 + C(z_2, w_0, w_2) z^3 + O(z^4)$$

$$.25 \leq Q \leq .5$$

as

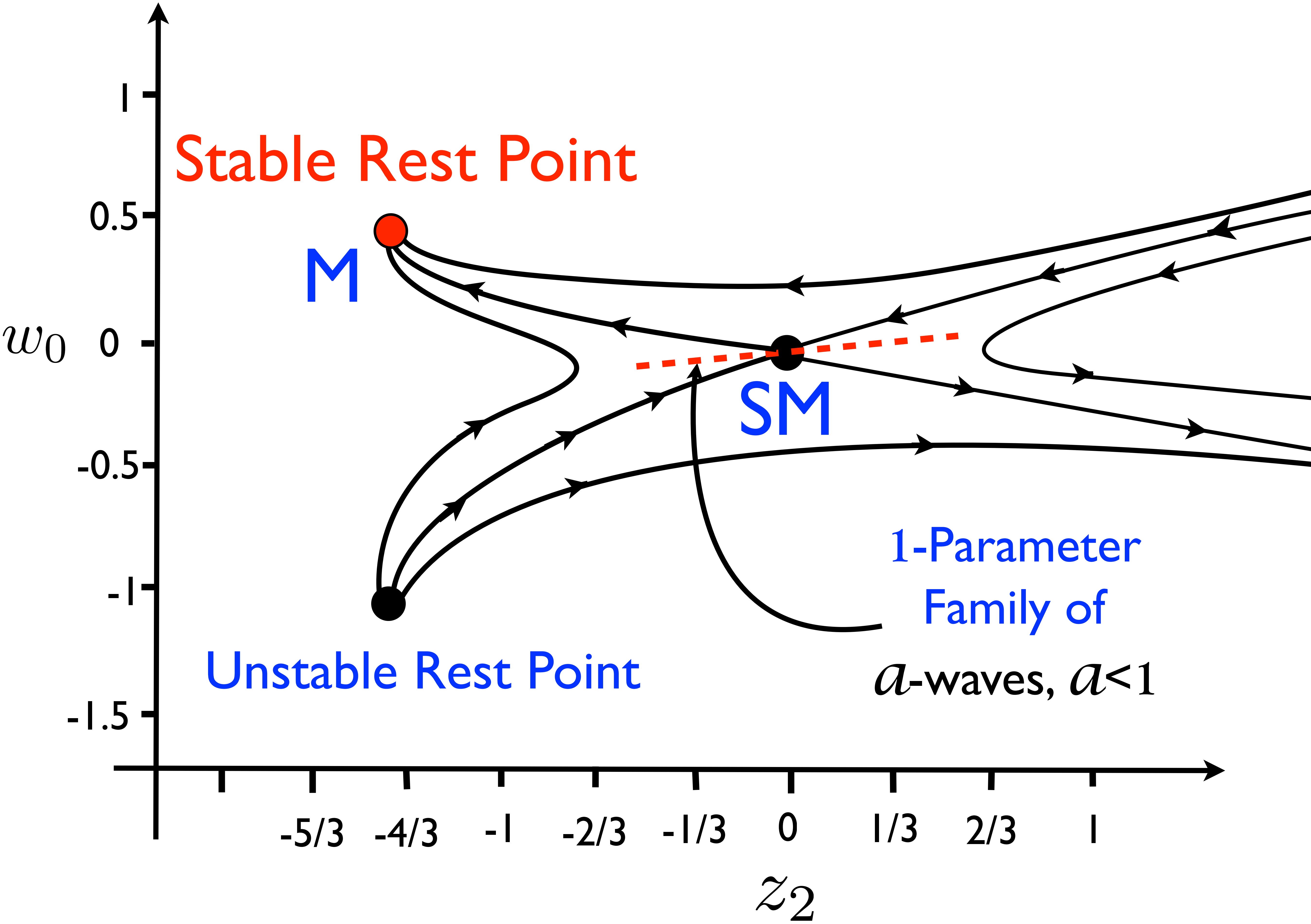
$$z_2' = -3w_0 \left(\tfrac{4}{3} + z_2\right)$$

$$w_0' = -\left(\tfrac{1}{6} z_2 + \tfrac{1}{3} w_0 + w_0^2\right)$$

Orbit evolves to a NEW STABLE REST POINT

- A Wave with Under-density: $\dfrac{\rho_{SM} - \rho_{ssw}}{\rho_{SM}} = 7.45 \times 10^{-6}$

$$H_0 d_\ell = z + .425 z^2 + .3591 z^3 + O(z^4)$$


Stable Rest Point
M
SM
1-Parameter
Family of
a-waves, a<1
Unstable Rest Point
w_0
1
0.5
0
-0.5
-1
-1.5
-5/3
-4/3
-1
-2/3
-1/3
0
1/3
2/3
1
z_2

Strategy: Use our equations to evolve the initial data for a-waves at the end of radiation to determine (a, T_*) that gives the correct anomalous acceleration.

I.e., (a, T_*) that give the observed quadratic correction to redshift vs luminosity at present time

- In the Standard Model p=0 at about

$t_* \approx$ 10,000-30,000 yrs

$T_* \approx 9000^0 K$

(Depending on theories of Dark Matter)

- Our simulation turns out to be entirely insensitive to the initial t_* , T_*

- I.e., we need only compute the value of the acceleration parameter that gives the correct anomalous acceleration.

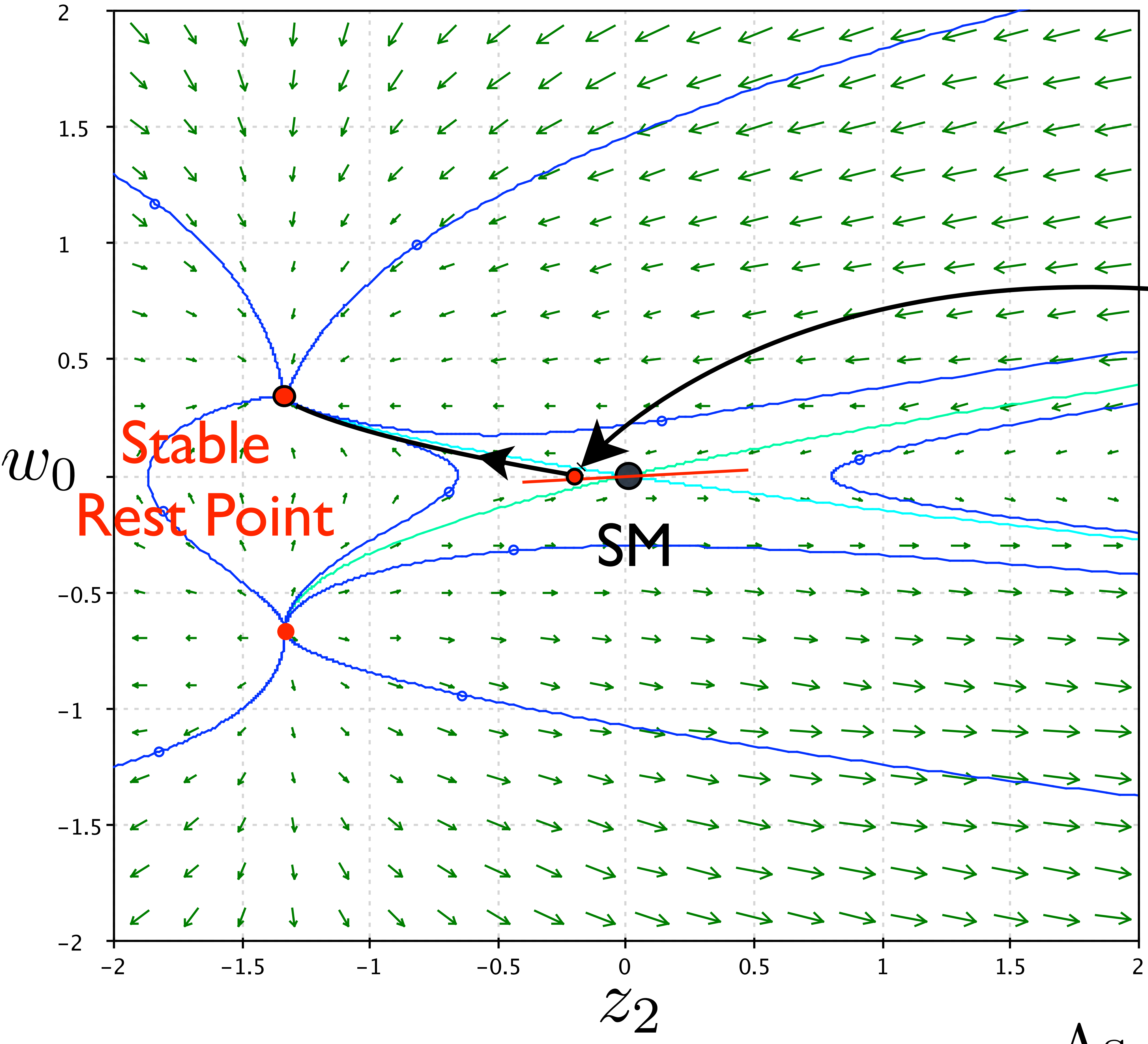
THE ANSWER: The value of the acceleration for the wave perturbation of SM that produces a quadradic correction of .425 at the present value of H_0 is:

- $\underline{a} = 0.99999957 = 1 - \left(4.3 \times 10^{-7}\right)$

$$H_0 d_\ell = z + .425 z^2 + .3591 z^3$$

- This corresponds to a relative under-density of

$$\frac{\rho_{SM} - \rho_{ssw}}{\rho_{SM}} = 7.45 \times 10^{-6}$$



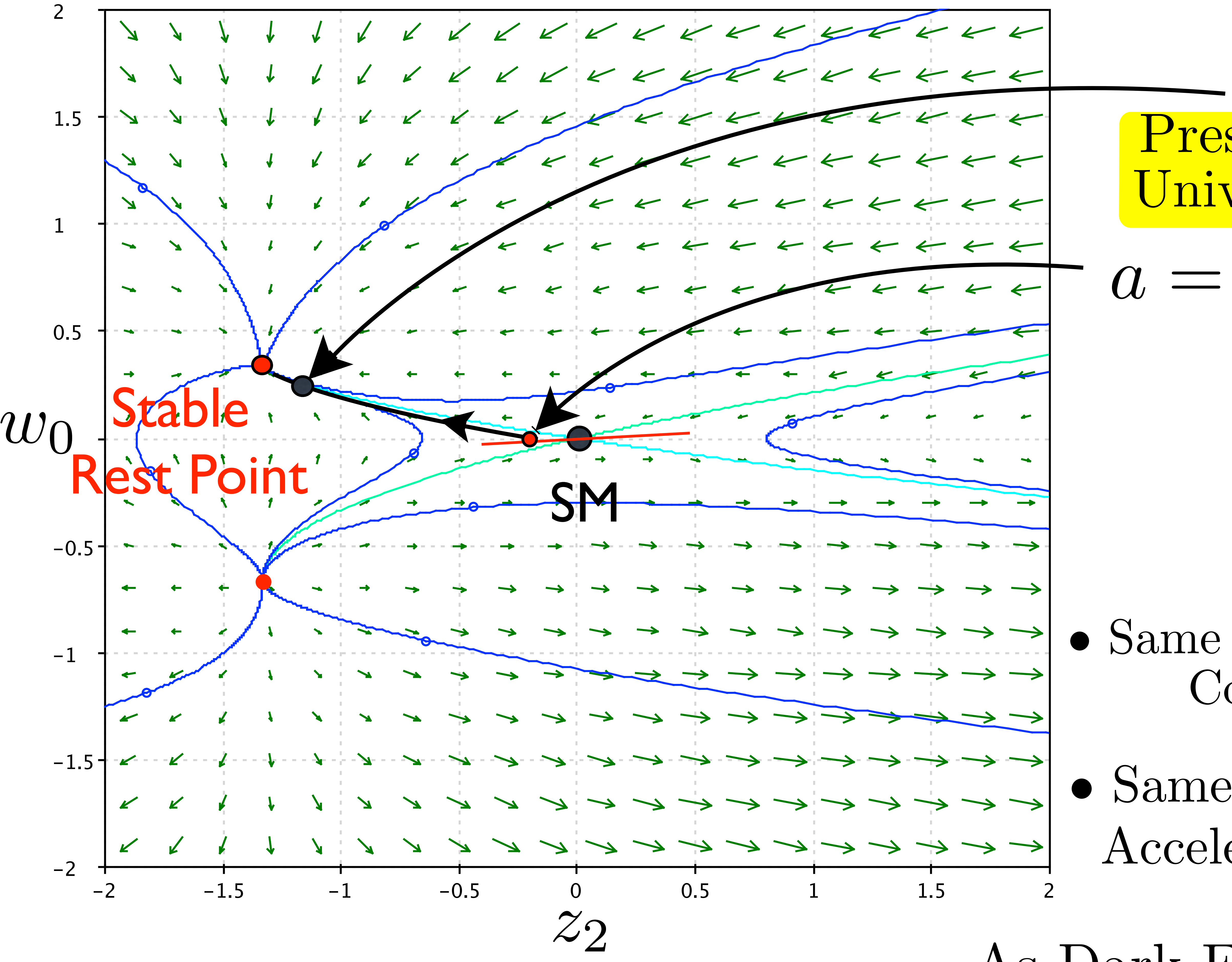
$a = \underline{a}$

$(\approx 1 - .00000043)$

Creates…

- Same Hubble Constant
- Same .425 Acceleration

As Dark Energy



Present Universe

$a = \underline{a}$

- Same Hubble Constant
- Same .425 Acceleration

As Dark Energy

- The relative underdensity at the end of radiation:

$$\boxed{\frac{\rho_{SM} - \rho_{ssw}}{\rho_{SM}} = 7.45 \times 10^{-6}}$$

- The relative underdensity at present time:

$$\boxed{\frac{\rho_{ssw}(t_0)}{\rho_{SM}(t_0)} = .1438 \approx \frac{1}{7}.}$$

Conclude:

An under-density of
one part in 10^6
at the end of radiation
produces a
seven-fold under-density

at present time…

CONCLUDE:

The Standard Model is Unstable to Perturbation by this 1-parameter family of Waves

Comparison with Dark Energy:

$$H_0 d_\ell = z + .425\, z^2 - .1804\, z^3$$

Dark Energy

$$H_0 d_\ell = z + .425 z^2 + .3591\, z^3$$

Wave Theory

$$z \sim \frac{d_\ell}{H_0} \sim \frac{r}{ct} \sim \xi$$

Measures Fractional Distance to Hubble Length

$z << 1$

A prediction:
The wave contributes MORE to the Anomalous Acceleration far from the center

Neglecting $O(\xi^4)$ errors: The spacetime near the center evolves toward the Stable Rest Point

- The metric tends to Flat Minkowski Spacetime which is not co-moving with the fluid

- BUT: The evolution creates a uniformly expanding density near the center, which, neglecting relativistic corrections, is *CENTER-INDEPENDENT* (like Friedmann Spacetimes)

CONCLUDE:

The wave creates a

UNIFORMLY EXPANDING SPACETIME

with an

ANOMALOUS ACCELERATION

in a

LARGE, FLAT, CENTER-INDEPENDENT

region near the center of the wave

Neglecting errors $O(\xi^4)$:

$$
\begin{aligned}
z(t,\xi) &= \left(\frac{4}{3}+z_2(t)\right)\xi^2+\xcancel{\left\{\frac{40}{27}+z_4(t)\right\}}\xi^4+\xcancel{O(\xi^6)}, \\
w(t,\xi) &= \left(\frac{2}{3}+w_0(t)\right)+\left\{\frac{2}{9}+w_2(t)\right\}\xi^2+\xcancel{O(\xi^4)}
\end{aligned}
$$

$z \sim density \qquad w \sim velocity$

$\xi = \dfrac{r}{t} \sim fractional\ distance\ to\ \text{Hubble Length}$

THEOREM: Neglecting $O(\xi^4)$ errors, as the orbit tends to the Stable Rest Point:

- The Density drops FASTER than SM:

$$\rho_{WAVE}(t) = \frac{k_0}{t^3(1+\bar{w})} \qquad \rho_{SM}(t) = \frac{4}{3t^2}$$

where $\bar{w}(t)$ and $k_0(t)$ change exponentially slowly.

- **The metric tends to FLAT MINKOWSKI:**

$$\mathrm{ds}^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$

Theorem: There exists a unique value $\underline{a} = 0.99999956 \approx 1 - 4.3 \times 10^{-7}$ such that:

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- The $p = 0$ evolution starting from this initial data evolves to $H = H_0,\ Q = .425$ at $t = t_0$, in agreement with Dark Energy at $t = t_{DE}$.

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- The cubic correction is $C = 0.3591$ at $t = t_0$, while Dark Energy is $C = -0.1804$ at $t = t_{DE}$.

Theorem: There exists a unique value

$\underline{a} = 0.99999956 \approx 1 - 4.3 \times 10^{-7}$ such that:

- The $p = 0$ evolution starting from this initial data evolves to $H - H_0$, $Q = .425$ at $t = t_0$, in agreement with Dark Energy at $t = t_{DE}$.

- The cubic correction is $C = 0.3591$ at $t = t_0$, while Dark Energy is $C = -0.1804$ at $t = t_{DE}$.

- The ages of the universe are related by:

$$t_0 \approx (.95)t_{DE} \approx 1.38 \times t_{SM} = 1.38 \times (9.8 \times 10^9 yr)$$

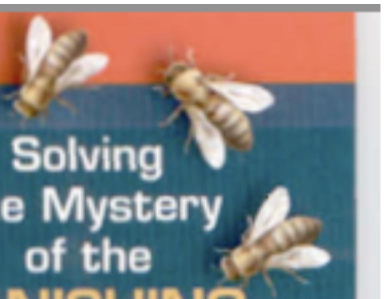
Around 2007:

Other research groups began exploring the possibility that the anomalous acceleration might be due to the earth lying near the center of a large region of Under-Density

We first saw publication in 2009

Virus Watch: Preventing the Next Pandemic

SCIENTIFIC AMERICAN
April 2009
www.SciAm.com

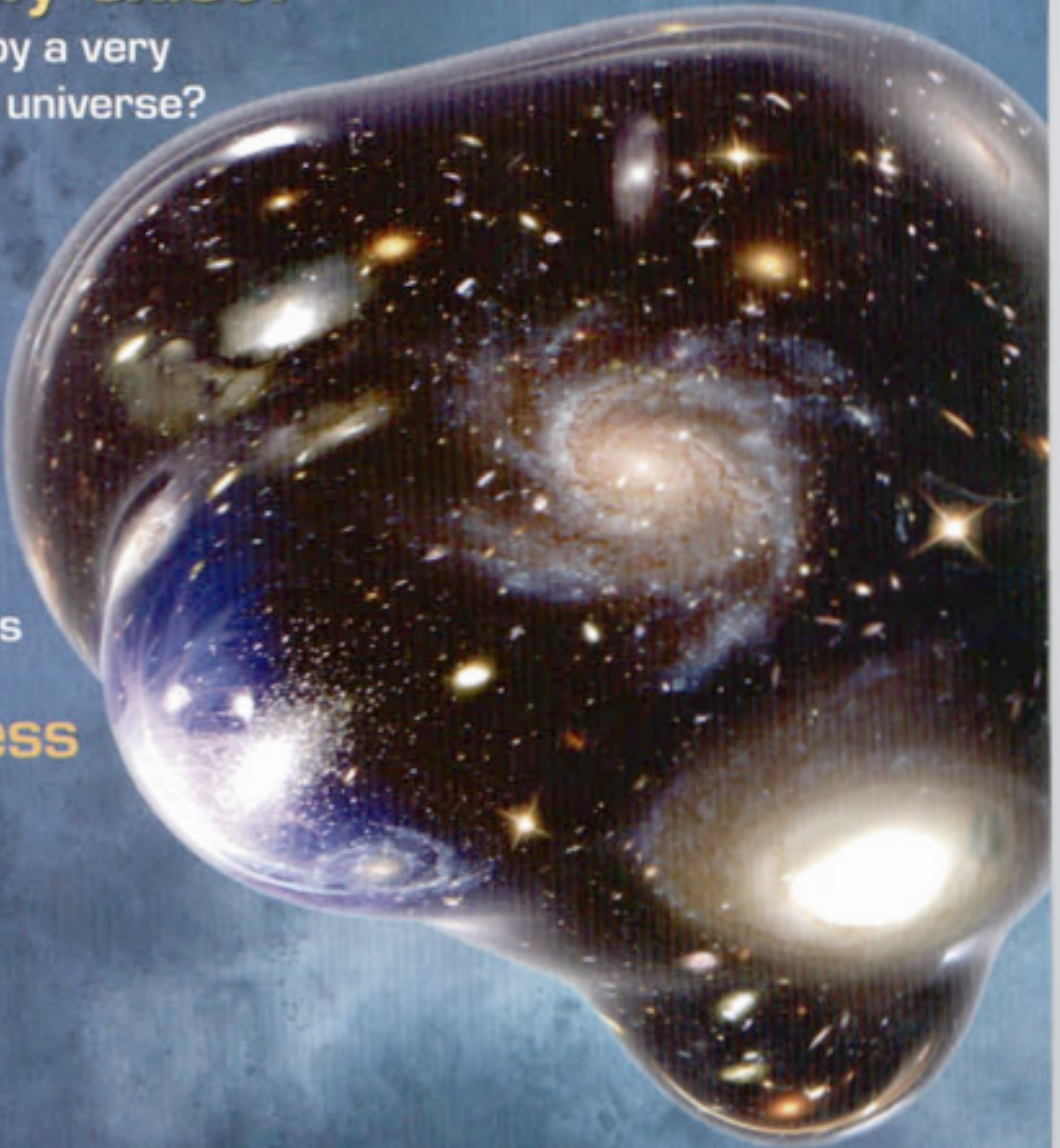


Solving the Mystery of the
VANISHING BEES
page 40

DARK ENERGY

Does it really exist?

Or does Earth occupy a very unusual place in the universe?



Color Vision
Our Eyes Reflect Primate Evolution

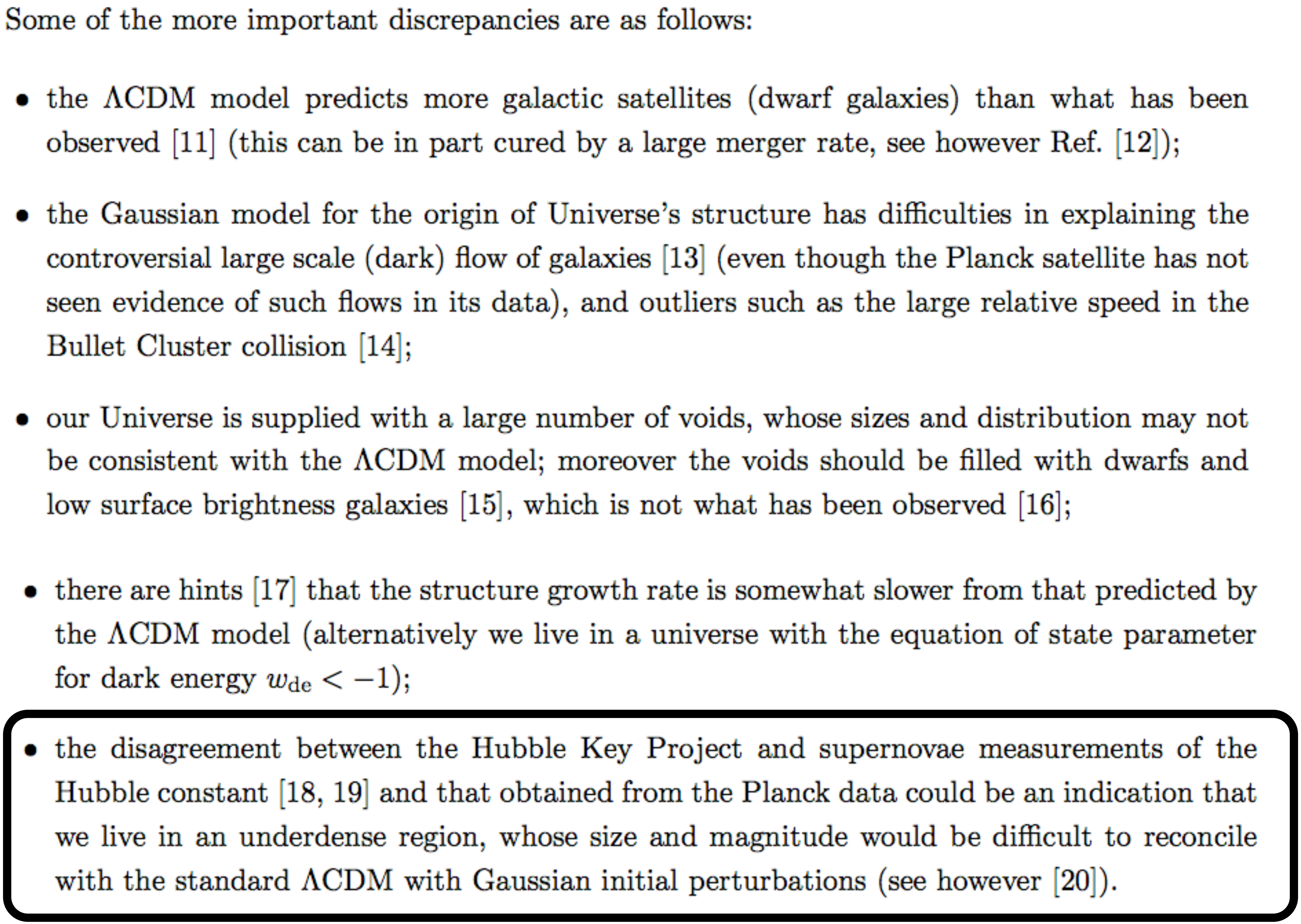
Green Lasers
The Next Innovation in Chip-Based Beams

Soldiers' Stress
What Doctors Get Wrong about PTSD

$5.99

This proposal is still taken seriously in Astrophysics

Prokopek...2013 (Astrophysicist, Utrecht University)



Some of the more important discrepancies are as follows:

- the ΛCDM model predicts more galactic satellites (dwarf galaxies) than what has been observed [11] (this can be in part cured by a large merger rate, see however Ref. [12]);
- the Gaussian model for the origin of Universe's structure has difficulties in explaining the controversial large scale (dark) flow of galaxies [13] (even though the Planck satellite has not seen evidence of such flows in its data), and outliers such as the large relative speed in the Bullet Cluster collision [14];
- our Universe is supplied with a large number of voids, whose sizes and distribution may not be consistent with the ΛCDM model; moreover the voids should be filled with dwarfs and low surface brightness galaxies [15], which is not what has been observed [16];
- there are hints [17] that the structure growth rate is somewhat slower from that predicted by the ΛCDM model (alternatively we live in a universe with the equation of state parameter for dark energy $w_{\rm de} < -1$);
- the disagreement between the Hubble Key Project and supernovae measurements of the Hubble constant [18, 19] and that obtained from the Planck data could be an indication that we live in an underdense region, whose size and magnitude would be difficult to reconcile with the standard ΛCDM with Gaussian initial perturbations (see however [20]).

Details of our Analysis

Main Steps:

(1) Derivation of the $p=0$ Einstein equations in a new coordinate system aligned with the structure of the waves.

(2) A new ansatz for the Corrections to SM such that the asymptotic equations close.

(3) Putting the Initial Data from the Radiation Epoch into the gauge of our asymptotics.

(4) The Redshift vs Luminosity determined by the Corrections.

I. A New Formulation of the p=0 Einstein Equations

The Einstein equations for
spherically symmetric
spacetimes take their
Simplest Form
in
Standard Schwarzschild
Coordinates
(SSC)

I.e.

I.e. A General Spherically Symmetric metric

$$\mathrm{ds}^2 = -D(t,\bar{r})d\bar{t}^2 + E(\bar{t},\bar{r})d\bar{t}d\bar{r} + F(\bar{t},\bar{r})d\bar{r}^2 + G(\bar{t},\bar{r})d\Omega^2$$

I.e. A General Spherically Symmetric metric

$$\mathrm{ds}^2 = -D(t,\bar{r})d\bar{t}^2 + E(\bar{t},\bar{r})d\bar{t}d\bar{r} + F(\bar{t},\bar{r})d\bar{r}^2 + G(\bar{t},\bar{r})d\Omega^2$$

Transforms to SSC form:

I.e. A General Spherically Symmetric metric

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Transforms to SSC form:

$$(\bar{t},\bar{r}) \longrightarrow (t,r)$$

⬇

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$$\mathrm{ds}^2 = -D(t,\bar{r})d\bar{t}^2 + E(\bar{t},\bar{r})d\bar{t}d\bar{r} + F(\bar{t},\bar{r})d\bar{r}^2 + G(\bar{t},\bar{r})d\Omega^2$$

Transforms to SSC form:

$$(\bar{t},\bar{r}) \longrightarrow (t,r)$$

⬇

$$ds^2 = -B(t,r)dt^2 + \frac{1}{A(t,r)}dr^2 + r^2 d\Omega^2$$

SSC

The Equations
In SSC

Standard Schwarzschild Coordinates

Four PDE's

$$\begin{aligned}
\left\{-r\frac{A_r}{A}+\frac{1-A}{A}\right\} &= \frac{\kappa B}{A}r^2T^{00} && (1)\\
\frac{A_t}{A} &= \frac{\kappa B}{A}rT^{01} && (2)\\
\left\{r\frac{B_r}{B}-\frac{1-A}{A}\right\} &= \frac{\kappa}{A^2}r^2T^{11} && (3)\\
-\left\{\left(\frac{1}{A}\right)_{tt}-B_{rr}+\Phi\right\} &= 2\frac{\kappa B}{A}r^2T^{22}, && (4)
\end{aligned}$$

where

$$\begin{aligned}
\Phi &= \frac{B_tA_t}{2A^2B}-\frac{1}{2A}\left(\frac{A_t}{A}\right)^2-\frac{B_r}{r}-\frac{BA_r}{rA}\\
&\qquad +\frac{B}{2}\left(\frac{B_r}{B}\right)^2-\frac{B}{2}\frac{B_r}{B}\frac{A_r}{A}.
\end{aligned}$$

(1)+(2)+(3)+(4) ⟺ (1)+(3)+div T=0

(weakly)

Theorem: (Te-Gr) The equations close in a "locally inertial" formulation of (1), (2) & Div T=0:

$$
\begin{aligned}
\left\{T_M^{00}\right\}_{,0}+\left\{\sqrt{AB}\,T_M^{01}\right\}_{,1} &= -\frac{2}{r}\sqrt{AB}\,T_M^{01}, && (1)\\
\left\{T_M^{01}\right\}_{,0}+\left\{\sqrt{AB}\,T_M^{11}\right\}_{,1} &= -\frac{1}{2}\sqrt{AB}\left\{\frac{4}{r}T_M^{11}+\frac{(1-A)}{Ar}\left(T_M^{00}-T_M^{11}\right)\right. && (2)\\
&\qquad \left.+\frac{2\kappa r}{A}\left(T_M^{00}T_M^{11}-\left(T_M^{01}\right)^2\right)-4rT^{22}\right\}, &&\\
rA_r &= (1-A)-\kappa r^2 T_M^{00}, && (3)\\
rB_r &= \frac{B(1-A)}{A}+\frac{B}{A}\kappa r^2 T_M^{11}. && (4)
\end{aligned}
$$

$$
T_M^{00}=\frac{\rho c^2+p}{1-\left(\frac{v}{c}\right)^2} \qquad T_M^{01}=\frac{\rho c^2+p}{1-\left(\frac{v}{c}\right)^2}\frac{v}{c}
$$

$$
T_M^{11}=\frac{p+\left(\frac{v}{c}\right)^2}{1-\left(\frac{v}{c}\right)^2}\rho c^2 \qquad T^{22}=\frac{p}{r^2} \qquad v=\frac{1}{\sqrt{AB}}\frac{u^1}{u^0}
$$

Setting p=0:

$$T^{00}_M = \frac{\rho c^2}{1-\left(\frac{v}{c}\right)^2} \ , \qquad T^{01}_M = \frac{\rho c^2}{1-\left(\frac{v}{c}\right)^2}\frac{v}{c}$$

$$T^{11}_M = \frac{\rho c^2}{1-\left(\frac{v}{c}\right)^2}\left(\frac{v}{c}\right)^2, \qquad T^{22} = 0$$

Everything can be written in terms of T^{00}_M and $\left(\frac{v}{c}\right)$:

$$T^{01}_M = T^{00}_M \left(\frac{v}{c}\right), \qquad T^{22}_M = T^{00}_M \left(\frac{v}{c}\right)^2$$

Substituting into the Equations gives:

$$\left(T_M^{00}\right)_t + \left\{\sqrt{AB}\left(\frac{v}{c}\right)\left(T_M^{00}\right)\right\}_r = -\frac{2\sqrt{AB}}{r}\left(\frac{v}{c}\right)\left(T_M^{00}\right)$$

$$\left(\left(\tfrac{v}{c}\right)T_M^{00}\right)_t + \left\{\sqrt{AB}\left(\tfrac{v}{c}\right)^2 T_M^{00}\right\}_r =$$

$$-\tfrac{\sqrt{AB}}{2r}\left\{4\left(\tfrac{v}{c}\right)^2 + \tfrac{1-A}{A}\left(1-\left(\tfrac{v}{c}\right)^2\right)\right\}T_M^{00}$$

$$\frac{A'}{A} = \frac{1}{r}\left(\frac{1}{A}-1\right) - \frac{\kappa r}{A}T_M^{00}$$

$$\frac{B'}{B} = \frac{1}{r}\left(\frac{1}{A}-1\right) + \frac{\kappa r}{A}T_M^{00}\left(\frac{v}{c}\right)^2$$

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$$\frac{B'}{B} = \frac{1}{r}\left(\frac{1}{A}-1\right) + \frac{\kappa r}{A}T_M^{00}\left(\frac{v}{c}\right)^2$$

Everything in terms of T_M^{00} and $\left(\frac{v}{c}\right)$

Substituting into the Equations gives:

$$\left(T_M^{00}\right)_t + \left\{\sqrt{AB}\left(\frac{v}{c}\right)\left(T_M^{00}\right)\right\}_r = -\frac{2\sqrt{AB}}{r}\left(\frac{v}{c}\right)\left(T_M^{00}\right) \tag{1}$$

$$\left(\left(\tfrac{v}{c}\right)T_M^{00}\right)_t + \left\{\sqrt{AB}\left(\tfrac{v}{c}\right)^2 T_M^{00}\right\}_r = -\frac{\sqrt{AB}}{2r}\left\{4\left(\tfrac{v}{c}\right)^2 + \frac{1-A}{A}\left(1-\left(\tfrac{v}{c}\right)^2\right)\right\}T_M^{00} \tag{2}$$

$$\frac{A'}{A} = \frac{1}{r}\left(\frac{1}{A}-1\right) - \frac{\kappa r}{A}T_M^{00}$$

$$\frac{B'}{B} = \frac{1}{r}\left(\frac{1}{A}-1\right) + \frac{\kappa r}{A}T_M^{00}\left(\frac{v}{c}\right)^2$$

Note: Equations are Singular at $r = 0$

The $1/r$ singularity reflects the fact that waves coming into $r = 0$ can amplify and blowup.

Since we are only interested in solutions representing outgoing, expanding waves, we look for natural changes of variables that regularize the equations at $r = 0$.

First: set $c = 1$, collect v/r, and assume v/r smooth at r=0:

$$\left(T_M^{00}\right)_t + r\left\{\sqrt{AB}\left(\frac{v}{r}\right)T_M^{00}\right\}_r = 3\sqrt{AB}\left(\frac{v}{r}\right)T_M^{00}$$

$$\left(\frac{v}{r}\right)_t + r\sqrt{AB}\left(\frac{v}{r}\right)\left(\frac{v}{r}\right)_r = -\sqrt{AB}\left\{\left(\frac{v}{r}\right)^2 + \frac{1-A}{2Ar^2}\left(1 - r^2\left(\frac{v}{r}\right)^2\right)\right\}$$

$$\frac{A'}{A} = \frac{1}{r}\left(\frac{1}{A} - 1\right) - \frac{\kappa r}{A}T_M^{00}$$

$$\frac{B'}{B} = \frac{1}{r}\left(\frac{1}{A} - 1\right) + \frac{\kappa r}{A}T_M^{00}\left(\frac{v}{c}\right)^2$$

Next: use (1) to eliminate T_M^{00} from (2)

$$\left(T_M^{00}\right)_t + \left\{\sqrt{AB}\left(\frac{v}{c}\right)\left(T_M^{00}\right)\right\}_r = -\frac{2\sqrt{AB}}{r}\left(\frac{v}{c}\right)\left(T_M^{00}\right) \qquad (1)$$

$$\left(\left(\tfrac{v}{c}\right) T_M^{00}\right)_t + \left\{\sqrt{AB}\left(\tfrac{v}{c}\right)^2 T_M^{00}\right\}_r =$$

$$-\tfrac{\sqrt{AB}}{2r}\left\{4\left(\tfrac{v}{c}\right)^2 + \tfrac{1-A}{A}\left(1-\left(\tfrac{v}{c}\right)^2\right)\right\} T_M^{00} \qquad (2)$$

$$\frac{A'}{A} = \frac{1}{r}\left(\frac{1}{A} - 1\right) - \frac{\kappa r}{A} T_M^{00}$$

$$\frac{B'}{B} = \frac{1}{r}\left(\frac{1}{A} - 1\right) + \frac{\kappa r}{A} T_M^{00}\left(\frac{v}{c}\right)^2$$

I.e.

$$\left(\left(\frac{v}{c}\right)T_M^{00}\right)_t+\left\{\sqrt{AB}\left(\frac{v}{c}\right)^2T_M^{00}\right\}_r= \qquad (2)$$

$$-\frac{\sqrt{AB}}{2r}\left\{4\left(\frac{v}{c}\right)^2+\frac{1-A}{A}\left(1-\left(\frac{v}{c}\right)^2\right)\right\}T_M^{00}$$

$$LHS=r\left(\frac{v}{r}\right)\left[\left(T_M^{00}\right)_t+\left\{\sqrt{AB}r\left(\frac{v}{r}\right)T_M^{00}\right\}_r\right]$$

$$+rT_M^{00}\left(\frac{v}{r}\right)_t+rT_M^{00}\sqrt{AB}\left(\frac{v}{r}\right)\left(r\left(\frac{v}{r}\right)\right)_r$$

$$\left(T_M^{00}\right)_t+\left\{\sqrt{AB}\left(\frac{v}{c}\right)\left(T_M^{00}\right)\right\}_r=-\frac{2\sqrt{AB}}{r}\left(\frac{v}{c}\right)\left(T_M^{00}\right) \qquad (1)$$

Substitute (1) into (2):

Obtain:

$$-2\sqrt{AB}\left(\frac{v}{r}\right)^2 rT_M^{00} + rT_M^{00}\left(\frac{v}{r}\right)_t + rT_M^{00}\sqrt{AB}\left(\frac{v}{r}\right)\left(r\left(\frac{v}{r}\right)\right)_r \qquad \textbf{(2)}$$

$$= \quad -\frac{\sqrt{AB}}{2r}\left\{4\left(\frac{v}{c}\right)^2 + \frac{1-A}{A}\left(1-\left(\frac{v}{c}\right)^2\right)\right\}T_M^{00}$$

Obtain:

$$\begin{aligned} & -2\sqrt{AB}\left(\frac{v}{r}\right)^2 rT_M^{00} + rT_M^{00}\left(\frac{v}{r}\right)_t \\ & \qquad + rT_M^{00}\sqrt{AB}\left(\frac{v}{r}\right)\left(r\left(\frac{v}{r}\right)\right)_r \\ = \quad & -\frac{\sqrt{AB}}{2r}\left\{4\left(\frac{v}{c}\right)^2 + \frac{1-A}{A}\left(1-\left(\frac{v}{c}\right)^2\right)\right\}T_M^{00} \end{aligned} \tag{2}$$

Linearity in T_M^{00} ➡ Divide by rT_M^{00}

Next: simplify and collect: $z = \kappa T_M^{00} r^2$

$$\left(\kappa T_M^{00} r^2\right)_t + \left\{\sqrt{AB}\,\frac{v}{r}\left(\kappa T_M^{00} r^2\right)\right\}_r = -2\sqrt{AB}\,\frac{v}{r}\left(\kappa T_M^{00} r^2\right)$$

$$\left(\tfrac{v}{r}\right)_t + r\sqrt{AB}\left(\tfrac{v}{r}\right)\left(\tfrac{v}{r}\right)_r = -\sqrt{AB}\left\{\left(\tfrac{v}{r}\right)^2 + \tfrac{1-A}{2Ar^2}\left(1 - r^2\left(\tfrac{v}{r}\right)^2\right)\right\}$$

$$r\frac{A'}{A} = \left(\frac{1}{A} - 1\right) - \frac{1}{A}\kappa T_M^{00} r^2$$

$$r\frac{B'}{B} = \left(\frac{1}{A} - 1\right) + \frac{1}{A}\left(\frac{v}{c}\right)^2 \kappa T_M^{00} r^2$$

Simplify and collect: $z = \kappa T^{00}_M r^2$

$$\left(\kappa T^{00}_M r^2\right)_t + \left\{\sqrt{AB}\,\frac{v}{r}\left(\kappa T^{00}_M r^2\right)\right\}_r = -2\sqrt{AB}\,\frac{v}{r}\left(\kappa T^{00}_M r^2\right)$$

$$\left(\tfrac{v}{r}\right)_t + r\sqrt{AB}\left(\tfrac{v}{r}\right)\left(\tfrac{v}{r}\right)_r = -\sqrt{AB}\left\{\left(\tfrac{v}{r}\right)^2 + \tfrac{1-A}{2Ar^2}\left(1 - r^2\left(\tfrac{v}{r}\right)^2\right)\right\}$$

$$r\frac{A'}{A} = \left(\frac{1}{A} - 1\right) - \frac{1}{A}\kappa T^{00}_M r^2$$

$$r\frac{B'}{B} = \left(\frac{1}{A} - 1\right) + \frac{1}{A}\left(\frac{v}{c}\right)^2 \kappa T^{00}_M r^2$$

(This is the self-similar variable in the waves from the radiation epoch!)

Final change of variables---

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$$(t, r) \longrightarrow (t, \xi)$$

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$$(t, r) \rightarrow (t, \xi) \qquad \xi = \frac{r}{t}$$

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$$z = \kappa T_M^{00}\, r^2, \quad w = \frac{v}{\xi}$$

Final change of variables---

$$(t, r) \longrightarrow (t, \xi) \qquad \xi = \frac{r}{t}$$

$$\left(T_M^{00}, v\right) \longrightarrow (z, w)$$

$$z = \kappa T_M^{00}\, r^2, \quad w = \frac{v}{\xi}$$

$$\frac{\partial}{\partial r} = \frac{1}{t}\,\frac{\partial}{\partial r}\,, \qquad \frac{\partial}{\partial r} f(t, r) = \left(\frac{\partial}{\partial t} - \frac{1}{t^2}\,\frac{\partial}{\partial \xi}\right) f(t, \xi)$$

Substituting into (1) and (2) we obtain the following dimensionless eqns:

$$t z_t + \xi \left\{(-1 + Dw) z\right\}_\xi = -Dwz, \tag{1}$$

$$t w_t + \xi \left(-1 + Dw\right) w_\xi = w - D\left\{w^2 + \tfrac{1-\xi^2 w^2}{2A}\left[\tfrac{1-A}{\xi^2}\right]\right\}, \tag{2}$$

Where: $D = \sqrt{AB}$

Take A and D instead of A and B:

Take A and D instead of A and B:

$$\xi A_\xi = (A-1) - z,$$

$$\xi \frac{B_\xi}{B} = \frac{1}{A}\left\{1 - A + \xi^2 w^2 z\right\},$$

$$\xi(\sqrt{AB})_\xi = \sqrt{AB}\left\{(1-A) - \frac{(1-\xi^2 w^2)}{2} z.\right.$$

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$$\xi(\sqrt{AB})_\xi = \sqrt{AB}\left\{(1-A) - \frac{(1-\xi^2 w^2)}{2} z.\right.$$

$$\xi A_\xi = (A-1) - z$$

$$\xi\,(D)_\xi = D\left\{(1-A) - \frac{(1-\xi^2 w^2)}{2} z\right\}$$

This leads to the following Dimensionless Formulation of the p=0 Einstein Equations:

Einstein Equations when p=0

$$tz_t + \xi\left\{(-1 + Dw)z\right\}_\xi = -Dwz,$$

$$tw_t + \xi\left(-1 + Dw\right)w_\xi = \\ w - D\left\{w^2 + \frac{1-\xi^2w^2}{2A}\left[\frac{1-A}{\xi^2}\right]\right\},$$

$$\xi A_\xi = (A - 1) - z,$$

$$\frac{\xi D_\xi}{D} = (1 - A) - \frac{(1-\xi^2w^2)}{2}z.$$

Einstein Equations when p=0

$$tz_t + \xi\left\{(-1+Dw)z\right\}_\xi = -Dwz,$$

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$$\xi A_\xi = (A-1) - z,$$

$$\tfrac{\xi D_\xi}{D} = (1-A) - \tfrac{(1-\xi^2w^2)}{2}z.$$

$$ds^2 = -Bdt^2 + \frac{1}{A}dr^2 + r^2d\Omega^2, \quad D = \sqrt{AB}, \quad z = \frac{\rho r^2}{(1-v^2)}, \quad w = \frac{v}{\xi}$$

2. The Ansatz and Asymptotics for the Corrections:

Our Ansatz for Corrections to the Standard Model

$$z(t,\xi) = z_F(\xi) + \Delta z(t,\xi)$$
$$w(t,\xi) = w_F(\xi) + \Delta w(t,\xi)$$
$$A(t,\xi) = A_F(\xi) + \Delta A(t,\xi)$$
$$D(t,\xi) = D_F(\xi) + \Delta D(t,\xi)$$

Our Ansatz for Corrections to the Standard Model

$$
\begin{aligned}
z(t,\xi) &= z_F(\xi) + \Delta z(t,\xi) \\
w(t,\xi) &= w_F(\xi) + \Delta w(t,\xi) \\
A(t,\xi) &= A_F(\xi) + \Delta A(t,\xi) \\
D(t,\xi) &= D_F(\xi) + \Delta D(t,\xi)
\end{aligned}
$$

- The Standard Model is Self-Similar:

$$
\begin{aligned}
z_F &= \tfrac{4}{3}\xi^2 + \tfrac{40}{27}\xi^4 + O(\xi^6) \\
w_F &= \tfrac{2}{3} + \tfrac{2}{9}\xi^2 + O(\xi^4) \\
A_F &= 1 - \tfrac{4}{9}\xi^2 - \tfrac{8}{27}\xi^4 + O(\xi^6) \\
D_F &= 1 - \tfrac{1}{9}\xi^2 + O(\xi^4)
\end{aligned}
$$

Our Ansatz for Corrections to the Standard Model

$$z(t,\xi) = z_F(\xi) + \Delta z(t,\xi)$$
$$w(t,\xi) = w_F(\xi) + \Delta w(t,\xi)$$
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Our Ansatz for Corrections to the Standard Model

$$z(t,\xi) = z_F(\xi) + \Delta z(t,\xi) \qquad \Delta z = z_2(t)\xi^2 + z_4(t)\xi^4$$

$$w(t,\xi) = w_F(\xi) + \Delta w(t,\xi) \qquad \Delta w = w_0(t) + w_2(t)\xi^2$$

$$A(t,\xi) = A_F(\xi) + \Delta A(t,\xi) \qquad \Delta A = A_2(t)\xi^2 + A_4(t)\xi^4$$

$$D(t,\xi) = D_F(\xi) + \Delta D(t,\xi) \qquad \Delta D = D_2(t)\xi^2$$

- Note: Corrections only involve even powers of ξ
- The Standard Model is Self-Similar:

$$z_F = \frac{4}{3}\xi^2 + \frac{40}{27}\xi^4 + O(\xi^6)$$

$$w_F = \frac{2}{3} + \frac{2}{9}\xi^2 + O(\xi^4)$$

$$A_F = 1 - \frac{4}{9}\xi^2 - \frac{8}{27}\xi^4 + O(\xi^6)$$

$$D_F = 1 - \frac{1}{9}\xi^2 + O(\xi^4)$$

Our Ansatz for Corrections to the Standard Model

$$\begin{aligned} z(t,\xi) &= \left(\frac{4}{3} + z_2(t)\right)\xi^2 + \left\{\frac{40}{27} + z_4(t)\right\}\xi^4 + O(\xi^6), \\ w(t,\xi) &= \left(\frac{2}{3} + w_0(t)\right) + \left\{\frac{2}{9} + w_2(t)\right\}\xi^2 + O(\xi^4), \end{aligned}$$

- Reiterate:

We don't use co-moving coordinates,

but rather write the SSC eqns in (t, ξ)-coordinates.

$$ds^2 = -B(t,r)dt^2 + \frac{1}{A(t,r)}dr^2 + r^2 d\Omega^2$$

$$\xi = r/t \qquad D = \sqrt{AB}$$

Equations for the Corrections to SM

- When we plug into the equations a remarkable simplification occurs:

$$\boxed{A_2 = -\frac{1}{3} z_2, \quad A_4 = -\frac{1}{5} z_4, \quad D_2 = -\frac{1}{12} z_2}$$

Equations for the Corrections to SM

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$$\boxed{A_2 = -\frac{1}{3} z_2, \quad A_4 = -\frac{1}{5} z_4, \quad D_2 = -\frac{1}{12} z_2}$$

- This is a coordinate gauge condition reflecting the serendipity of our (t, ξ)-coordinate system!!

Plugging Ansatz into Equations...

Plugging

$$\boxed{A_2 = -\frac{1}{3} z_2, \quad A_4 = -\frac{1}{5} z_4, \quad D_2 = -\frac{1}{12} z_2}$$

and

$$\begin{aligned}
z(t,\xi) &= z_F(\xi) + z_2(t)\xi^2 + z_4(t)\xi^4 \\
w(t,\xi) &= w_F(\xi) + w_0(t) + w_2(t)\xi^2 \\
A(t,\xi) &= A_F(\xi) + A_2(t)\xi^2 + A_4(t)\xi^4 \\
D(t,\xi) &= D_F(\xi) + D_2(\xi)\xi^2
\end{aligned}$$

into equations:

$$tz_t + \xi\left\{(-1+Dw)z\right\}_\xi = -Dwz$$

$$tw_t + \xi\left(-1+Dw\right)w_\xi = w - D\left\{w^2 + \tfrac{1-\xi^2 w^2}{2A}\left[\tfrac{1-A}{\xi^2}\right]\right\}$$

Gives:

THEOREM: The $p = 0$ waves take the asymptotic form

$$\begin{aligned} z(t,\xi) &= \left(\frac{4}{3} + z_2(t)\right)\xi^2 + \left\{\frac{40}{27} + z_4(t)\right\}\xi^4 + O(\xi^6), \\ w(t,\xi) &= \left(\frac{2}{3} + w_0(t)\right) + \left\{\frac{2}{9} + w_2(t)\right\}\xi^2 + O(\xi^4), \end{aligned}$$

where $z_2(t), z_4(t), w_0(t), w_2(t)$ evolve according to the equations

$$\begin{aligned} -t\dot{z}_2 &= 3w_0\left(\frac{4}{3} + z_2\right), \\ -t\dot{z}_4 &= -5\left\{\frac{2}{27}z_2 + \frac{4}{3}w_2 - \frac{1}{18}z_2^2 + z_2w_2\right\} \\ &\qquad -5w_0\left\{\frac{4}{3} - \frac{2}{9}z_2 + z_4 - \frac{1}{12}z_2^2\right\}, \\ -t\dot{w}_0 &= \frac{1}{6}z_2 + \frac{1}{3}w_0 + w_0^2, \\ -t\dot{w}_2 &= \frac{1}{10}z_4 + \frac{4}{9}w_0 - \frac{1}{3}w_2 + \frac{1}{24}z_2^2 - \frac{1}{3}z_2w_0 \\ &\qquad -\frac{1}{3}w_0^2 + 4w_0w_2 - \frac{1}{4}w_0^2z_2. \end{aligned}$$

The Corrections to SM evolve according to

$$
\begin{aligned}
-t\dot{z}_2 &= 3w_0\left(\frac{4}{3}+z_2\right), \\
-t\dot{z}_4 &= -5\left\{\frac{2}{27}z_2+\frac{4}{3}w_2-\frac{1}{18}z_2^2+z_2w_2\right\} \\
&\qquad -5w_0\left\{\frac{4}{3}-\frac{2}{9}z_2+z_4-\frac{1}{12}z_2^2\right\}; \\
-t\dot{w}_0 &= \frac{1}{6}z_2+\frac{1}{3}w_0+w_0^2, \\
-t\dot{w}_2 &= \frac{1}{10}z_4+\frac{4}{9}w_0-\frac{1}{3}w_2+\frac{1}{24}z_2^2-\frac{1}{3}z_2w_0 \\
&\qquad -\frac{1}{3}w_0^2+4w_0w_2-\frac{1}{4}w_0^2z_2,
\end{aligned}
$$

- Note: RHS is Autonomous!

We can make LHS Automomous too!

$$
\begin{aligned}
-z_2' &= -t\dot{z}_2 = 3w_0\left(\frac{4}{3}+z_2\right), \\
-z_4' &= -t\dot{z}_4 = -5\left\{\frac{2}{27}z_2+\frac{4}{3}w_2-\frac{1}{18}z_2^2+z_2w_2\right\} \\
&\qquad -5w_0\left\{\frac{4}{3}-\frac{2}{9}z_2+z_4-\frac{1}{12}z_2^2\right\}; \\
-w_0' &= -t\dot{w}_0 = \frac{1}{6}z_2+\frac{1}{3}w_0+w_0^2, \\
-w_2' &= -t\dot{w}_2 = \frac{1}{10}z_4+\frac{4}{9}w_0-\frac{1}{3}w_2+\frac{1}{24}z_2^2-\frac{1}{3}z_2w_0 \\
&\qquad -\frac{1}{3}w_0^2+4w_0w_2-\frac{1}{4}w_0^2z_2.
\end{aligned}
$$

$$\tau = ln(t) \Rightarrow t\frac{d}{dt} = \frac{d}{d\tau} \equiv {}'$$

➡ LHS Autonomous

Autonomous Eqns for Corrections to SM

$$
\begin{aligned}
-z_2' &= 3w_0\left(\frac{4}{3}+z_2\right), \\
-z_4' &= -5\left\{\frac{2}{27}z_2+\frac{4}{3}w_2-\frac{1}{18}z_2^2+z_2w_2\right\} \\
&\qquad -5w_0\left\{\frac{4}{3}-\frac{2}{9}z_2+z_4-\frac{1}{12}z_2^2\right\}, \\
-w_0' &= \frac{1}{6}z_2+\frac{1}{3}w_0+w_0^2, \\
-w_2' &= \frac{1}{10}z_4+\frac{4}{9}w_0-\frac{1}{3}w_2+\frac{1}{24}z_2^2-\frac{1}{3}z_2w_0 \\
&\qquad -\frac{1}{3}w_0^2+4w_0w_2-\frac{1}{4}w_0^2z_2.
\end{aligned}
$$

$$\mathrm{t}_* \le t \le 10^{14}\, yr$$

$$\ln(\mathrm{t}_*) \le \tau \le 14\cdot\ln(10)$$

Trivializes the large time simulation problem!

The Equations for the Corrections

$$
\begin{aligned}
-z_2' &= 3w_0\left(\frac{4}{3}+z_2\right), \\
-z_4' &= -5\left\{\frac{2}{27}z_2+\frac{4}{3}w_2-\frac{1}{18}z_2^2+z_2w_2\right\} \\
&\qquad -5w_0\left\{\frac{4}{3}-\frac{2}{9}z_2+z_4-\frac{1}{12}z_2^2\right\}; \\
-w_0' &= \frac{1}{6}z_2+\frac{1}{3}w_0+w_0^2, \\
-w_2' &= \frac{1}{10}z_4+\frac{4}{9}w_0-\frac{1}{3}w_2+\frac{1}{24}z_2^2-\frac{1}{3}z_2w_0 \\
&\qquad -\frac{1}{3}w_0^2+4w_0w_2-\frac{1}{4}w_0^2z_2.
\end{aligned}
$$

Everything is dimensionless involving only pure numbers!

The Equations for the Corrections

Leading order (z_2, w_0)

$$\begin{aligned}
-z_2' &= 3w_0\left(\frac{4}{3}+z_2\right), \\
-z_4' &= -5\left\{\frac{2}{27}z_2+\frac{4}{3}w_2-\frac{1}{18}z_2^2+z_2w_2\right\} \\
&\qquad -5w_0\left\{\frac{4}{3}-\frac{2}{9}z_2+z_4-\frac{1}{12}z_2^2\right\}, \\
-w_0' &= \frac{1}{6}z_2+\frac{1}{3}w_0+w_0^2, \\
-w_2' &= \frac{1}{10}z_4+\frac{4}{9}w_0-\frac{1}{3}w_2+\frac{1}{24}z_2^2-\frac{1}{3}z_2w_0 \\
&\qquad -\frac{1}{3}w_0^2+4w_0w_2-\frac{1}{4}w_0^2z_2.
\end{aligned}$$

- Note: Leading order Eqns Uncouple!

The Leading Order Corrections...

$$
\begin{aligned}
z(t,\xi) &= \left(\frac{4}{3} + z_2(t)\right)\xi^2 + O(\xi^4), \\
w(t,\xi) &= \left(\frac{2}{3} + w_0(t)\right) + O(\xi^2),
\end{aligned}
$$

...And Their Equations

$$
\begin{aligned}
-z_2' &= -t\dot{z}_2 = 3w_0\left(\frac{4}{3} + z_2\right), \\
-w_0' &= -t\dot{w}_0 = \frac{1}{6}z_2 + \frac{1}{3}w_0 + w_0^2.
\end{aligned}
$$

- Keep in mind that ξ is on the order of fractional distance to the Hubble Length:

$$\xi = r/ct \approx \frac{\text{arclength distance at fixed time}}{\text{distance of light travel since Big Bang}}$$

- For example: At 1/10 way across the visible universe, about 1.1 billion light-years out:

$$\xi^4 \approx \frac{1}{10{,}000} = .0001$$

Hubbles Law:

$$\mathrm{H}_0\, d_\ell = z$$

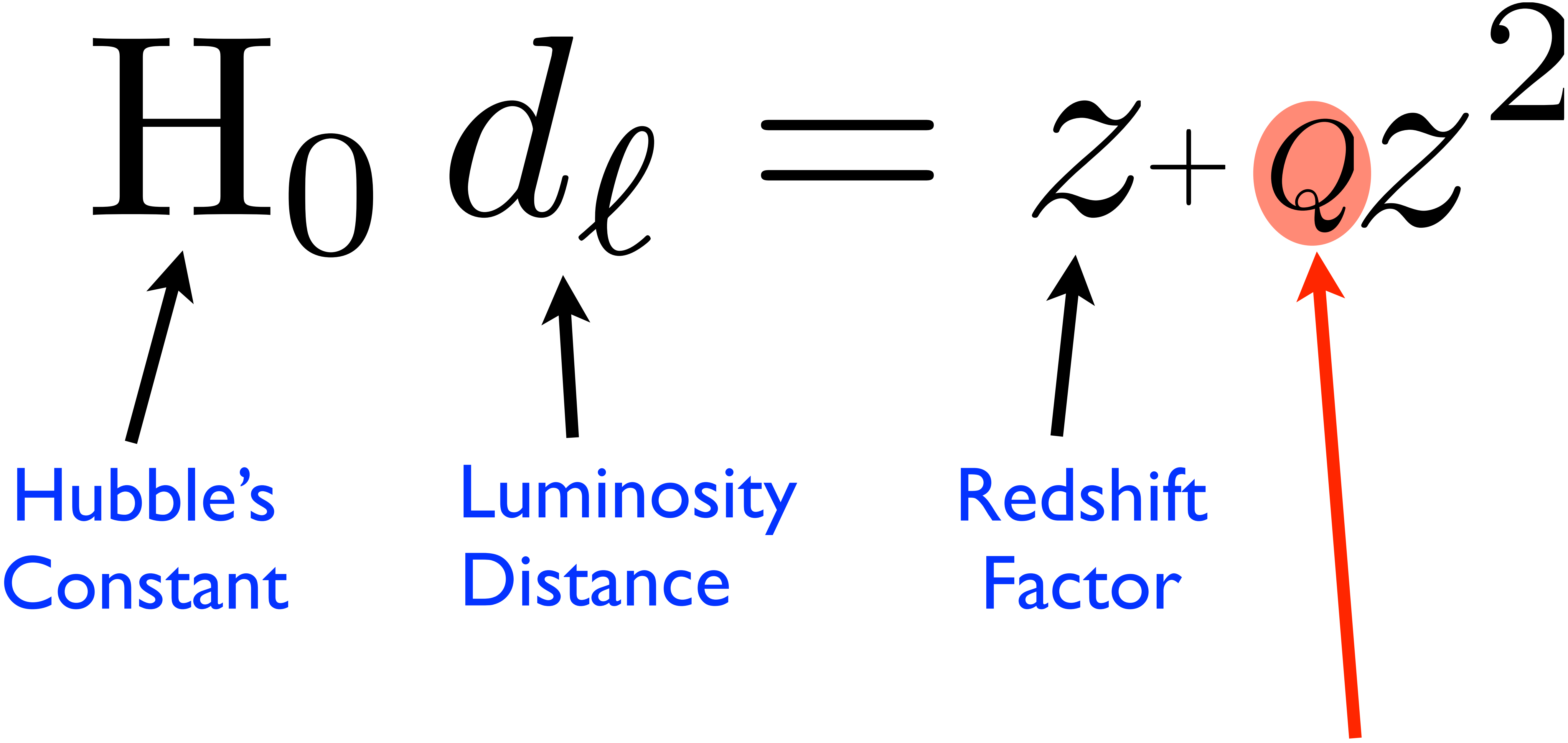
Hubble's Constant

Luminosity Distance

Redshift Factor

1929: Linear relation between redshift and luminosity

Hubbles Law:



$$\mathrm{H}_0\, d_\ell = z + Q z^2$$

Hubble's Constant

Luminosity Distance

Redshift Factor

1999: There is an anomalous acceleration

- In Fact: ξ is on the order of the redshift factor, and (z_2, w_0) determines the quadratic correction to redshift vs luminosity =anomalous acceleration

$$H_0 d_\ell = z + Q(z_2, w_0)\, z^2 + O(z^3)$$

This term accounts for the corrections to the Standard Model Observed in the Supernova Data (Nobel Prize)

- In Fact: ξ is on the order of the redshift factor, and (z_2, w_0) determines the quadratic correction to redshift vs luminosity =anomalous acceleration

$$H_0 d_\ell = z + Q(z_2, w_0)\, z^2 + O(z^3)$$

Determined by the value
of the so-called
"Deceleration Parameter" q

- The cubic correction is determined by (z_2, w_0, w_2)

$$H_0 d_\ell = z + Q(z_2, w_0) z^2 + \boxed{C(z_2, w_0, w_2)}\, z^3 + O(z^3)$$

Determined by solving
our system of four equations
for (z_2, z_4, w_0, w_4)

- The cubic correction is determined by (z_2, w_0, w_2)

$$H_0 d_\ell = z + Q(z_2, w_0) z^2 + \boxed{C(z_2, w_0, w_2)}\, z^3$$

$$+O(z^3)$$

A prediction

Beyond experimental precision

- The quadratic correction is determined by our equations for (z_2, w_0)

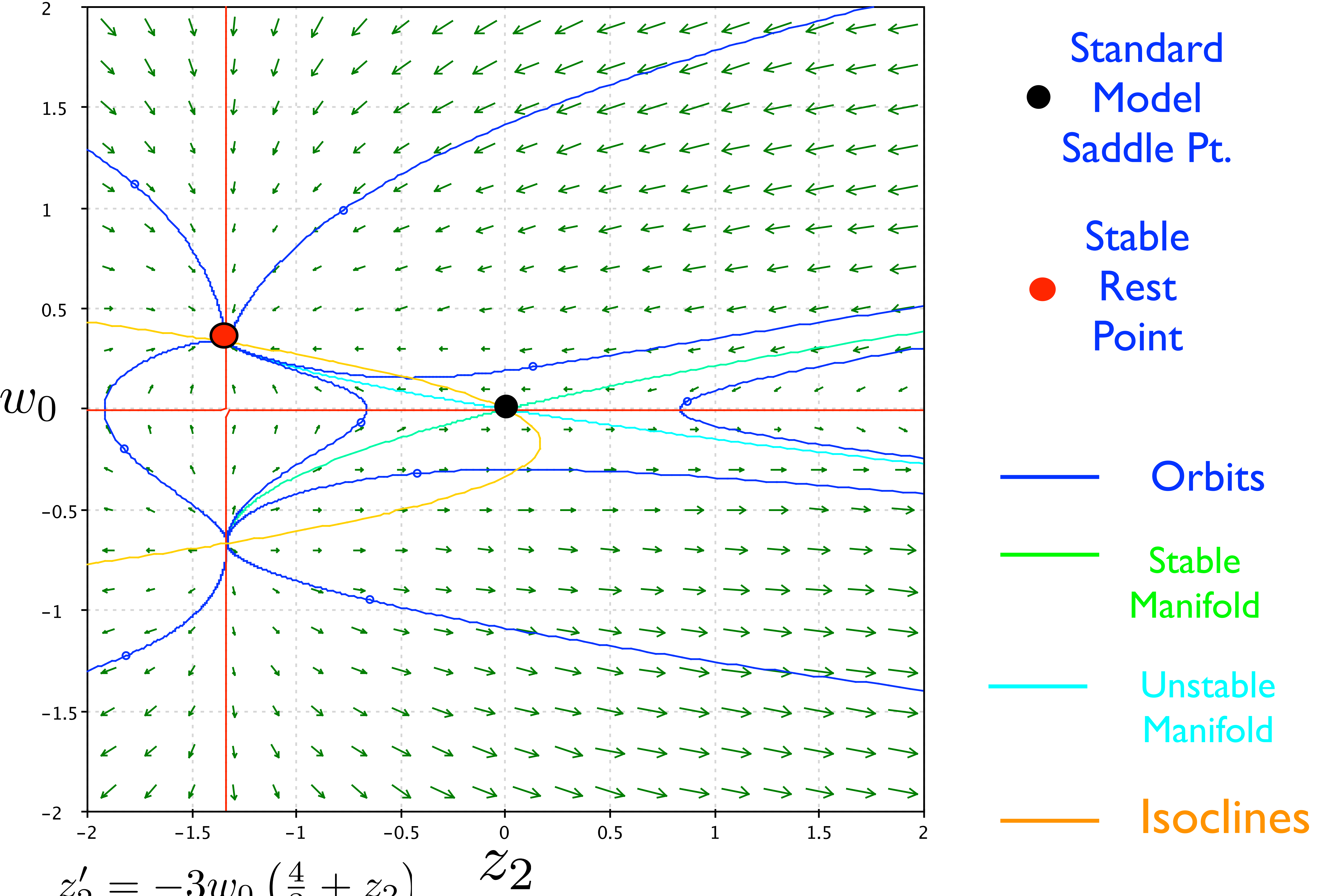
$$H_0 d_\ell = z + Q(z_2, w_0)\, z^2 + O(z^3)$$

$$\begin{aligned} -z_2' &= -t\dot{z}_2 = 3w_0\left(\frac{4}{3} + z_2\right), \\ -w_0' &= -t\dot{w}_0 = \frac{1}{6}z_2 + \frac{1}{3}w_0 + w_0^2. \end{aligned}$$

Numerical Simulation

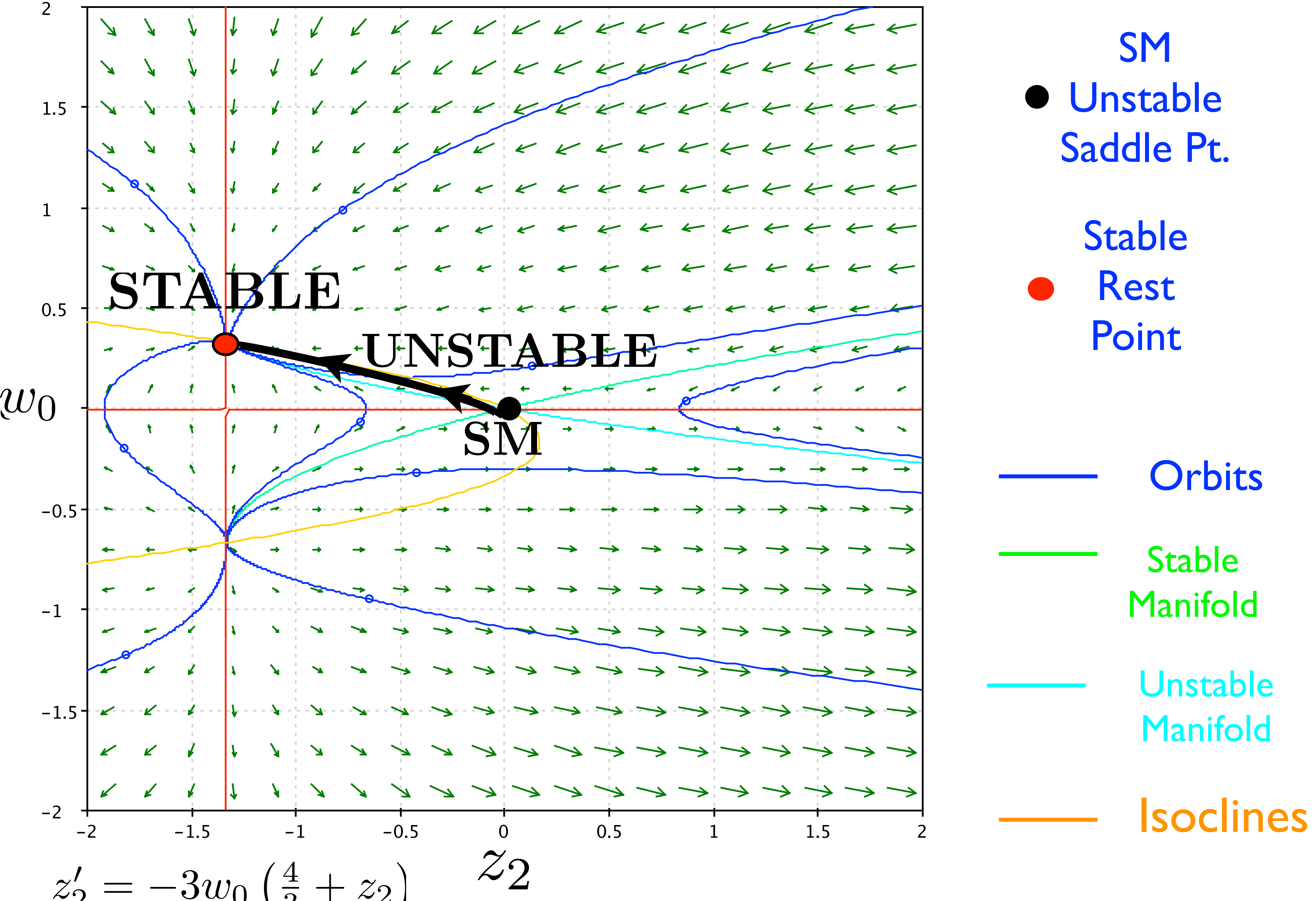
The (z_2, w_0) phase portrait:

Thanks to: *pplane* Rice University



$$z_2' = -3w_0\left(\tfrac{4}{3} + z_2\right)$$

$$w_0' = -\left(\tfrac{1}{6}z_2 + \tfrac{1}{3}w_0 + w_0^2\right)$$



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$$w_0' = -\left(\frac{1}{6}z_2 + \frac{1}{3}w_0 + w_0^2\right)$$

3. The Initial Data determined by the Self-Similar Waves from the Radiation Epoch

A SSC Self-Similar Formulation of the k=0 Friedmann Spacetimes when

$$p = \sigma^2 \rho:$$

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FRW Co-moving: $ds^2 = -dt^2 + R(t)^2 \left\{dr^2 + r^2 d\Omega^2\right\}$

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FRW Self-Similar:
$$\bar{t} = F(\eta) t; \quad \bar{r} = \eta t,$$

$$\xi \equiv \frac{\bar{r}}{\bar{t}} = \frac{\eta}{F(\eta)}; \qquad \eta \equiv \frac{\bar{r}}{t}; \quad F(\eta) = \left(1 - \frac{1-3\sigma}{9(1+\sigma)^2}\eta^2\right)^{\frac{3(1+\sigma)}{2(1+3\sigma)}}$$

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FRW Self-Similar: $$\bar{t} = F(\eta) t; \quad \bar{r} = \eta t,$$

$$ds^2 = -\frac{F(\eta)^{-\frac{1+3\sigma}{3(1+\sigma)}}}{1 - \left(\frac{2}{3(1+\sigma)\eta^2}\right)^2} d\bar{t}^2 + \frac{1}{1 - \left(\frac{2}{3(1+\sigma)\eta^2}\right)^2} d\bar{r}^2 + \bar{r}^2 d\Omega^2$$

$$\xi \equiv \frac{\bar{r}}{\bar{t}} = \frac{\eta}{F(\eta)}; \quad \eta \equiv \frac{\bar{r}}{t}; \quad F(\eta) = \left(1 - \frac{1 - 3\sigma}{9(1+\sigma)^2} \eta^2\right)^{\frac{3(1+\sigma)}{2(1+3\sigma)}}$$

$$\sigma = 0$$

$$p = 0$$

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$$ds^2 = -B_F(\xi)d\bar{t}^2 + \frac{1}{A_F(\xi)}d\bar{r}^2 + \bar{r}^2 d\Omega^2$$

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$$p = 0$$

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$$A_F(\xi) = 1 - \tfrac{4}{9}\xi^2 - \tfrac{8}{27}\xi^4 + O(\xi^6)$$
$$D_F(\xi) \equiv \sqrt{A_F B_F} = 1 - \frac{1}{9}\xi^2 + O(\xi^4).$$

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- Note: $$\xi = \frac{\bar{r}}{\bar{t}} = \frac{\bar{r}}{ct} + O(\xi^2)$$

$$\begin{aligned}\sigma &= 0\\ p &= 0\end{aligned}$$

$$ds^2 = -B_F(\xi)d\bar{t}^2 + \frac{1}{A_F(\xi)}d\bar{r}^2 + \bar{r}^2 d\Omega^2$$

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- Note: $$\xi = \frac{\bar{r}}{\bar{t}} = \frac{\bar{r}}{ct} + O(\xi^2)$$

Where: $$\frac{\bar{r}}{ct} = \frac{\text{arclength distance at fixed time}}{\text{distance of light travel since Big Bang}}$$

in co-moving coordinates

$$\begin{matrix} \sigma = 0 \\ p = 0 \end{matrix}$$

$$ds^2 = -B_F(\xi)d\bar{t}^2 + \frac{1}{A_F(\xi)}d\bar{r}^2 + \bar{r}^2 d\Omega^2$$

$$A_F(\xi) = 1 - \tfrac{4}{9}\xi^2 - \tfrac{8}{27}\xi^4 + O(\xi^6)$$

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Where: $$\frac{\bar{r}}{ct} = \frac{\text{arclength distance at fixed time}}{\text{distance of light travel since Big Bang}}$$

in co-moving coordinates

- Conclude: $\xi \approx$ fractional distance to Hubble Length

$$\sigma = 0$$
$$p = 0$$

$$ds^2 = -B_F(\xi)d\bar{t}^2 + \frac{1}{A_F(\xi)}d\bar{r}^2 + \bar{r}^2 d\Omega^2$$

$$A_F(\xi) = 1 - \tfrac{4}{9}\xi^2 - \tfrac{8}{27}\xi^4 + O(\xi^6)$$
$$D_F(\xi) \equiv \sqrt{A_F B_F} = 1 - \frac{1}{9}\xi^2 + O(\xi^4).$$

$$z_F(\xi) = \frac{4}{3}\xi^2 + \frac{40}{27}\xi^4 + O(\xi^6)$$
$$w_F \equiv \frac{v}{\xi} = \frac{2}{3} + \frac{2}{9}\xi^2 + O(\xi^4)$$

The p=0 Friedmann Universe in Self-Similar Coordinates

Thus our equations are for the corrections to the Standard Model:

$$\begin{aligned} z(t,\xi) &= \left(\frac{4}{3}+z_2(t)\right)\xi^2+\left\{\frac{40}{27}+z_4(t)\right\}\xi^4+O(\xi^6), \\ w(t,\xi) &= \left(\frac{2}{3}+w_0(t)\right)+\left\{\frac{2}{9}+w_2(t)\right\}\xi^2+O(\xi^4) \end{aligned}$$

$$(p=0)$$

$$p = \frac{c^2}{3}\rho$$

$$\sigma = \frac{1}{\sqrt{3}}$$

Self-similar coordinates for Friedmann with Pure Radiation

$$\bar{\xi} \neq \xi$$

$$\begin{aligned}
z_{1/3} &\equiv z^1_{1/3}(\bar{t},\bar{\xi}) = \frac{3}{4}\bar{\xi}^2 + \frac{9}{16}\bar{\xi}^4 + O(\bar{\xi}^6), \\
v_{1/3} &\equiv v^1_{1/3}(\bar{t},\bar{\xi}) = \frac{1}{2}\bar{\xi} + \frac{1}{8}\bar{\xi}^3 + O(\bar{\xi}^5), \\
A_{1/3} &\equiv A^1_{1/3}(\bar{t},\bar{\xi}) = 1 - \frac{1}{4}\bar{\xi}^2 - \frac{1}{8}\bar{\xi}^4 + O(\bar{\xi}^6), \\
D_{1/3} &\equiv D^1_{1/3}(\bar{t},\bar{\xi}) = 1 + O(\bar{\xi}^4).
\end{aligned}$$

The $p = \frac{c^2}{3}\rho$ Friedmann Universe admits a 1-parameter family of Self-Similar spacetimes that perturb the Standard Model during the Radiation Epoch:

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The $p = 0$ Friedmann Universe DOES NOT admit Self-Similar perturbations!

The $p = \frac{c^2}{3}\rho$ Friedmann Universe is embedded in 1-parameter family of Self-Similar spacetimes that perturb the Standard Model during the Radiation Epoch:

The $p = 0$ Friedmann Universe DOES NOT admit Self-Similar perturbations!

(The topic of our PNAS and MEMOIR)

First Discovered by Cahill and Taub:

Commun Math Phys., 21, 1-40 (1971)

Extended by others, esp. Carr and Coley, Survey:

Physical Review D, 62, 044023-1-25 (1999)

Our interest is in the possible connection between these waves and the Anomalous Acceleration.

We extract properties of the waves from a system of ODE's we derived, that defines them:

The perturbations are describe by ODE’s:

$$
\begin{aligned}
\xi A_\xi &= -\left[\frac{4(1-A)v}{(3+v^2)G-4v}\right] \\
\xi G_\xi &= -G\left\{\left(\frac{1-A}{A}\right)\frac{2(1+v^2)G-4v}{(3+v^2)G-4v}-1\right\} \\
\xi v_\xi &= -\left(\frac{1-v^2}{2\left\{\cdot\right\}_D}\right)\left\{(3+v^2)G-4v+\frac{4\left(\frac{1-A}{A}\right)\left\{\cdot\right\}_N}{(3+v^2)G-4v}\right\}
\end{aligned}
$$

$$
\begin{aligned}
\left\{\cdot\right\}_N &= \left\{-2v^2+2(3-v^2)vG-(3-v^4)G^2\right\} \\
\left\{\cdot\right\}_D &= \left\{(3v^2-1)-4vG+(3-v^2)G^2\right\}
\end{aligned}
$$

$$\boxed{G=\frac{\xi}{\sqrt{AB}} \quad ; \quad \xi=\frac{r}{t}}$$

$$p = \frac{c^2}{3}\rho$$

$$\sigma = \frac{1}{\sqrt{3}}$$

Self-Similar perturbations of Friedmann for Pure Radiation

(The topic of our PNAS and MEMOIR)

$$z^a_{1/3} = \frac{3a^2}{4}\bar{\xi}^2 + \frac{3a^2(2+a^2)}{16}\bar{\xi}^4 + O(\bar{\xi}^6)$$

$$v^a_{1/3} = \frac{1}{2}\bar{\xi} + \frac{2-a^2}{8}\bar{\xi}^3 + O(\bar{\xi}^5)$$

$$A^a_{1/3} = 1 - \frac{a^2}{4}\bar{\xi}^2 + \frac{a^2(1-3a^2)}{16}\bar{\xi}^4 + O(\bar{\xi}^6)$$

$$D^a_{1/3} = 1 + O(\bar{\xi}^4)$$

A 1-parameter family of solutions depending on the Acceleration Parameter $0 < a < \infty$

$$z^a_{1/3} = \frac{3a^2}{4}\bar{\xi}^2 + \frac{3a^2(2+a^2)}{16}\bar{\xi}^4 + O(\bar{\xi}^6)$$

$$v^a_{1/3} = \frac{1}{2}\bar{\xi} + \frac{2-a^2}{8}\bar{\xi}^3 + O(\bar{\xi}^5)$$

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$$D^a_{1/3} = 1 + O(\bar{\xi}^4)$$

$a = 1$ is the Standard Model for Pure Radiation

$$z^a_{1/3} = \frac{3a^2}{4}\bar{\xi}^2 + \frac{3a^2(2+a^2)}{16}\bar{\xi}^4 + O(\bar{\xi}^6)$$

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The initial data created by self-similar waves at the end of the Radiation Epoch depends on:

(1) The temperature T_* at which $p = 0$

(2) The value of the acceleration parameter a

OUR GOAL NOW: Use our equations to evolve the initial data at the end of radiation to determine

$$(a, T_*)$$

that gives the correct anomalous acceleration.

I.e., (a, T_*) that give the observed quadratic correction to redshift vs luminosity at present time

- In the Standard Model p=0 at about

$t_* \approx$ 10,000-30,000 yrs

$T_* \approx 9000^0 K$

(Depending on theories of Dark Matter)

- Our simulation turns out to be entirely insensitive to the initial t_*, T_*

- I.e., we need only compute the value of the acceleration parameter that gives the correct anomalous acceleration.

- Technical Problem: The self-similar waves at the end of radiation are in the wrong gauge due to the fact that time since the Big Bang changes between $p = 0$ and $p = \frac{c^2}{3}\rho$

- That is: The initial data for the self-similar waves does not meet the gauge conditions for our p=0 ansatz

$$A_2 = -\frac{1}{3}z_2, \quad A_4 = -\frac{1}{5}z_4, \quad D_2 = -\frac{1}{12}z_2$$

- (Resolving this held us back for close to a year!)

- Resolution: We post-process the initial data by a gauge transformation of the form---

$$t = \bar{t} + \frac{1}{2} q (\bar{t} - \bar{t}_*)^2 - t_B$$

- That is: The initial data for the self-similar waves does not meet the gauge conditions for our p=0 ansatz

$$A_2 = -\frac{1}{3} z_2, \quad A_4 = -\frac{1}{5} z_4, \quad D_2 = -\frac{1}{12} z_2$$

- (Resolving this held us back for close to a year!)

THEOREM: Let the transformation $\bar{t} \to t$ be defined by

$$t = \bar{t} + \frac{1}{2}q(\bar{t} - \bar{t}_*)^2 - t_B,$$

where q and t_B are given by

$$t_B = \bar{t}_*(1 - \alpha),$$

$$q = \frac{a^2}{16\bar{\gamma}} = \frac{a^2}{2(1 + a^2)},$$

where

$$\alpha = \frac{1}{5}\left(\frac{1 + a^2}{1.3 - a^2}\right).$$

Then, on the constant temperature surface $T = T_*$, the initial data from the self-similar waves at the end of the radiation epoch meets the gauge conditions in $(\bar{t}, \bar{\xi})$.

- **2nd Technical Problem**: The $T = T_*,\ \rho = \rho_*$ surfaces are distinct from the constant time $t = t_*$ surfaces

- **Resolution**: To get the asymptotics correct we have to pull the initial data back to

$$t = t_*$$

The initial data created by self-similar waves on a constant temperature surface at the end of the Radiation Epoch

THEOREM: The initial data for oiur $p = 0$ evolution at time $t = t_*$ is given as a function of the acceleration parameter a and start temperature $\rho_* = \mathrm{a}T_*^4$ by

$$\begin{aligned} z_2(t_*) &= \hat{z}_2, \\ z_4(t_*) &= \hat{z}_4 + 3\hat{w}_0\left(\frac{4}{3} + \hat{z}_2\right)\gamma, \\ w_0(t_*) &= \hat{w}_0, \\ w_2(t_*) &= \hat{w}_2 + \left(\frac{1}{6}\hat{z}_2 + \frac{1}{3}\hat{w}_0 + \hat{w}_0^2\right)\gamma, \end{aligned}$$

$$\begin{aligned} \hat{z}_2 &= \left\{\frac{3a^2\alpha^2}{4} - \frac{4}{3}\right\}_{z2} \\ \hat{z}_4 &= \left\{\frac{15a^2(\frac{3}{2} - a^2)\alpha^4}{16} - \frac{40}{27}\right\}_{z4} \\ \hat{w}_0 &= \left\{\frac{\alpha}{2} - \frac{2}{3}\right\}_{v1} \\ \hat{w}_2 &= \left\{\frac{\alpha^3}{16}(9.5 - 8a^2) - \frac{2}{9}\right\}_{v3} \end{aligned}$$

where

$$t_* = \alpha\hat{t}_* = \frac{a\alpha}{2}\sqrt{\frac{3}{\kappa\rho_*}}, \qquad \gamma = \alpha\bar{\gamma} = \frac{(1+a^2)\alpha}{8}, \qquad \alpha = \frac{(1+a^2)}{5(1.3-a^2)}$$

4. Redshift vs Luminosity as a function of our corrections

A (long) Calculation gives:

$$H_0 d_\ell = z\left\{1 + \underbrace{\left[\frac{1}{4} + E_2\right]}_{\text{Anomalous Acceleration}} z + \underbrace{\left[-\frac{1}{8} + E_3\right]}_{\text{Cubic Correction}} z^2\right\} + O(z^4)$$

$$E_2 = \frac{24w_0 + 45w_0^2 + 3z_2}{4(2+3w_0)^2} = E_2(z_2, w_0),$$

$$E_3 = E_3(z_2, w_0, w_3)$$

$E_3(z_2, w_0, w_2)$ is quite complicated:

$$H_0 d_\ell = z \left\{ 1 + \left[\frac{1}{4} + E_2 \right] z + \underbrace{\left[-\frac{1}{8} + E_3 \right]}_{\text{Cubic Correction}} z^2 \right\} + O(z^4)$$

A calculation gives:

$$E_3 = 2I_2 + I_3,$$

$$I_2 = H_2 + \frac{9w_0}{2(2+3w_0)}$$

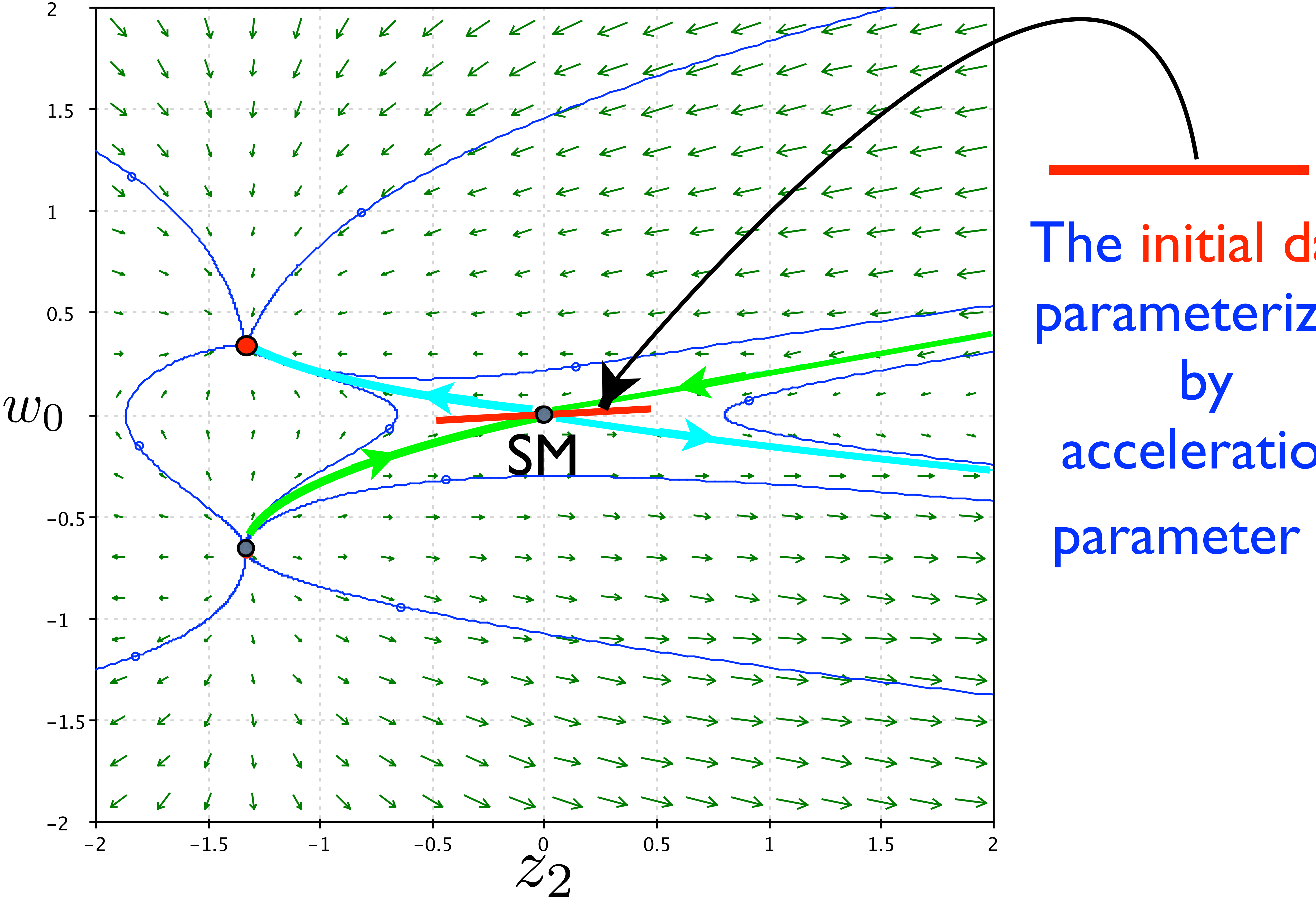
$$I_3 = H_3 + 3\left[-1 + \left(\frac{8 - 8H_2 + 3w_0 - 12H_2 w_0}{2(2+3w_0)^2}\right)\right],$$

$$H_2 = \frac{1}{4}\left\{1 - \frac{1 + 9\left(\frac{2}{3}w_0 + \frac{1}{2}w_0^2 - \frac{1}{12}z_2\right)}{(1 + \frac{3}{2}w_0)^2}\right\},$$

$$H_3 = \frac{5}{8}\left\{1 - \frac{1 - \frac{18}{5}Q_2 - \frac{81}{5}Q_2^2 + \frac{9}{5}w_0 + \frac{27}{5}Q_3 + \frac{81}{10}Q_3 w_0}{\left(1 + \frac{3}{2}w_0\right)^4}\right\}$$

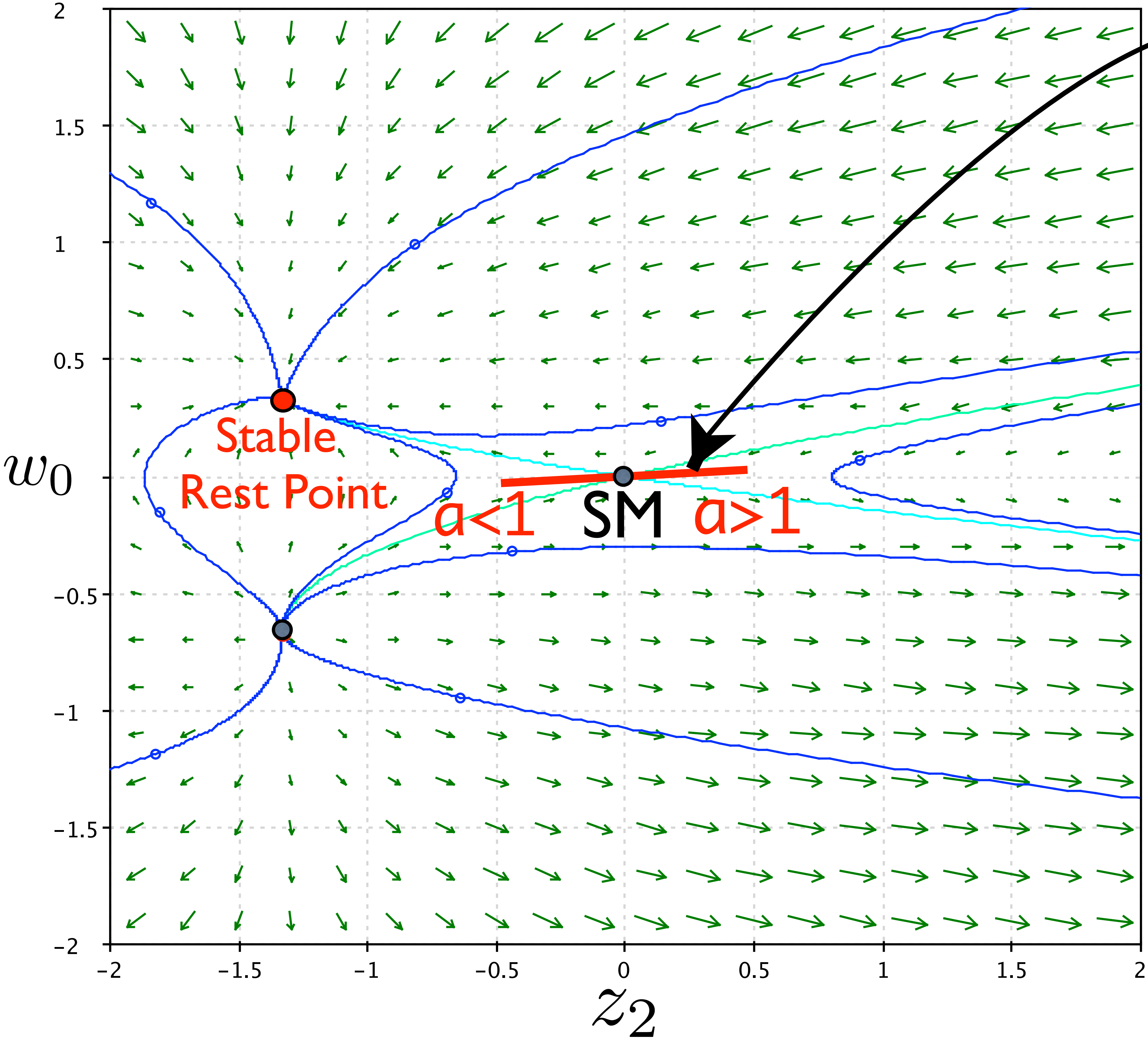
$$\begin{aligned} Q_2 &= \frac{2}{3}w_0 + \frac{1}{2}w_0^2 - \frac{1}{12}z_2 \\ Q_3 &= \frac{2}{9}w_0 + w_0^2 + \frac{1}{2}w_0^3 + w_2 - \frac{1}{18}z_2 - \frac{1}{3}z_2 w_0 \end{aligned}$$

(Each term represents a different effect...)



The initial data parameterized by acceleration parameter a

The initial data cuts between the stable and unstable manifold of SM



The initial data parameterized by acceleration parameter a

Under-densities a<1 are within the domain of attraction of the Stable Rest Point

3. Comparison with the Standard Model

- Redshift vs Luminosity for k=0 Friedmann can be obtained from exact formulas: $p = \sigma\rho$

$$H_0 d_\ell = \frac{2}{1+3\sigma}\left\{(1+z) - (1+z)^{\frac{1-3\sigma}{2}}\right\}.$$

- In the case $p = \sigma = 0$, we get

$$H_0 d_\ell = z + \frac{1}{4}z^2 - \frac{1}{8}z^3 + O(z^4)$$

- C.f. our formula:

$$H_0 d_\ell = z\left\{1 + \left[\frac{1}{4} + E_2\right]z + \left[-\frac{1}{8} + E_3\right]z^2\right\} + O(z^4)$$

Cosmology now assumes a Cosmological Constant with Seventy Percent Dark Energy

$$H_0 d_\ell = (1+z)\int_0^z \frac{dy}{(1+z)\sqrt{1+\Omega_M y}}. \qquad \Omega_M + \Omega_\Lambda = 1$$

Taylor expanding gives:

$$H_0 d_\ell = z + \frac{1}{2}\left(-\frac{\Omega_M}{2}+1\right)z^2 + \frac{1}{6}\left(-1-\frac{\Omega_M}{2}+\frac{3\Omega_M^2}{4}\right)z^3 + O(z^4)$$

In the case $\Omega_M = .3,\ \Omega_\Lambda = .7$ this gives

$$\boxed{H_0 d_\ell = z + .425\, z^2 - .1804\, z^3 + O(z^4)}$$

CONCLUDE: $k = 0$, $p = 0$ Friedmann with and without Dark Energy

$$\Omega_M + \Omega_\Lambda = 1$$

$$H_0 d_\ell = z + .425\, z^2 - .1804\, z^3 + O(z^4)$$

Standard Model
with
Dark Energy

$$\Omega_\Lambda = .7$$

The
Anomalous
Acceleration

$$H_0 d_\ell = z + .25\, z^2 - .125\, z^3 + O(z^4)$$

Standard Model
Without
Dark Energy

$$\Omega_\Lambda = 0$$

IN FACT: As the Dark Energy Parameter ranges from 0 to 1, the Anomalous Acceleration ranges from .25 to .5

$$H_0 d_\ell = z + \underbrace{\frac{1}{2}\left(-\frac{\Omega_M}{2}+1\right)} z^2 + \frac{1}{6}\left(-1-\frac{\Omega_M}{2}+\frac{3\Omega_M^2}{4}\right) z^3 + O(z^4)$$

Range: .25 to .5

as

$$0 \leq \Omega_M \leq 1$$

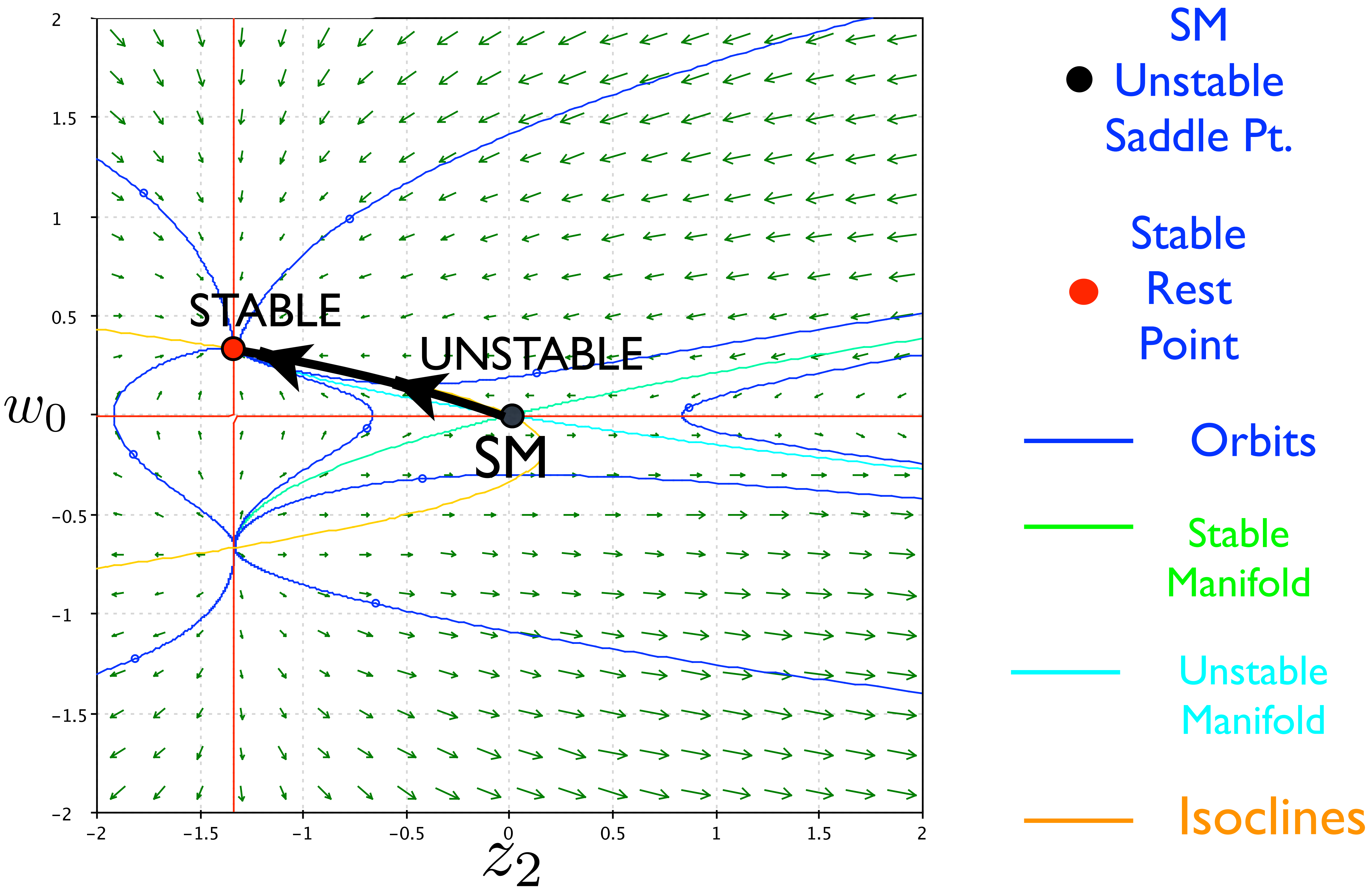
We get the Same Conclusion in the Wave Theory!

$$H_0 d_\ell = z\left\{1 + \underbrace{\left[\frac{1}{4} + E_2\right]}_{} z + \left[-\frac{1}{8} + E_3\right] z^2\right\} + O(z^4)$$

Range: .25 to .5

$$E_2 = \frac{24w_0 + 45w_0^2 + 3z_2}{4(2 + 3w_0)^2}$$

along the orbit
from the Standard Model
to the
Stable Rest Point



The Anomalous Acceleration ranges from .25 to .5 along orbit from SM to Stable Rest Point≈ *Dark Energy*

5. Determination of the value of the Acceleration Parameter that matches the Anomalous Acceleration

We simulate our equations starting from the self-similar wave data at the end of radiation $T = T_*$, to find the value of (a, T_*) that gives the same Anomalous Acceleration as seventy percent Dark Energy when $H = H_0$:

$$H_0 d_\ell = z + \underbrace{.425}\, z^2 - .1804\, z^3 + O(z^4)$$

Dark Energy
$\Omega_\Lambda = .7$

$$H_0 d_\ell = z + \overbrace{[.24 + E_2]}\, z^2 + [-.125 + E_3]\, z^3 + O(z^4)$$

Our Wave Model

$$\begin{aligned}
-z_2' &= -t\dot{z}_2 = 3w_0\left(\frac{4}{3} + z_2\right), \\
-z_4' &= -t\dot{z}_4 = -5\left\{\frac{2}{27}z_2 + \frac{4}{3}w_2 - \frac{1}{18}z_2^2 + z_2w_2\right\} \\
&\qquad -5w_0\left\{\frac{4}{3} - \frac{2}{9}z_2 + z_4 - \frac{1}{12}z_2^2\right\}, \\
-w_0' &= -t\dot{w}_0 = \frac{1}{6}z_2 + \frac{1}{3}w_0 + w_0^2, \\
-w_2' &= -t\dot{w}_2 = \frac{1}{10}z_4 + \frac{4}{9}w_0 - \frac{1}{3}w_2 + \frac{1}{24}z_2^2 - \frac{1}{3}z_2w_0 \\
&\qquad -\frac{1}{3}w_0^2 + 4w_0w_2 - \frac{1}{4}w_0^2z_2.
\end{aligned}$$

$$E_2 = \frac{24w_0 + 45w_0^2 + 3z_2}{4(2+3w_0)^2}$$

THE ANSWER: The value of the acceleration for the wave perturbation of SM that produces a quadradic correction of .425 at the present value of H_0 is:

- $$\underline{a} = 0.99999957 = 1 - \left(4.3 \times 10^{-7}\right)$$

$$H_0 d_\ell = z + .425z^2 + .3591z^3$$

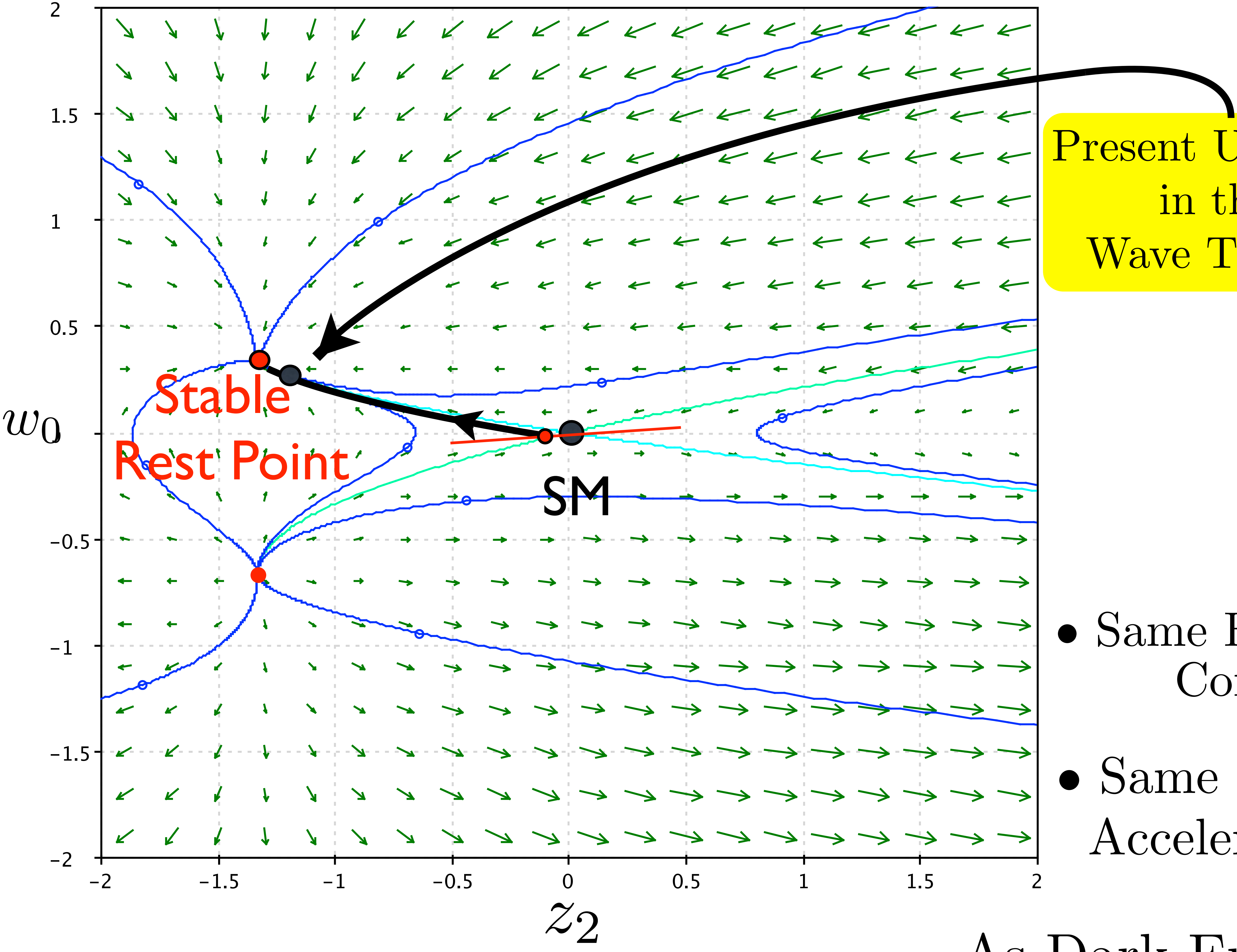
THE ANSWER: The value of the acceleration for the wave perturbation of SM that produces a quadradic correction of .425 at the present value of H_0 is:

- $$\underline{a} = 0.99999957 = 1 - \left(4.3 \times 10^{-7}\right)$$

$$H_0 d_\ell = z + .425z^2 + .3591z^3$$

- This corresponds to an relative underdensity of

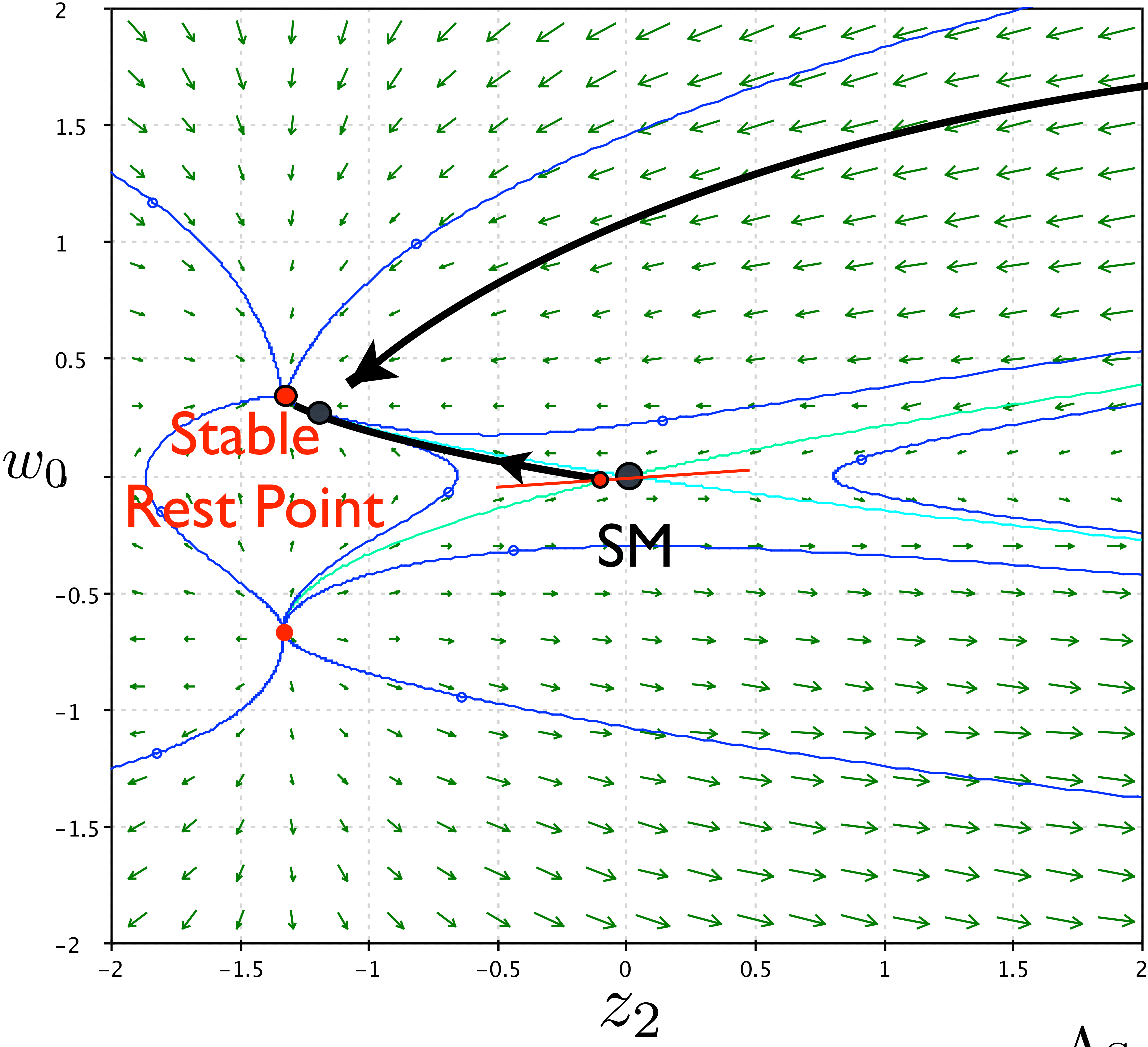
$$\frac{\rho_{SM} - \rho_{ssw}}{\rho_{SM}} = 7.45 \times 10^{-6}$$



Present Universe
in the
Wave Theory

- Same Hubble Constant
- Same .425 Acceleration

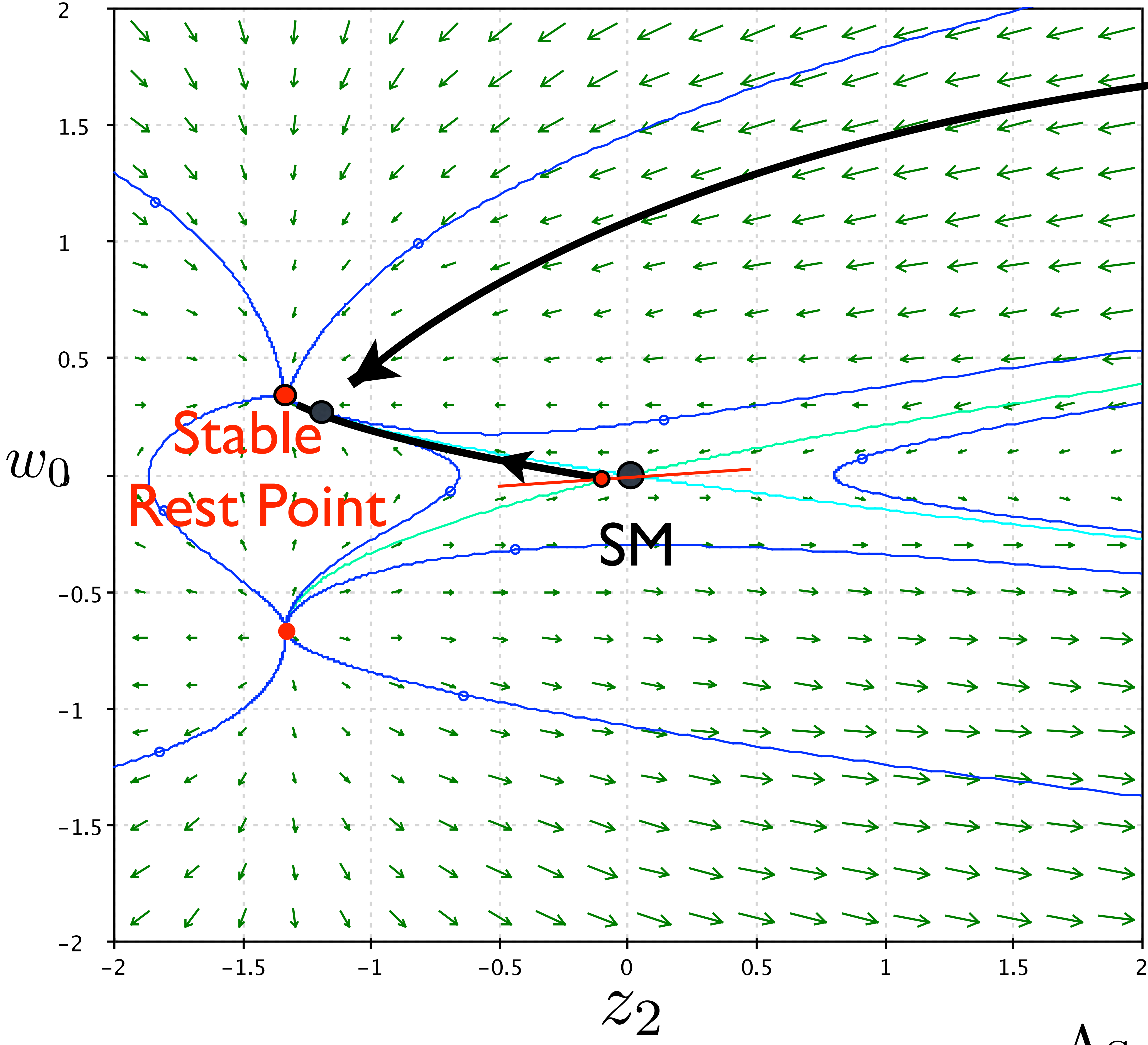
As Dark Energy



Present Universe
in the
Wave Theory

- Same Hubble Constant
- Same .425 Acceleration

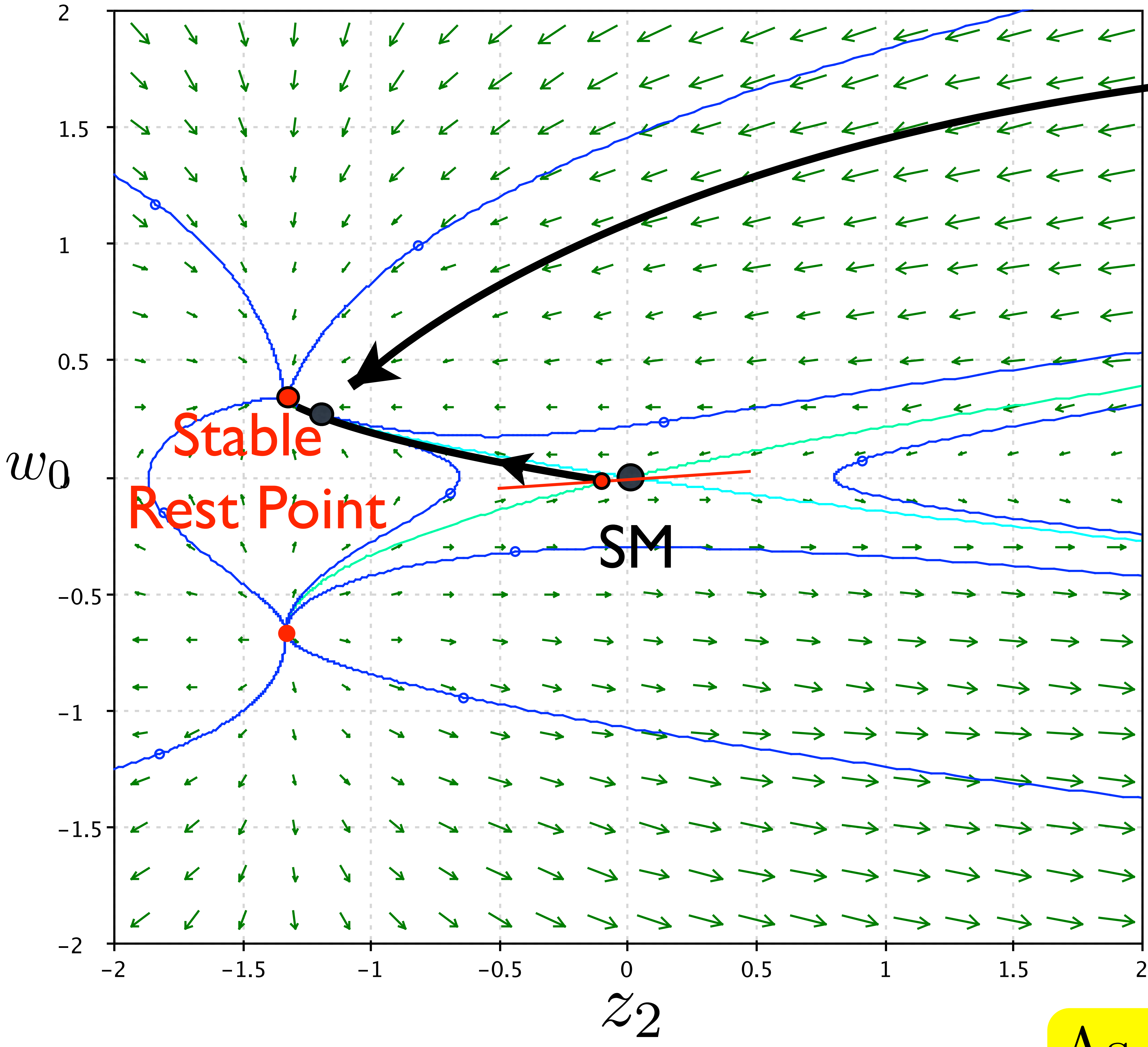
As Dark Energy



Present Universe
in the
Wave Theory

- Same Hubble Constant
- Same .425 Acceleration

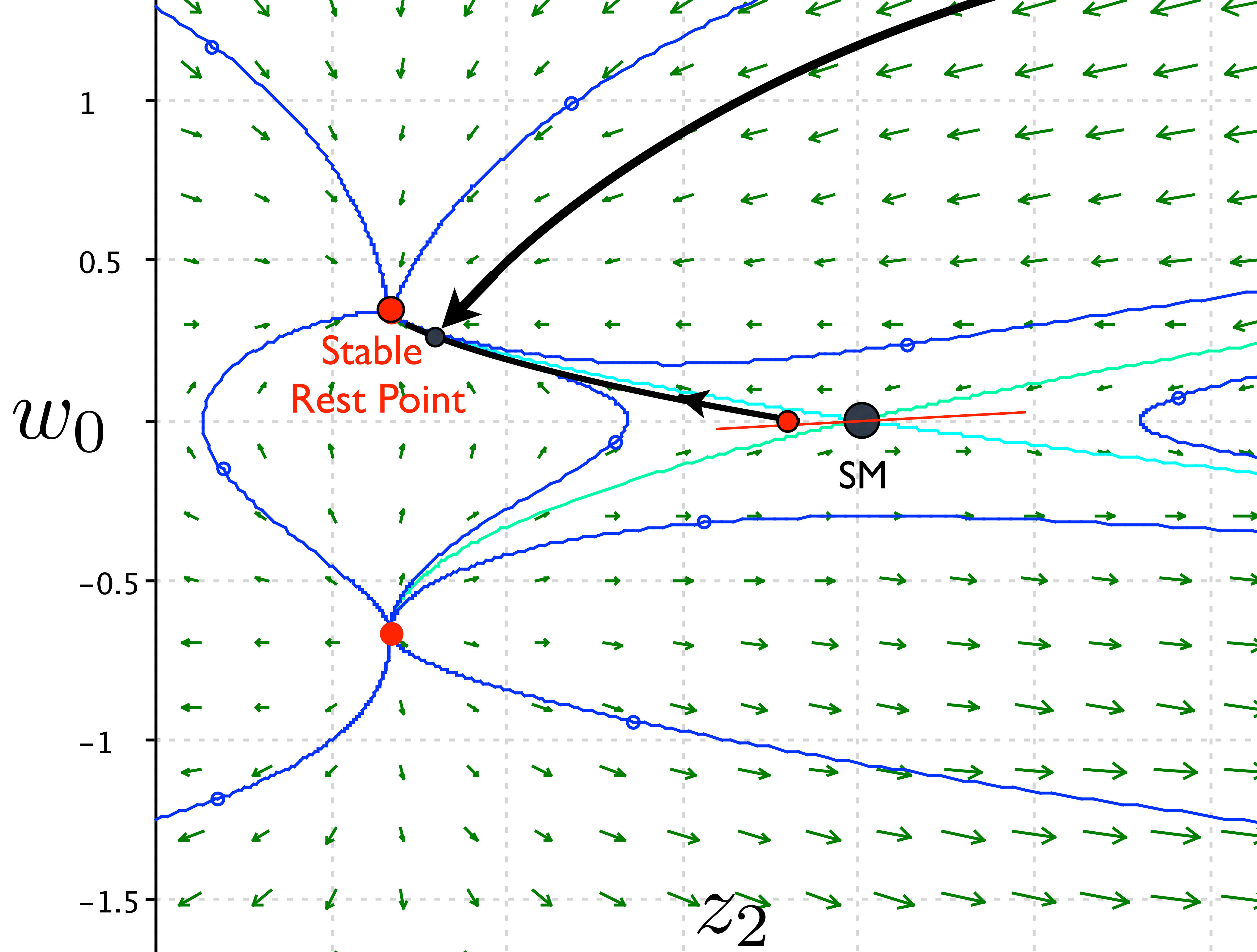
As Dark Energy



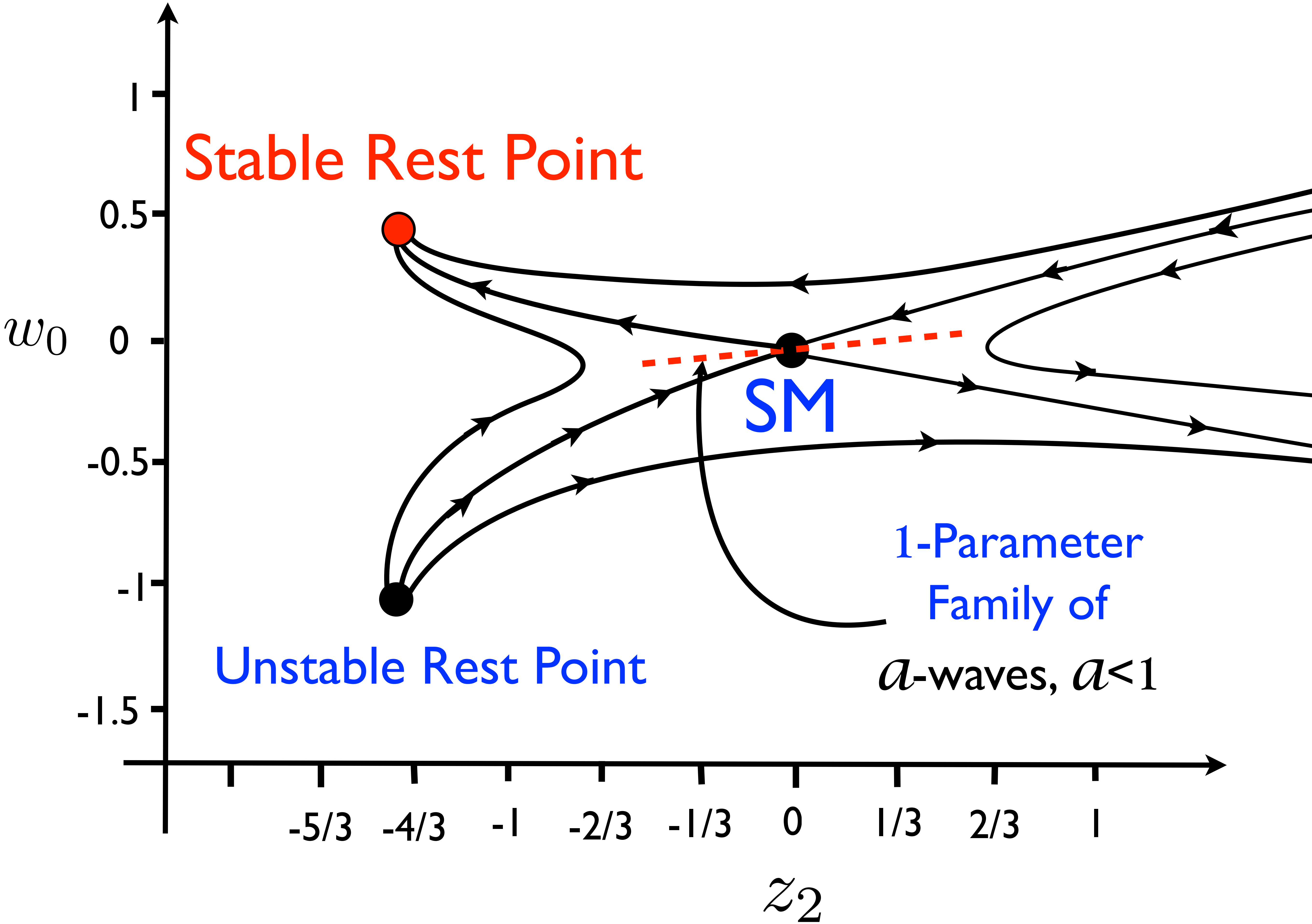
Present Universe
in the
Wave Theory

- Same Hubble Constant
- Same .425 Acceleration

As Dark Energy



1
0.5
w_0
−0.5
−1
−1.5
z_2
Stable
Rest Point
SM


Stable Rest Point
w_0
1
0.5
0
-0.5
-1
-1.5
SM
1-Parameter
Family of
a-waves, a<1
Unstable Rest Point
-5/3
-4/3
-1
-2/3
-1/3
0
1/3
2/3
1
z_2

- The relative underdensity at the end of radiation:

$$\frac{\rho_{SM}-\rho_{ssw}}{\rho_{SM}} = 7.45 \times 10^{-6}$$

- Numerical Simulation gives the relative under-density at present time as:

$$\frac{\rho_{ssw}(t_0)}{\rho_{SM}(t_0)} = .1438 \approx \frac{1}{7}.$$

Conclude: An under-density of one part in 10^6at the end of radiation produces a seven-fold under-density at present time!

Conclude: The Standard Model is
Unstable to Perturbation
by this family of Waves…

Comparison with Dark Energy:

$$H_0 d_\ell = z + .425\, z^2 - .1804\, z^3$$

Dark Energy

$$H_0 d_\ell = z + .425 z^2 + .3591 z^3$$

Wave Theory

Comparison with Dark Energy:

$$H_0 d_\ell = z + .425\, z^2 - .1804\, z^3$$

Dark Energy

$$H_0 d_\ell = z + .425 z^2 + .3591 z^3$$

Wave Theory

The Wave Theory predicts a Larger Anomalous Acceleration far from the center than Dark Energy

Comparison with Dark Energy:

$$H_0 d_\ell = z + .425\, z^2 - .1804\, z^3$$

Dark Energy

$$H_0 d_\ell = z + .425 z^2 + .3591 z^3$$

Wave Theory

Wave Theory takes More Time to $H = H_0$:

$$t_{DE} \approx 13.8 \text{ Billion years} \approx (1.45)\, t_{SM}$$

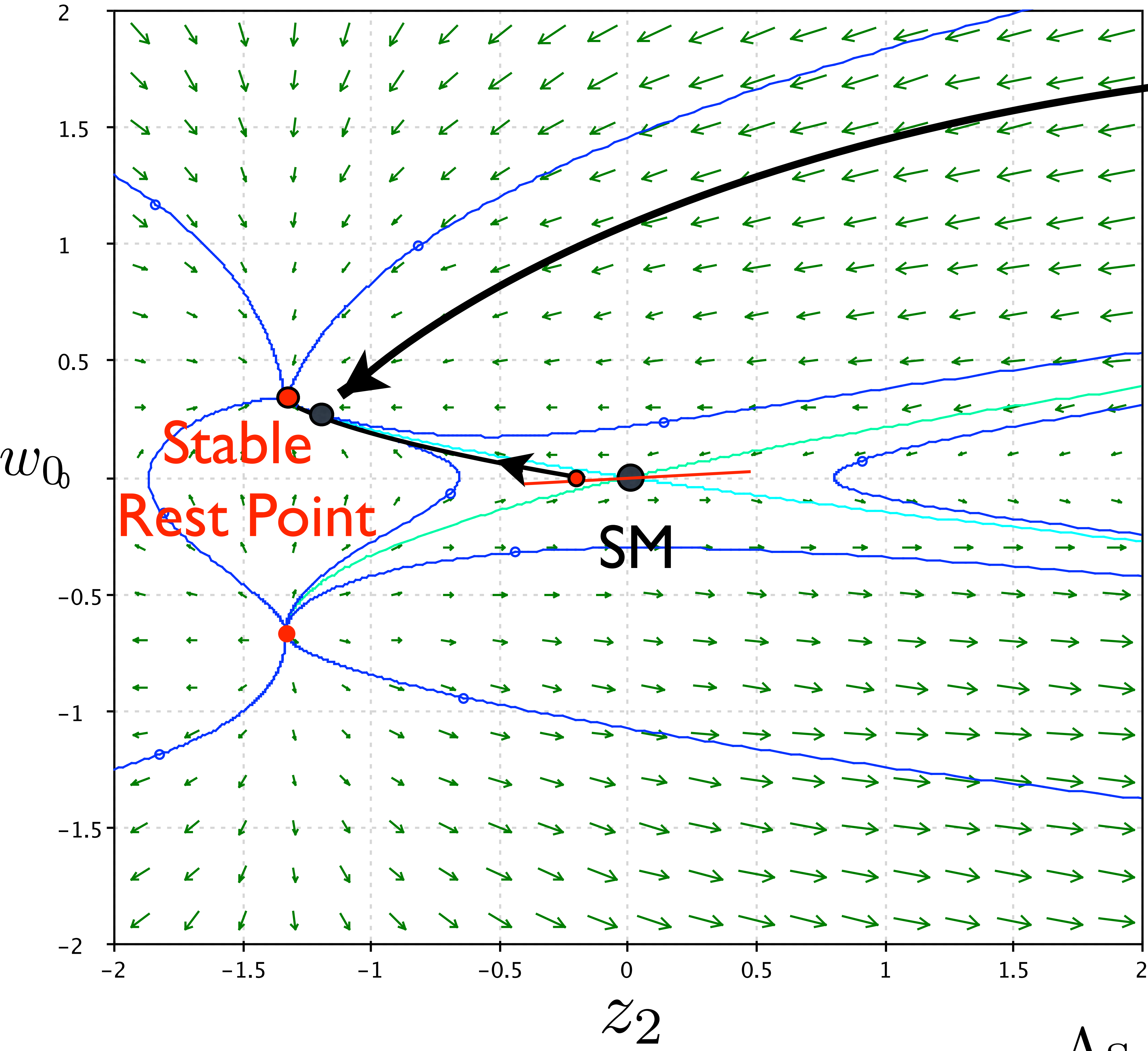
$$t_0 \approx (.98) t_{DE}$$

In Fact: A slight over-density will also create the Anomalous Acceleration

$$\bar{a} = 1.0000006747 = 1 + \left(6.747 \times 10^{-7}\right)$$

$$H_0 d_\ell = z + .425z^2 - 2.7555z^3$$

A different
cubic correction

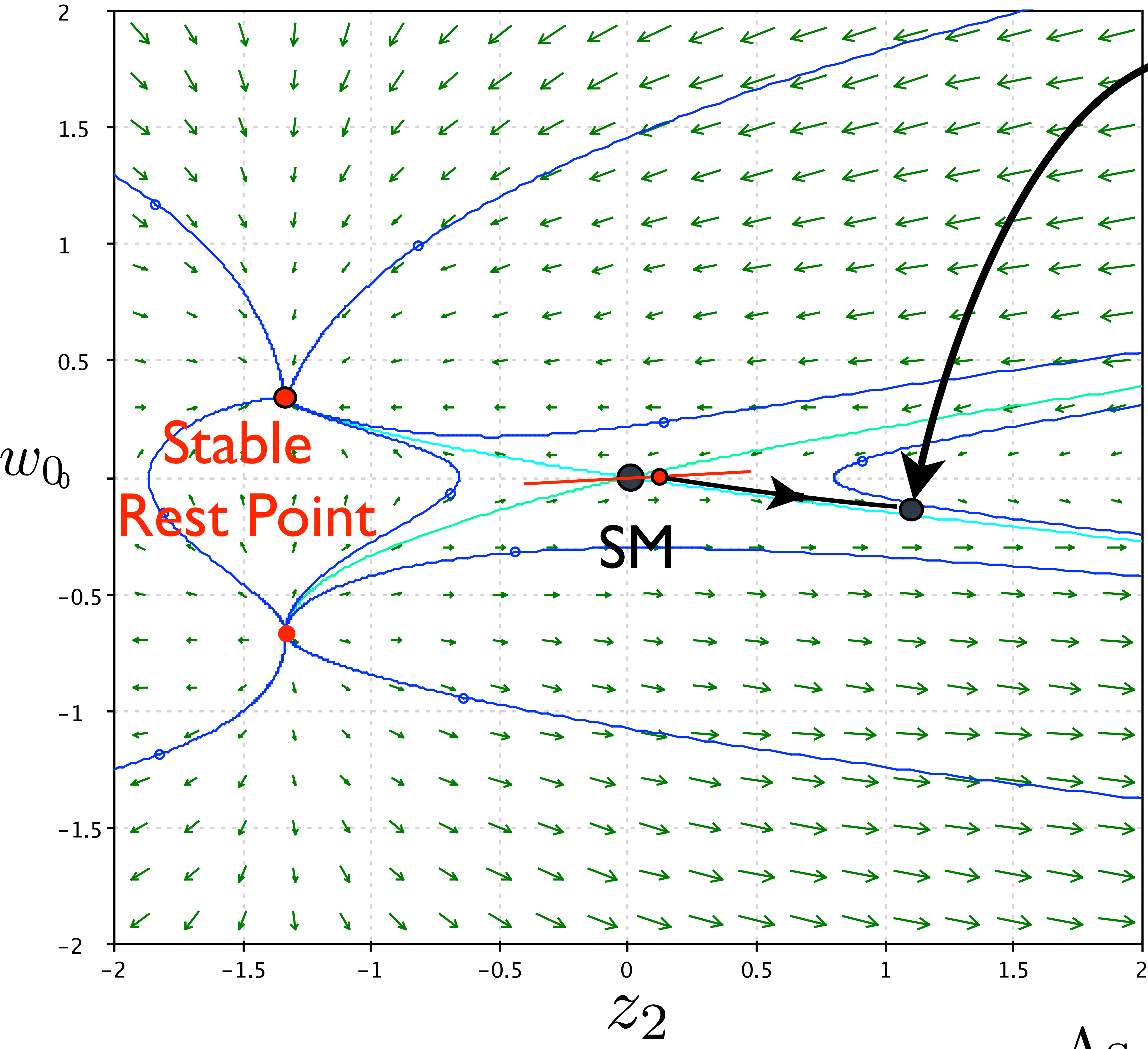


UNDER-DENSITY

Present Universe
Under-Dense
Case

- Same Hubble Constant
- Same .425 Acceleration

As Dark Energy



Present Universe
Over-Dense
Case

- Same Hubble Constant
- Same .425 Acceleration

OVER-DENSITY

As Dark Energy

Conclude: The Standard Model
is
Unstable to Perturbation
by this
Family of Waves,
and under-densities create an
Anomalous Acceleration

Theorem: Let $t = t_0$ denote present time since the Big Bang in the wave model and $t = t_{DE}$ present time since the Big Bang in the Dark Energy model. Then there exists a unique value of the acceleration parameter $\underline{a} = 0.99999959 \approx 1 - 4.3 \times 10^{-7}$ corresponding to an under-density relative to the SM at the end of radiation, such that the subsequent $p = 0$ evolution starting from this initial data evolves to time $t = t_0$ with $H = H_0$ and $Q = .425$, in agreement with the values of H and Q at $t = t_{DE}$ in the Dark Energy model. The cubic correction at $t = t_0$ in the wave theory is then $C = 0.3591$, while Dark Energy theory gives $C = -0.1804$ at $t = t_{DE}$. The times are related by $t_0 \approx (.98)t_{DE}$

6. The Flat Uniformly Expanding Spacetime at the Center of the Wave

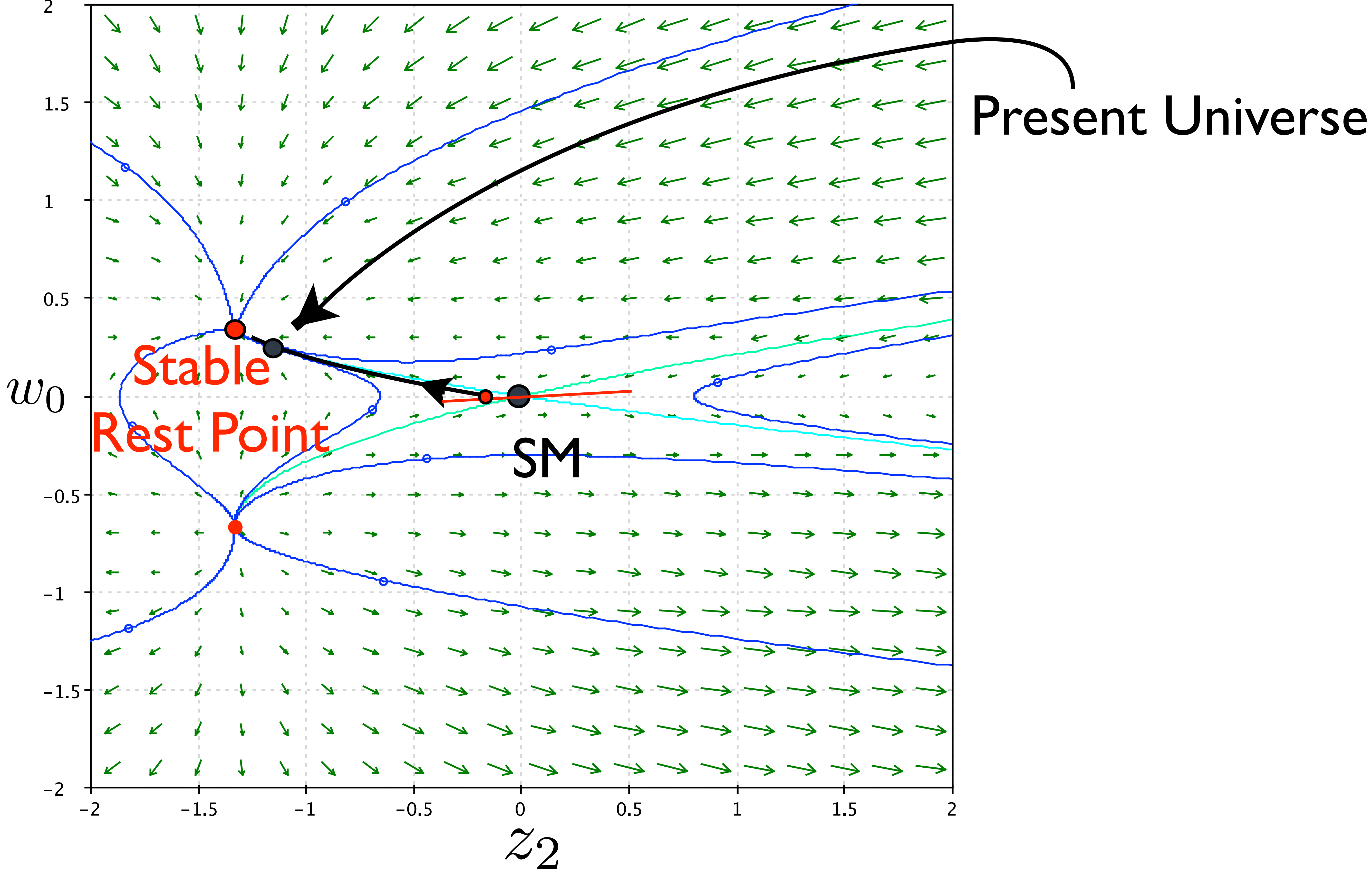
(Under-Dense Case: $\underline{a} < 1$)

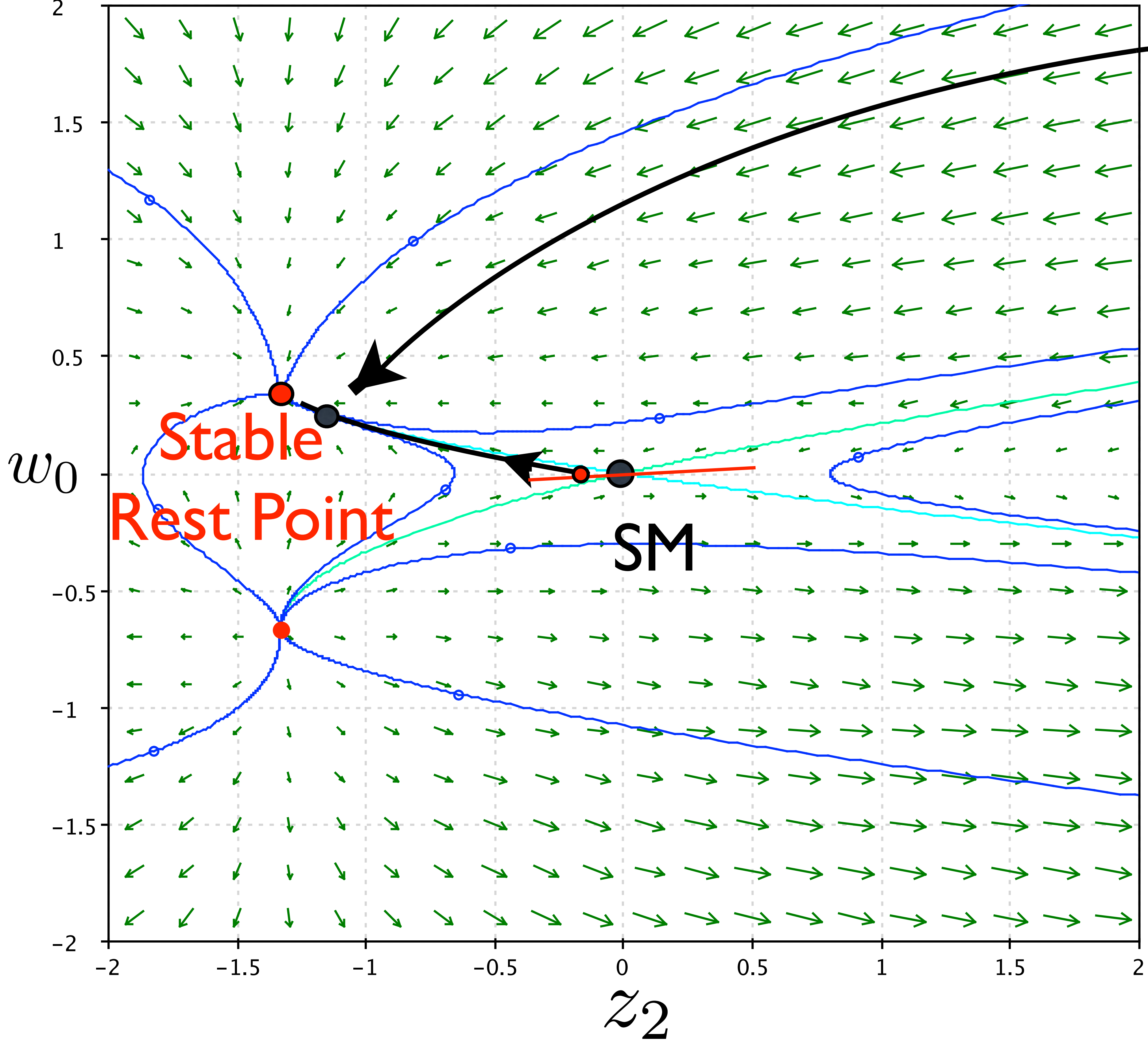
Consider the evolution of the spactime at the center obtained by **neglecting all errors** of order

$$\boxed{O(\xi^4)}$$

The spacetime near the center evolves toward the

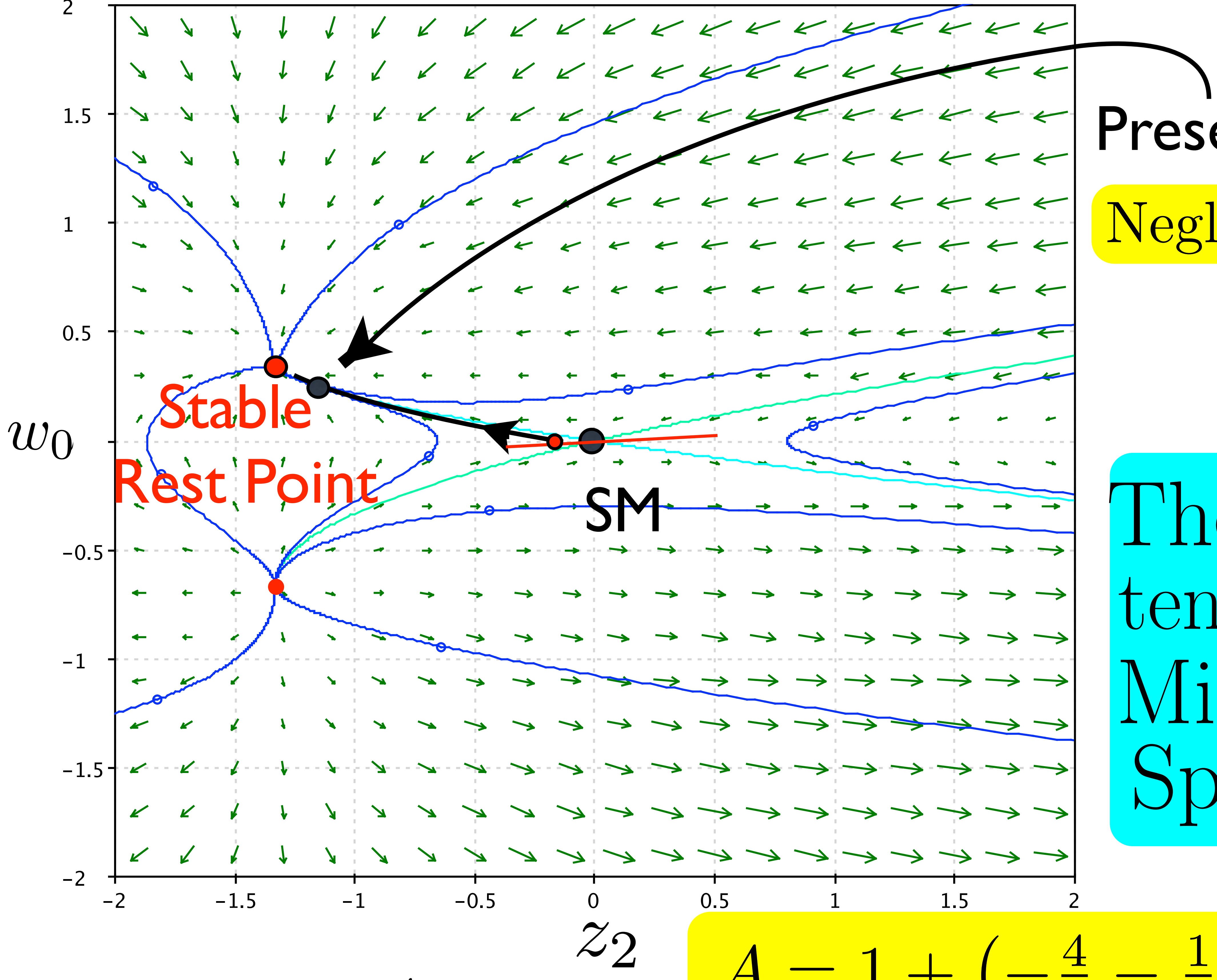
Stable Rest Point


Present Universe
Stable
Rest Point
SM
w_0
z_2
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2



Present Universe

Neglecting $O(\xi^4)$



Present Universe

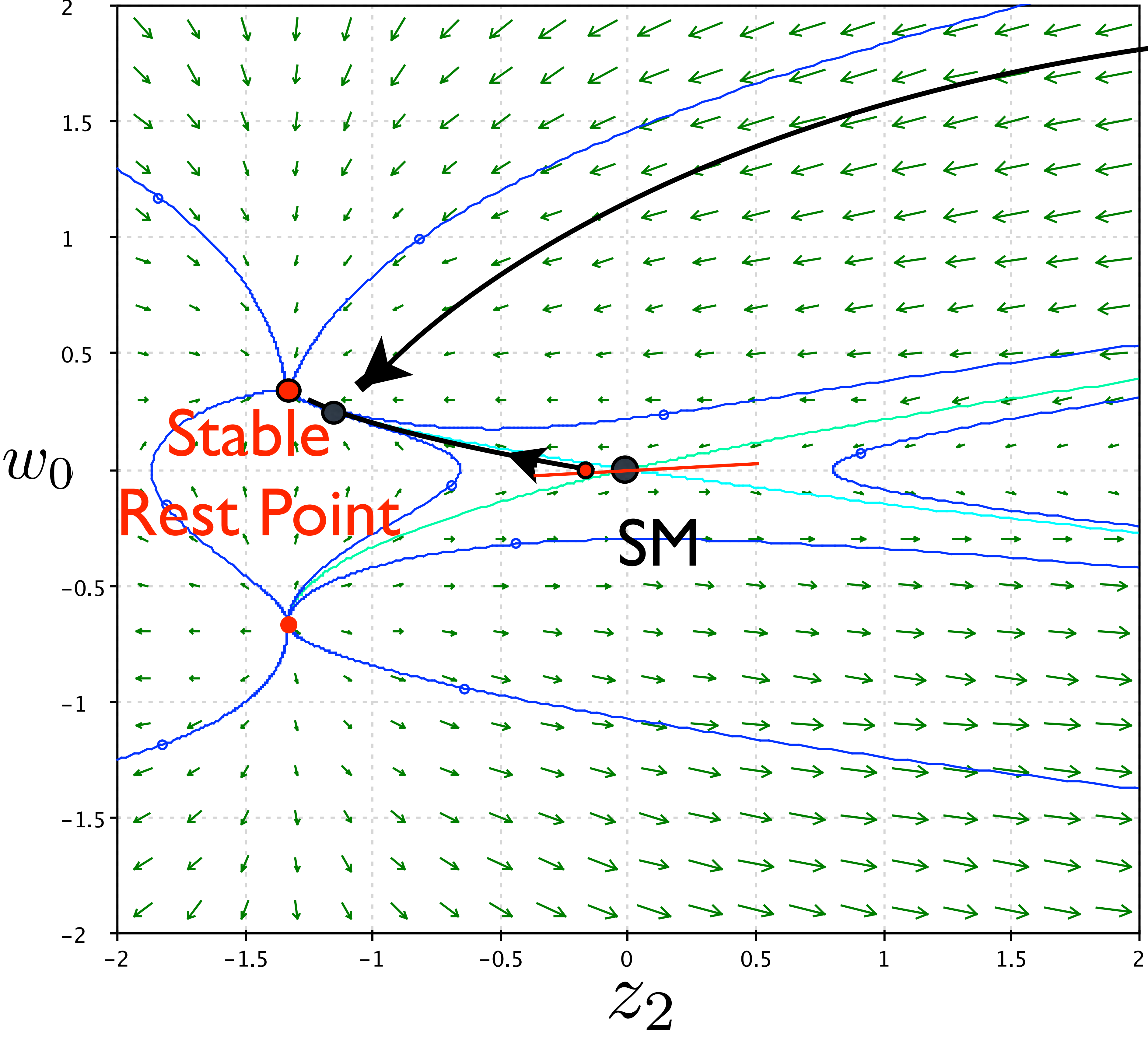
Neglecting $O(\xi^4)$

The Metric tends to flat Minkowski Spacetime

$$z_2 \rightarrow -\frac{4}{3}$$

$$A = 1 + \left(-\tfrac{4}{9} - \tfrac{1}{3} z_2\right) \xi^2 \rightarrow 1$$

$$D = 1 + \left(-\tfrac{1}{9} - \tfrac{1}{12} z_2\right) \xi^2 \rightarrow 1$$



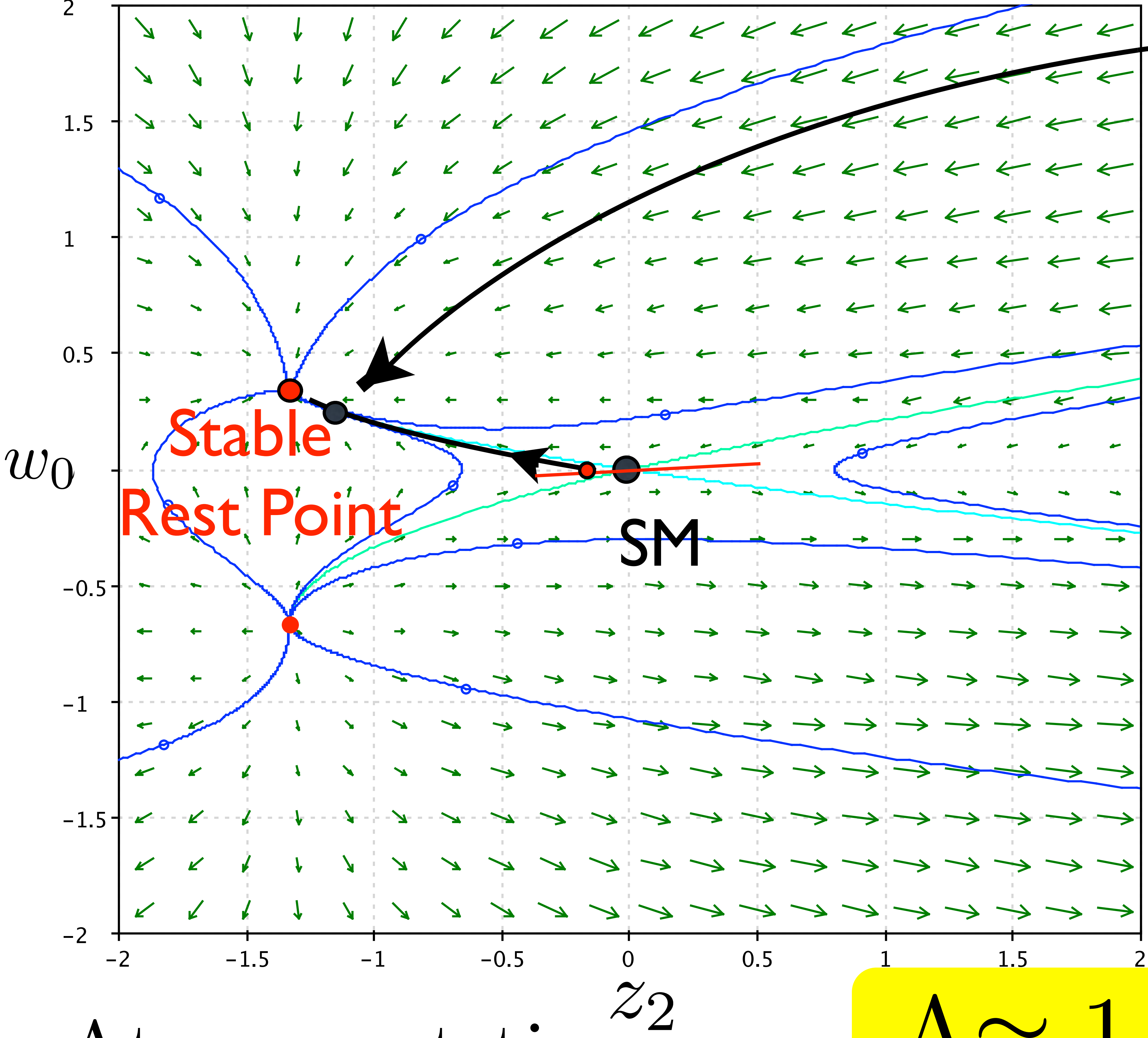
Present Universe

Neglecting $O(\xi^4)$

The Metric tends to flat Minkowski Spacetime

$$\mathrm{ds}^2 = -Bdt^2 + \frac{1}{A}dr^2 + r^2 d\Omega^2$$

➡ $$\mathrm{ds}^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$



Present Universe

Neglecting $O(\xi^4)$

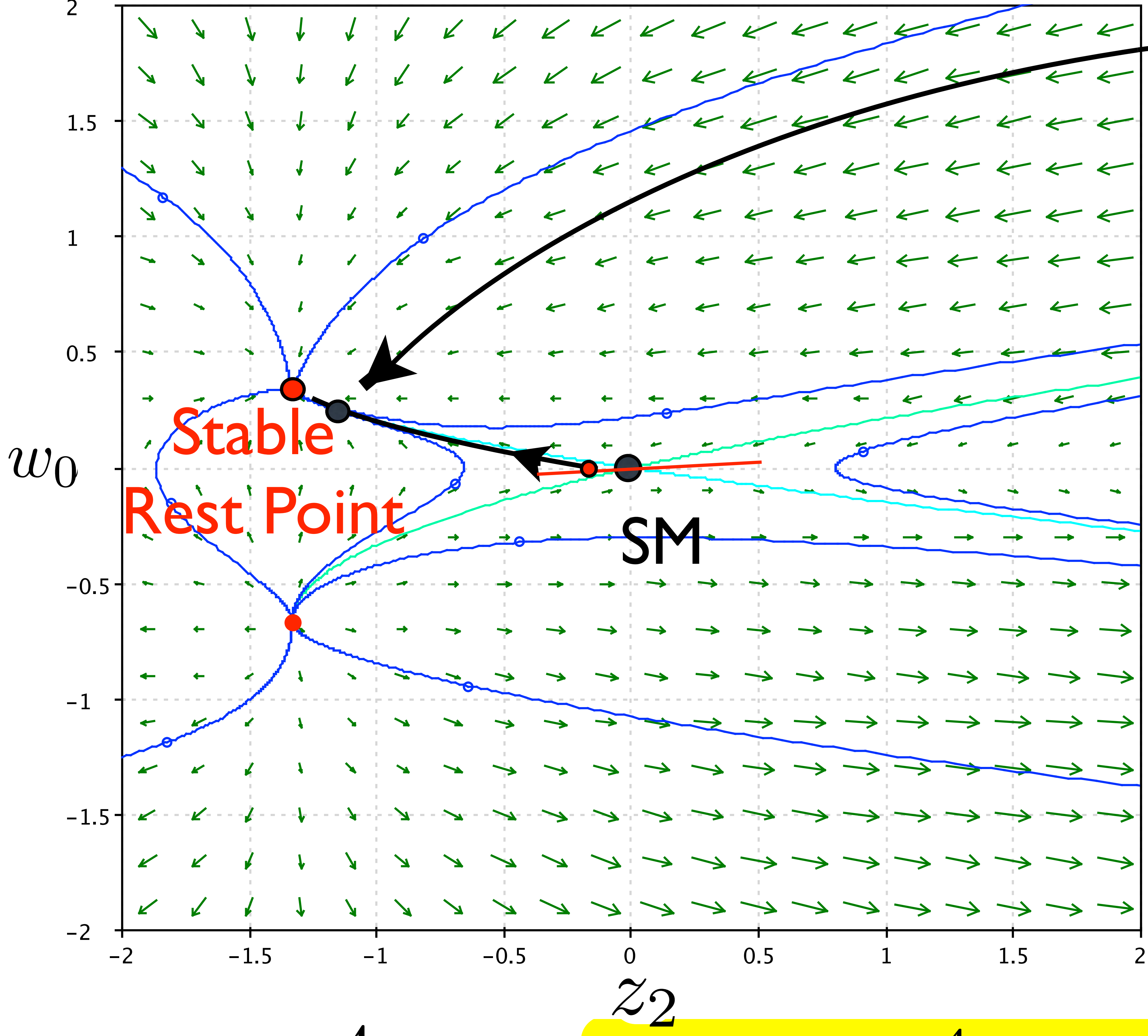
The Metric tends to flat Minkowski Spacetime

At present time:

$$ds^2 = -\frac{D^2}{A}dt^2 + \frac{1}{A}dr^2 + r^2 d^2$$

$$\text{A} \approx 1 - (.063)\,\xi^2$$

$$\text{D} \approx 1 - (.016)\,\xi^2$$



Present Universe

Neglecting $O(\xi^4)$

The density drops *faster* than the $O\left(\frac{1}{t^2}\right)$ of the Standard Model

$$z_2 \longrightarrow -\frac{4}{3} \quad \Rightarrow \quad \rho(t) = \frac{\frac{4}{3}+z_2(t)}{t^2} = O\left(\frac{1}{t^3}\right)$$

Neglecting $O(\xi^4)$ errors:
The spacetime near the center evolves toward the Stable Rest Point

- The metric tends to Flat Minkowski Spacetime which is not co-moving with the fluid

- BUT: The evolution creates a uniformly expanding density near the center, which, neglecting relativistic corrections, is *CENTER-INDEPENDENT* (like Friedmann Spacetimes)

THEOREM: Neglecting $O(\xi^4)$, as the orbit tends to the Stable Rest Point, the density drops *FASTER* than SM,

$$\rho(t) = \frac{k_0}{t^{3(1+\bar{w})}}, \qquad \rho_{SM}(t) = \frac{4}{3t^2},$$

where $\bar{w}(t)$ and $k_0(t)$ change exponentially slowly.

CONCLUDE: The wave creates a

UNIFORMLY EXPANDING SPACETIME

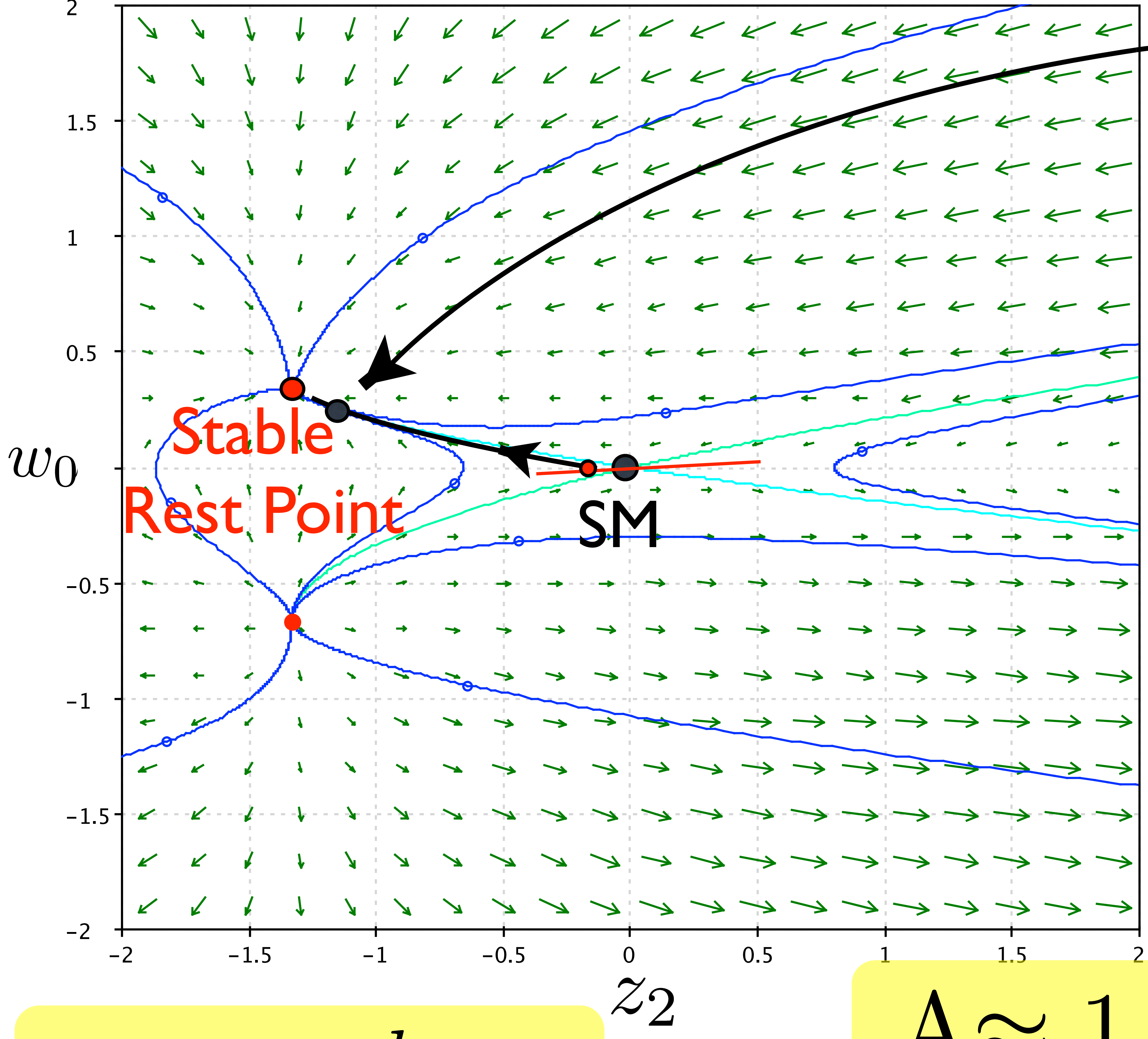
with an

ANOMALOUS ACCELERATION

in a

LARGE, FLAT, CENTER-INDEPENDENT

region near in the center of the wave.



Present Universe

Neglecting $O(\xi^4)$

$$\rho(t) = \frac{k_0}{t^{3(1+\bar{w})}},$$

$$\mathrm{A} \approx 1 - (.063)\,\xi^2$$

$$\mathrm{D} \approx 1 - (.016)\,\xi^2$$

CONCLUSIONS:

Our Proposal: The AA is due to a local underdensity on the scale of the supernova data, created by a self-similar wave from the radiation epoch that triggers an instability in the SM when the pressure drops to zero.

We have made no assumptions regarding the space-time far from the center of the perturbations. The consistency of this model with other observations in astrophysics would require additional assumptions.

CONCLUSIONS:

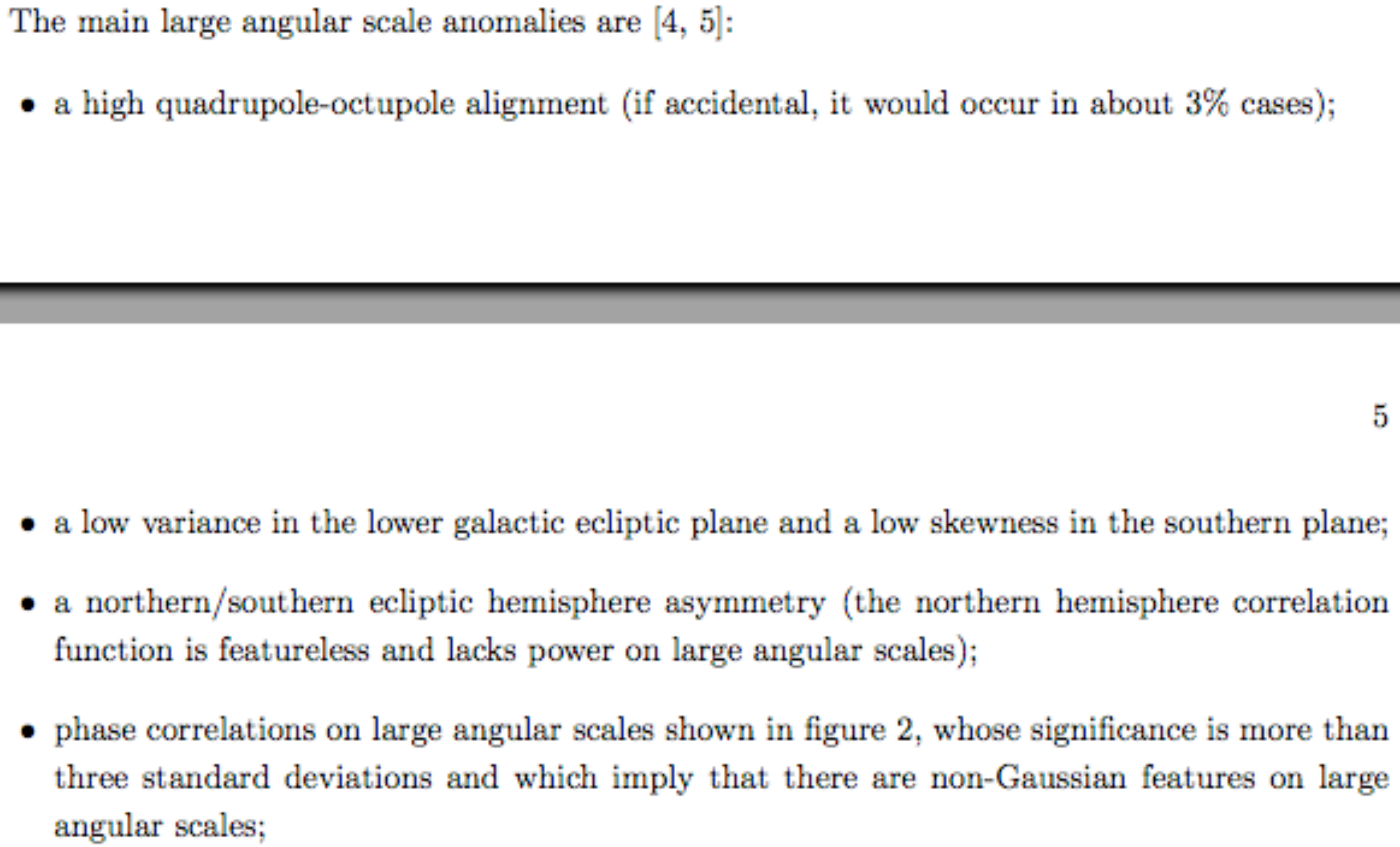
- This is arguably the simplest explanation for the anomalous acceleration within Einstein's original theory of GR, without requiring Dark Energy.
- It demonstrates that any local center of the Standard Model of Cosmology is unstable on the largest length scale, to perturbation by exact solutions from the Radiation Epoch.
- These perturbations are stabilized by a nearby stable rest point that generates the same accelerations as Dark Energy.
- It makes testable predictions.

QUESTIONS:

- On what scale would such waves apply?
- If these came from time-asymptotic wave patterns created in an earlier epoch, would we expect secondary transitional waves far from the center?
- How does cosmology address the instability? Can Dark Energy help? (NO!)
- Implications of a preferred center?
- Is this more fine-tuned than Dark Energy?

Prokopek...2013 (Astrophysicist, Utrecht University)

There are large scale anomalies in the data indicating a lack of uniformity on the largest length scale

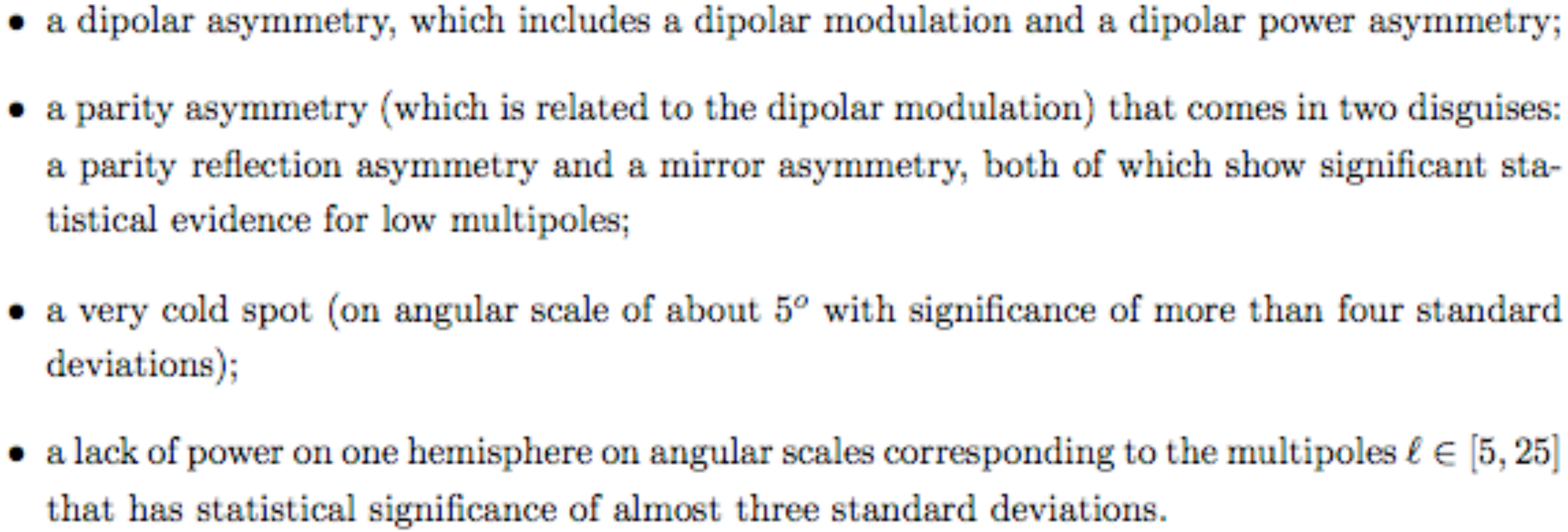


The main large angular scale anomalies are [4, 5]:

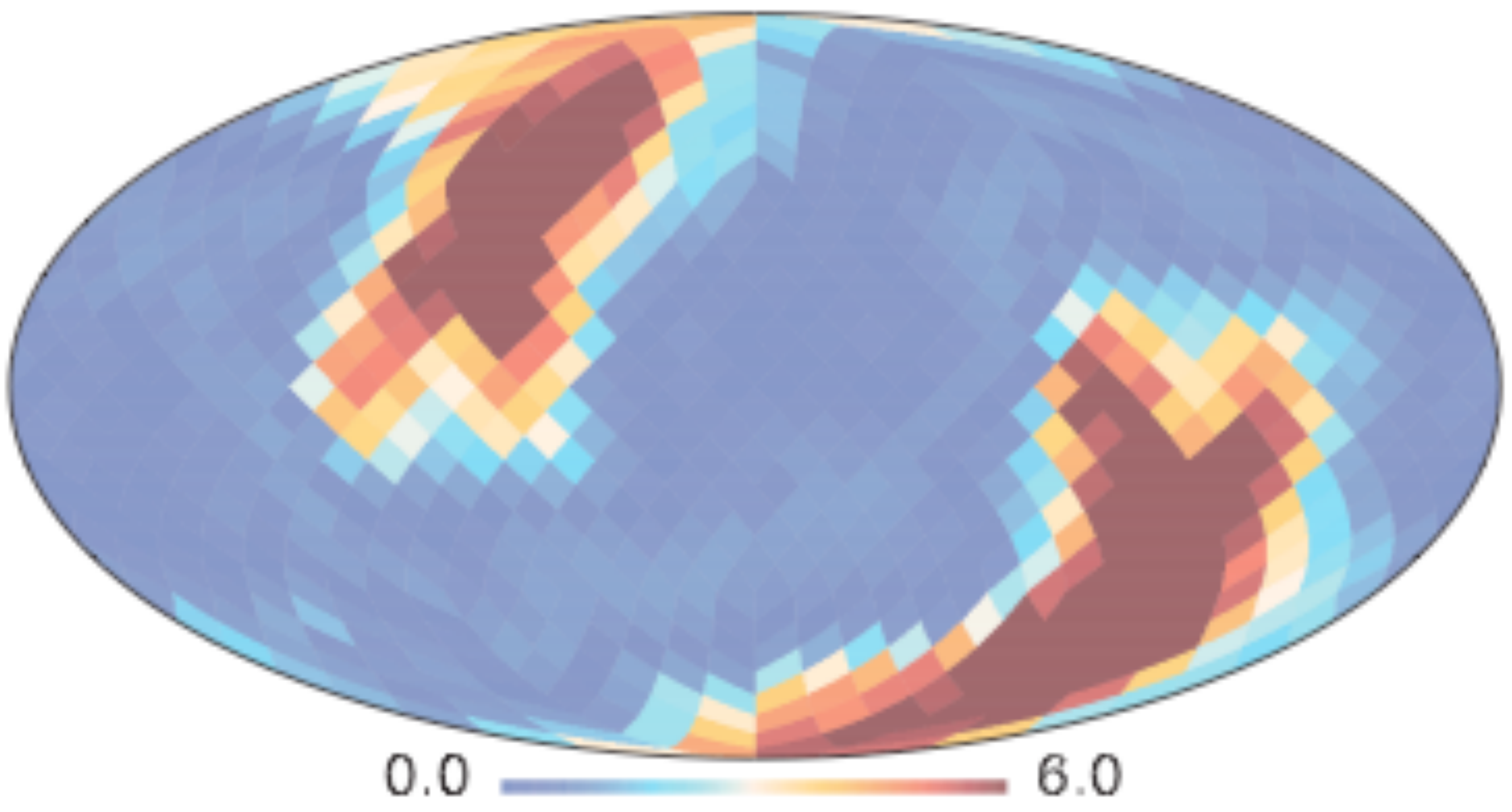
- a high quadrupole-octupole alignment (if accidental, it would occur in about 3% cases);

- a low variance in the lower galactic ecliptic plane and a low skewness in the southern plane;
- a northern/southern ecliptic hemisphere asymmetry (the northern hemisphere correlation function is featureless and lacks power on large angular scales);
- phase correlations on large angular scales shown in figure 2, whose significance is more than three standard deviations and which imply that there are non-Gaussian features on large angular scales;

Prokopek...2013 (Astrophysicist, Utrecht University)



- a dipolar asymmetry, which includes a dipolar modulation and a dipolar power asymmetry;
- a parity asymmetry (which is related to the dipolar modulation) that comes in two disguises: a parity reflection asymmetry and a mirror asymmetry, both of which show significant statistical evidence for low multipoles;
- a very cold spot (on angular scale of about 5^o with significance of more than four standard deviations);
- a lack of power on one hemisphere on angular scales corresponding to the multipoles $\ell \in [5, 25]$ that has statistical significance of almost three standard deviations.



FINAL COMMENT

Every aspect of this work
came from
Applied Mathematics,

Whatever its implications to Physics,
it stands on its own as a self-contained
model in Applied Mathematics

Mathematics is part of physics…
…the part of physics
where experiments are cheap.

—Arnold, Paris, 1997

End

P. PAPIER

Big Bang blunder bursts the multiverse bubble

Premature hype over gravitational waves highlights gaping holes in models for the origins and evolution of the Universe, argues **Paul Steinhardt***.*

When a team of cosmologists announced at a press conference in March that they had detected gravitational waves generated in the first instants after the Big Bang, the origins of the Universe were once again major news. The reported discovery created a worldwide sensation in the scientific community, the media and the public at large (see *Nature* **507**, 281–283; 2014).

According to the team at the BICEP2 South Pole telescope, the detection is at the 5–7 sigma level, so there is less than one chance in two million of it being a random occurrence. The results were hailed as proof of the Big Bang inflationary theory and its progeny, the multiverse. Nobel prizes were predicted and scores of theoretical models spawned. The announcement also influenced decisions about academic appointments and the rejections of papers and grants. It even had a role in governmental planning of large-scale projects.

THE INFLATIONARY PARADIGM IS FUNDAMENTALLY **UNTESTABLE,** AND HENCE SCIENTIFICALLY **MEANINGLESS.**

The BICEP2 team identified a twisty (B-mode) pattern in its maps of polarization of the cosmic microwave background, concluding that this was a detection of primordial gravitational waves. Now, serious flaws in the analysis have been revealed that transform the sure detection into no detection. The search for gravitational waves must begin anew. The problem is that other effects, including light scattering from dust and the synchrotron radiation generated by electrons moving around galactic magnetic fields within our own Galaxy, can also produce these twists.

The BICEP2 instrument detects radiation at only one frequency, so cannot distinguish the cosmic contribution from other sources. To do so, the BICEP2 team used measurements of galactic dust collected by the Wilkinson Microwave Anisotropy Probe and Planck satellites, each of which operates over a range of other frequencies. When the BICEP2 team did its analysis, the Planck dust map had not yet been published, so the team extracted data from a preliminary map that had been presented several months earlier. Now a careful reanalysis by scientists at Princeton University and the Institute for Advanced Study, also in Princeton, has concluded that the BICEP2 B-mode pattern could be the result mostly or entirely of foreground effects without any contribution from gravitational waves. Other dust models considered by the BICEP2 team do not change this negative conclusion, the Princeton team showed (R. Flauger, J. C. Hill and D. N. Spergel, preprint at http://arxiv.org/abs/1405.7351; 2014).

The sudden reversal should make the scientific community contemplate the implications for the future of cosmology experimentation and theory. The search for gravitational waves is not stymied. At least eight experiments, including BICEP3, the Keck Array and Planck, are already aiming at the same goal.

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This time, the teams can be assured that the world will be paying close attention. This time, acceptance will require measurements over a range of frequencies to discriminate from foreground effects, as well as tests to rule out other sources of confusion. And this time, the announcements should be made after submission to journals and vetting by expert referees. If there must be a press conference, hopefully the scientific community and the media will demand that it is accompanied by a complete set of documents, including details of the systematic analysis and sufficient data to enable objective verification.

The BICEP2 incident has also revealed a truth about inflationary theory. The common view is that it is a highly predictive theory. If that was the case and the detection of gravitational waves was the 'smoking gun' proof of inflation, one would think that non-detection means that the theory fails. Such is the nature of normal science. Yet some proponents of inflation who celebrated the BICEP2 announcement already insist that the theory is equally valid whether or not gravitational waves are detected. How is this possible?

The answer given by proponents is alarming: the inflationary paradigm is so flexible that it is immune to experimental and observational tests. First, inflation is driven by a hypothetical scalar field, the inflaton, which has properties that can be adjusted to produce effectively any outcome. Second, inflation does not end with a universe with uniform properties, but almost inevitably leads to a multiverse with an infinite number of bubbles, in which the cosmic and physical properties vary from bubble to bubble. The part of the multiverse that we observe corresponds to a piece of just one such bubble. Scanning over all possible bubbles in the multiverse, everything that can physically happen does happen an infinite number of times. No experiment can rule out a theory that allows for all possible outcomes. Hence, the paradigm of inflation is unfalsifiable.

This may seem confusing given the hundreds of theoretical papers on the predictions of this or that inflationary model. What these papers typically fail to acknowledge is that they ignore the multiverse and that, even with this unjustified choice, there exists a spectrum of other models which produce all manner of diverse cosmological outcomes. Taking this into account, it is clear that the inflationary paradigm is fundamentally untestable, and hence scientifically meaningless.

Cosmology is an extraordinary science at an extraordinary time. Advances, including the search for gravitational waves, will continue to be made and it will be exciting to see what is discovered in the coming years. With these future results in hand, the challenge for theorists will be to identify a truly explanatory and predictive scientific paradigm describing the origin, evolution and future of the Universe. ■

Paul Steinhardt *is professor of physics at Princeton University.*
e-mail: steinh@princeton.edu