Answers to Questions Posed by Reporters:
Temple-Smoller GR Expanding Waves
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In this we give careful answers to the questions most frequently asked by reporters covering our PNAS paper Expanding Wave Solutions of the Einstein Equations That Induce an Anomalous Acceleration Into the Standard Model of Cosmology, released from embargo August 17, 2009 at 3 pm.

To set the stage, here is a 200 word summary of our work:

“We introduce a new family of expanding wave solutions of the Einstein equations, and argue that they could account for the observed anomalous acceleration of the galaxies without Dark Energy. Currently, the anomalous acceleration is accounted for by adding in the cosmological constant (Dark Energy) to the Einstein equations, an artificial correction term that speeds up the expansion rate. In contrast, we prove that this new family of self-similar solutions of the Einstein equations will speed up or slow down the expansion rate relative to the standard model of
cosmology\textsuperscript{1} without assuming Dark Energy, by adjustment of a free parameter in the family. Based on a general principle of the mathematical theory of conservation laws, (that nonlinearities alone can generate dissipation that, in time, will drive a chaotic disturbance into a non-interacting self-similar expansion), we argue that it is physically plausible that these new expanding waves could emerge in time from the initial disturbance of the Big Bang, something like the orderly concentric circles of waves that emerge in time on the surface of a pond after a rock disturbs the surface with a chaotic “plunk”.

**To Begin:** Let us say at the start that what is definitive about our work is the construction of a new one-parameter family of exact, self-similar expanding wave solutions to Einstein’s equations of general relativity. They apply during the radiation phase of the Big Bang, and approximate the standard model of cosmology arbitrarily well. For this we have complete mathematical arguments that are not controvertible. Our intuitions that led us to these, and their physical significance to the anomalous acceleration problem are based on lessons learned from the mathematical theory of nonlinear conservation laws, and only this interpretation is subject to debate.

\textsuperscript{1}By *standard model of cosmology* we mean the critical ($k = 0$) Friedman spacetime that was the baseline model for cosmology before dark energy.
1) Could you explain - in simple terms - what an expanding wave solution is and what other phenomena in nature can be explained through this mathematics?

To best understand what an expanding wave is, imagine a stone thrown into a pond, making a splash as it hits the water. The initial “plunk” at the start creates chaotic waves that break every which way, but after a short time the whole disturbance settles down into orderly concentric circles of waves that radiate outward from the center—think of the resulting final sequence of waves as the “expanding wave”. In fact, it is the initial breaking of waves that dissipates away all of the disorganized motion, until all that is left is the orderly expansion of waves. For us, the initial “plunk” of the stone is the chaotic Big Bang at the start of the radiation phase, and the expansion wave is the orderly expansion that emerges at the end of the radiation phase. What we have found is that the standard model of cosmology is not the only expanding wave that could emerge from the initial “plunk”. In fact, we constructed a whole family of possible expanding waves that could emerge; and we argue that which one would emerge depends delicately on the nature of the chaos in the initial “plunk”. That is, one expanding wave in the family is equally likely to emerge as another. Our family depends on a freely assignable number \( a \) which we call the acceleration parameter, such that if we pick \( a = 1 \), then we get the standard model of cosmology, but if \( a > 1 \) we get an expanding wave that looks a lot like the standard model, but expands faster, and if \( a < 1 \), then it expands slower. So an “anomalous acceleration”
would result if $a > 1$.

**Summary:** By “expanding wave” we mean a wave that expands outward in a “self-similar” orderly way in the sense that at each time the wave looks like it did at an earlier time, but more “spread out”. The importance of an expanding wave is that it is the end state of a chaotic disturbance because it is what remains after all the complicated breaking of waves is over...one part of the expanding solution no longer effects the other parts. Our thesis, then, is that we can account for the anomalous acceleration of the galaxies without dark energy by taking $a > 1$.

2) *Could you explain how and why you decided to apply expanding wave solutions to this particular issue?*

We (Temple) got the idea that the anomalous acceleration of the galaxies might be explained by a secondary expansion wave reflected backward from the shock wave in our earlier construction of a shock wave in the standard model of cosmology, and proposed to numerically simulate such a wave. Temple got this idea while giving a public lecture to the National Academy of Sciences in Bangalore India, in 2006. We set out together to simulate this wave while Temple was Gehring Professor in Ann Arbor in 2007, and in setting up the simulation, we subsequently discovered exact formulas for a family of such waves, without the need for the shock wave model.

3) *Do you think this provides the strongest evidence yet that dark energy is a redundant idea?*

At this stage we personally feel that this gives the most plausible explanation for the anomalous acceleration of the galaxies that
does not invoke dark energy. Since we don’t believe in “dark energy”...
[more detail in (12) below]

We emphasize that our model implies a verifiable prediction, so it remains to be seen whether the model fits the red-shift vs luminosity data better than the dark energy theory. (We are working on this now.)

4) *Is this the first time that expanding wave solutions of the Einstein equations have been realized?*

As far as we know, this is the first time a family of self-similar expanding wave solutions of the Einstein equations has been constructed for the radiation phase of the Big Bang, such that the members of the family can approximate the standard model of cosmology arbitrarily well. Our main point is *not* that we have self-similar expanding waves, but that we have self-similar expanding waves during the radiation phase when (1) decay to such waves is possible because $p \neq 0$, and (2) they are close to the standard model. We are not so interested in self-similar waves when $p = 0$ because we see no reason to believe that self-similar waves during the time when $p = \rho c^2/3 \neq 0$ will evolve into exact self-similar waves in the present era when $p = 0$. That is, they should evolve into some sort of expanding spacetime when $p = 0$, but not a pure (self-similar) expansion wave.

5) *How did you reach the assumption that $p = [\rho][c]^2/3$, a wise one?*

We are mathematicians, and in the last several decades, a theory for how highly nonlinear equations can decay to self-similar waves was worked out by mathematicians, starting with fun-
damental work of Peter Lax and Jim Glimm. The theory was worked out for model equations much simpler than the Einstein equations. We realized that only during the radiation phase of the expansion were the equations “sufficiently nonlinear” to expect sufficient breaking of waves at the start to create enough dissipation to drive a chaotic disorganized disturbance into an orderly self-similar expansion wave at the end. The subtle point is that even though no mechanisms for dissipation are put into the model, the nonlinearities alone can cause massive dissipation via the breaking of waves that would drive a chaotic disturbance into an orderly expansion wave.

6) How does your suggestion – that the observed anomalous acceleration of the galaxies could be due to our view into an expansion wave – compare with an idea that I heard Subir Sarkar describe recently that the Earth could be in a void that is expanding faster that the outer parts of the Universe.

We became aware of this work in the fall of ’08, and forwarded our preprints. Our view here is that after the radiation phase is over, and the pressure drops to zero, there is no longer any nonlinear mechanism that can cause the breaking of waves that can cause dissipation into an expansion wave. Thus during the recent \( p = 0 \) epoch, (after some 300,000 years after the Big Bang), you might model the evolution of the remnants of such an expanding wave or underdensity, (in their terms a local “void”), but there is no mechanism in the \( p = 0 \) phase to explain the constraints under which such a void could form. (When \( p = 0 \), everything is in “freefall”, and there can be no breaking of waves). The expanding wave theory we present provides a possible quantitative explanation for the formation of such a void.
7) How do you intend to develop your research from here?

Our present paper demonstrates that there is some choice of the number $a$, (we proved it exists, but still do not know its precise value), such that the member of our family of expanding waves corresponding to that value of the acceleration parameter $a$, will account for the leading order correction of the anomalous acceleration. That is, it can account for how the plot of redshift-vs luminosity of the galaxies curves away from a straight line at the center. But once the correct value of $a$ is determined exactly, that value will give a prediction of how the plot should change beyond the first breaking of the curve. (There are no more free parameters to adjust!) We are currently working on finding that exact value of $a$ consistent with the observed anomalous acceleration, so that from this we can calculate the next order correction it predicts, all with the goal of comparing the expanding wave prediction to the observed redshift vs luminosity plot, to see if it does better than the prediction of dark energy.

8) What is your view on the relevance of the Copernican Principle to these new expanding waves?

These self-similar expanding waves represent possible end states of the expansion of the Big Bang that we propose could emerge at the end of the radiation phase when there exists a mechanism for their formation. We imagine that decay to such an expanding wave could have occurred locally in the vicinity of the earth, over some length scale, but we can only conjecture as to what length scale that might be—the wave could extend out to some fraction of the distance across the visible universe or it could extend even beyond, we cannot say, but to explain the
anomalous acceleration the earth must lie within some proximity of the center. That is, for the $a > 1$ wave to account for the anomalous acceleration observed in the galaxies, we would have to lie in some proximity of the center of such a wave to be consistent with no observed angular dependence in the redshift vs luminosity plots. (The void theory has the same implication.)

Now one might argue that our expanding waves violate the so-called *Copernican* or *Cosmological Principle* which states that *on the largest length scale* the universe looks the same everywhere. This has been a simplifying assumption taken in cosmology since the mid thirties when Howard Robertson and Geoffrey Walker proved that the Friedman spacetimes of the standard model, (constructed by Alexander Friedman a decade earlier), are the unique spacetimes that are spatially *homogeneous* and *isotropic* about every point—a technical way of saying there is no special place in the universe. The introduction of dark energy via the cosmological constant is the only way to preserve the Copernican Principle and account for the anomalous acceleration on the largest scale, everywhere. The stars, galaxies and clusters of galaxies are evidence of small scale variations that violate the Copernican Principle on smaller length scales. We are arguing that there could be an even larger length scale than the clusters of galaxies on which local decay to one of our expanding waves has occurred, and we happen to be near the center of one. This would violate the Copernican Principle if these expanding waves describe the entire universe—but our results allow for the possibility that on a scale even larger than the scale of the expanding waves, the universe may look everywhere the same like the standard model. Thus our view is that the Copernican principle is really a moot issue here. But it does beg the question as to how
big the effective center can be for the value of $a$ that accounts for the anomalous acceleration. This is a problem we hope to address in the future.

Another way to look at this is, if you believe there is no cosmological constant or dark energy, [see (12) below], then the anomalous acceleration may really be the first definitive evidence that in fact, by accident, we just happen to lie near the center of a great expansion wave of cosmic dimensions. We believe our work at this stage gives strong support for this possibility.

9) How large would the displacement of matter caused by the expanding wave be, and how far out would it extend?

For our model, the magnitude of the displacement depends on the value of the acceleration parameter $a$. It can be very large or very small, and we argue that somewhere in between it can be right on for the first breaking of the observed redshift vs luminosity curve near the center. To meet the observations, it has to displace the position of a distant galaxy the right amount to displace the straight line redshift vs luminosity plot of the standard model, into the curved graph observed. The paper by Clifton and Ferreira referenced in our paper, (exposition of this appeared as the cover article in Scientific American a few months ago), quote that the bubble of underdensity observed today should extend out to about one billion lightyears, about a tenth of the distance across the visible universe, and the size of the center consistent with no angular variation is about 15 megaparsecs, about 50 million lightyears, and this is approximately the distance between clusters of galaxies, a distance about $1/200$
across the visible universe.²

10) How do the spacetimes associated with the expanding waves compare to the spacetime of the standard model of cosmology?

Interestingly, we prove that the spacetimes associated with the expanding waves when \( a \neq 1 \) actually have properties surprisingly similar to the standard model \( a = 1 \). Firstly, the expanding spacetimes \( (a \neq 1) \) look more and more like the standard model \( a = 1 \) as you approach the center of expansion. (That is why you have to go far out to see an anomalous acceleration.) Moreover, out to a great distance from the center, say

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²The following back of the envelope calculation provides a ballpark estimate for what we might expect the extent of the remnants of one of our expanding waves might be today. Our thesis is that the self-similar expanding waves that can exist during the pure radiation phase of the standard model, can emerge at the end of the radiation phase by the dissipation created by the strong nonlinearities. Now matter becomes transparent with radiation at about 300,000 years after the Big Bang, so we might estimate that our wave should have emerged by about \( t_{\text{endrad}} \approx 10^5 \) years after the Big Bang. At this time, the distance of light-travel since the Big Bang is about \( 10^5 \) lightyears. Since the sound speed \( c/\sqrt{3} \approx 0.58c \) during the radiation phase is comparable to the speed of light, we could estimate that dissipation that drives decay to the expanding wave might reasonably be operating over a scale of \( 10^5 \) lightyears by the end of the radiation phase. Now in the \( p = 0 \) expansion that follows the radiation phase, the scale factor (that gives the expansion rate) evolves like

\[ R(t) = t^{2/3}, \]

so a distance of \( 10^5 \) lightyears at \( t = t_{\text{endrad}} \) years will expand to a length \( L \) at present time \( t_{\text{present}} \approx 10^{10} \) years by a factor of

\[ \frac{R(t_{\text{present}})}{R(t_{\text{endrad}})} = \frac{(10^{10})^{2/3}}{(10^5)^{2/3}} = 10^{4.7} \geq 5 \times 10^4. \]

It follows then that we might expect the scale of the wave at present time to extend over a distance of about

\[ L = 5 \times 10^5 \times 10^4 = 5 \times 10^9 \text{ lightyears}. \]

This is a third to a fifth of the distance across the visible universe, and agrees with the extent of the under-density void region quoted in the Clifton-Ferriera paper, with room to spare.
out to about 1/3 to 1/2 the distance across the visible universe, (where the anomalous acceleration is apparent), we prove that (to within negligible errors) there is a time coordinate $t$ such that the 3-space at each fixed $t$ has zero curvature, just like the standard model of cosmology, and observers fixed in time or at a fixed distance from the center, will measure distances and times exactly the same as in a Friedman universe, the spacetime of the standard model of cosmology. In technical terms, only line elements changing in space and time will measure dilation of distances and times relative to the standard model. This suggests that it would be easy to mistake one of these expanding waves for the Friedman spacetime itself until you did a measurement of redshift vs luminosity far out where the differences are highly apparent, (that is, you measured the anomalous acceleration).

11) Your expanding wave theory is more complicated than a universe filled with dark energy, and we have to take into account the Occam’s razor principle. What do you think about this assertion?

To quote Wikipedia, Occam’s razor states “the explanation of any phenomenon should make as few assumptions as possible, eliminating those that make no difference in the observable predictions of the explanatory hypothesis or theory.”

We could say that our theory does not require the extra hypothesis of dark energy or a cosmological constant to explain the anomalous acceleration. Since there is no obvious reason why an expansion wave with one value of $a$ over another would come out locally at any given location at the end of the radiation phase, and since we don’t need dark energy in the expanding wave ex-
planation, we could argue that the expanding wave explanation of the anomalous acceleration is simpler than dark energy. But a better answer is that our theory has an observable prediction, and only experiments, not the 14th century principle of Occam, can resolve the physics. Occams razor will have nothing whatsoever to say about whether we are, or are not, near the center of a cosmic expansion wave.

12) If, as you suggest, dark energy doesn’t exist, what is the ingredient of 75% of the mass-energy in our universe?

In short, nothing is required to replace it. The term “anomalous acceleration” of the galaxies begs the question “acceleration relative to what?” The answer is that the anomalous acceleration of the galaxies is an acceleration relative to the prediction of the standard model of cosmology. In the expanding wave theory, we prove that there is no “acceleration” because the anomalous acceleration can be accounted for in redshift vs luminosity by the fact that the galaxies in the expanding wave are displaced from their anticipated position in the standard model. So the expanding wave theory requires only classical sources of mass-energy for the Einstein equations.

13) If dark energy doesn’t exist, it would be just an invention. What do you think about dark energy theory?

Keep in mind that Einstein’s equations have been confirmed without the need for the cosmological constant or dark energy, in every physical setting except in cosmology.

Dark energy is the physical interpretation of the cosmological constant. The cosmological constant is a source term with a free parameter, (similar to but different from our $a$), that can be
added to the original Einstein equations and still preserve the frame independence, the "general relativity" if you will, of Einstein's equations. Einstein's equations express that mass-energy is the source of spacetime curvature. So if you interpret the cosmological constant as the effect of some exotic mass-energy, then you get dark energy. For the value of the cosmological constant required to fit the anomalous acceleration observed in the redshift vs luminosity data, this dark energy must account for some 73 percent of the mass-energy of the universe, and it has to have the physical property that it \textit{anti-gravitates}—that is, it gravitationally repels instead of attracts. Since no one has ever observed anything that has this property, (it would not fall to earth like an apple, it would fly up like a balloon), it seems rather suspect that such mass-energy could possibly exist. If it does exist, then it also is not like any other mass-energy in that the density of it stays constant, stuck there at the same value forever, even as the universe expands and spreads all the other mass-energy out over larger and larger scales—and there is no principle that explains why it has the value it has.\footnote{In the expanding wave theory, the principle for determining $a$ is that all values of $a$ near $a = 1$ should be (roughly) equally likely to appear, and one of them did...} On the other hand, if you put the cosmological constant on the other side of the equation with the curvature then there is always some (albeit very small) baseline curvature permeating spacetime, and the zero curvature spacetime is no longer possible; that is, the empty space Minkowski spacetime of special relativity no longer solves the equations. So when the cosmological constant is over on the curvature side of Einstein's equation, the equations no longer express the physical principle that led Einstein to discover them in the first place—that mass-energy should be the
sole source of spacetime curvature.

Einstein put the cosmological constant into his equations shortly after he discovered them in 1915, because this was the only way he could get the possibility of a static universe. (Anti-gravity holds the static universe up!) After Hubble proved that the universe was expanding in 1929, Einstein took back the cosmological constant, declaring it was the greatest blunder of his career, as he could have predicted the expansion ahead of time without it. At the time, taking out the cosmological constant was interpreted as a great victory for general relativity. Since then, cosmologists have become more comfortable putting the cosmological constant back in. There are many respected scientists who see no problem with dark energy.

14) How does the coincidence in the value of the cosmological constant in the dark energy theory compare to the coincidence that the Milky Way must lie near a local center of expansion in the expanding wave theory?

The dark energy explanation of the anomalous acceleration of the galaxies requires a value of the cosmological constant that accounts for some 73 percent of the mass-energy of the universe. That is, to correct for the anomalous acceleration in the supernova data, you need a value of the cosmological constant that is just three times the energy-density of the rest of the mass-energy of the universe. Now there is no principle that determines the value of the cosmological constant ahead of time, so its value could apriori be anything. Thus it must be viewed as a great coincidence that it just happens to be so close to the value of the energy density of the rest of the mass-energy of the universe. (Keep in mind that the energy-density of all the classical
sources decreases as the universe spreads out, while the cosmological constant stays constant.) So why does the value of the cosmological constant come out so close to, just 3 times, the value of the rest of the mass-energy of the universe, instead of $10^{10}$ larger or $10^{-10}$ smaller? This raises a very suspicious possibility. Since the magnitude of the sources sets the scale for the overall oomph of the solution, when you need to adjust the equations by an amount on the order of the sources present in order to fit the data, that smacks of the likelihood that you are really just adding corrections to the wrong underlying solution. So to us it looks like the coincidence in the value of the cosmological constant in the dark energy theory may well be greater than the coincidence that we lie near a local center of expansion in the expanding wave theory.

**In summary:** Our view is that the Einstein equations make more physical sense without dark energy or the cosmological constant, and dark energy is most likely an unphysical *fudge factor*, if you will, introduced into the theory to meet the data. But ultimately, whether dark energy or an expanding wave correctly explains the anomalous acceleration of the galaxies can only be decided by experiments, not the Copernican Principle or Occam’s Razor.

Blake Temple and Joel Smoller, August 17, 2009.