Applied Mathematics vs Dark Energy

Collaborators: Joel Smoller and Zeke Vogler

Blake Temple, UC-Davis

Special Session on Nonlinear Conservation Laws
Spring Western Sectional Meeting of AMS
April 18, 2015
GR Simple-Waves Trigger
An Instability in SM
which creates the
Anomalous Acceleration
without
Dark Energy

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The 1999 observations of redshift vs luminosity for type Ia supernovae in nearby galaxies won the Nobel Prize because they discovered the Anomalous Acceleration:

The universe is expanding faster than the Standard Model of Cosmology (SM), based on Einstein's original theory of General Relativity, allows.
The only way to preserve the Cosmological Principle—that on the largest length scale the universe is described by a Friedmann Space-Time which holds no special place—is to add the Cosmological Constant to Einstein's equations as a source term. Its interpretation is Dark Energy.
A best fit among Friedmann Space-Times with Dark Energy leads to the conclusion that the universe is a critical $k=0$ Friedmann Space-Time with Seventy Percent Dark Energy

$$\Omega_\Lambda \approx 0.7$$
2007 PI talk in Relativity Session at AMS National Meeting in New Orleans:

We proposed the idea that a Simple Wave from the Radiation Epoch of the Big Bang might account for the Anomalous Acceleration of the Galaxies Without Dark Energy.
Our Motivation

The Radiation Epoch: After Inflation until about 30,000 years after the Big Bang is evolution by Relativistic Compressible Euler Equations

The $p$-system with $p = \frac{c^2}{3} \rho$
Every characteristic field contributes to Decay in the sense of Glimm and Lax

Stefan-Bolzmann Law: \( \rho = aT^4 \) (No Contact Discontinuities)

\[ p = \frac{c^2}{3} \rho \]

The \( p \)-system with:

- Enormous sound speed \( \sigma \approx 0.57c \)
- Enormous modulus of Genuine Nonlinearity
It is reasonable to expect fluctuations would decay to simple wave patterns by the End of Radiation.

This is our Starting Assumption.
Stages of the Standard Model:

- **Inflation**
  - $10^{-35}$ s to $10^{-30}$ s

- **Big Bang**

  **Pure Radiation**
  - $10^{-30}$ to $3 \times 10^5$ yrs

  \[ p = \frac{c^2}{3} \rho \]  
  (Relativistic $p$-system)

Uncoupling of Matter and Radiation

\[ t \approx 3 \times 10^5 \]

(Neglect Radiation Pressure)

\[ p \approx 0 \]

Time of CMB 379,000 yr
Pursuing this Idea...

...we identified a 1-parameter family of Self-Similar Waves that perturb the Standard Model during the Radiation Epoch.
And proposed that these might induce an Anomalous Acceleration at a later time.

We set out our ideas in PNAS in 2009 and Memoirs of the AMS in 2011
Our interest is in the possible connection between these waves and the Anomalous Acceleration.

In Fact: This family of self-similar solutions was already known to exist

Cahill and Taub:

Extended by others, esp. Carr and Coley, Survey:
Around 2007:
Other research groups began exploring the possibility that the anomalous acceleration might be due to the earth lying near the center of a large region of Under-Density.

We first saw publication in 2009.
The record is clear on one thing:

No one before us proposed this family of waves as a mechanism that could account for the Anomalous Acceleration without Dark Energy.
We have now accomplished our goal of bringing the effects of these waves up to present time to compare with Dark Energy.

There are several surprises... in this talk I present what we have found...
We identify an instability in the SM based on a new (closed) asymptotic ansatz for local perturbations of the critical $k=0$ Friedmann Spacetime when $p=0$.

The instability naturally creates a region of accelerated uniform expansion on the scale of the supernova data within Einstein's original theory, without Dark Energy.

The region is one order of magnitude larger in extent than expected.
The instability is triggered by our time asymptotic perturbations of SM from the Radiation Epoch when: \[ p = \frac{c^2}{3} \rho \]

Surprisingly—The perturbations at the end of radiation do not directly cause the Anomalous Acceleration as we originally conjectured in PNAS.

Rather—It is the non-trivial phase portrait of the instability they trigger when \( p=0 \) that creates the later accelerations.
A phase portrait of the instability places the SM at a classic... **Unstable Saddle Rest Point**

![Phase Portrait Diagram]

- **Stable Rest Point**
- **Unstable Rest Point**
- **SM**
- **1-Parameter Family of \( a \)-waves**
\[ w' = -\left(\frac{1}{6}z + \frac{1}{3}w + w^2\right) \]

**Present Universe in the Wave Theory**

- Stable Rest Point
- Same Hubble Constant
- Same .425 Acceleration

As Dark Energy
The region of accelerated uniform expansion introduces precisely the same range of quadratic corrections to red-shift vs luminosity as does the cosmological constant in the theory of DE.

\[ H_0 d_\ell = z + Q z^2 + O(z^3) \]

\[ .25 \leq Q \leq .425 \leq .5 \]
The results lead naturally to a testable alternative to Dark Energy within Einstein's original theory…

Without the Cosmological Constant.

Our Proposal: The AA is due to a local under-dense perturbation of the SM on the scale of the supernova data, arising from time-asymptotic perturbations of SM from the Radiation Epoch that trigger an instability in the SM when the pressure drops to zero.
A calculation shows the cubic correction is of the same order, but of a different sign, than the cubic correction in DE theory...

\[ H_0 d_\ell = z + 0.425 z^2 - 0.1804 z^3 \]  

\[ H_0 d_\ell = z + 0.425 z^2 + 0.3591 z^3 \]
We address ONLY the anomalous acceleration... further assumptions regarding space-time far from the center would be required to connect the theory with other measurements...
INTRODUCTION TO COSMOLOGY
Edwin Hubble (1889-1953)

- Hubble’s Law (1929):
  
  "The galaxies are receding from us at a velocity proportional to distance"

- Universe is Expanding

- Based on Redshift vs Luminosity
Universe measured to 1% accuracy

By James Morgan
Science reporter, BBC News, Washington DC

Astronomers have measured the distances between galaxies in the universe to an accuracy of just 1%.

This staggeringly precise survey - across six billion light-years - is key to mapping the cosmos and determining the nature of dark energy.

The new gold standard was set by BOSS (the Baryon Oscillation Spectroscopic Survey) using the Sloan Foundation Telescope in New Mexico, US.
**Frozen ripples**
The BOSS team used baryon acoustic oscillations (BAOs) as a "standard ruler" to measure intergalactic distances.

BAOs are the "frozen" imprints of pressure waves that moved through the early universe - and help set the distribution of galaxies we see today.

"Nature has given us a beautiful ruler," said Ashley Ross, an astronomer from the University of Portsmouth.

"The ruler happens to be half a billion light years long, so we can use it to measure distances precisely, even from very far away."
Conclude: The universe appears (and is assumed) uniform on a scale of about 1/20th the distance across the visible universe

\[ \xi = \frac{r}{ct} \approx 0.05 \]
10 billion light-years \(\approx\) Visible Universe

500 million light-years \(\approx\) Uniform Density

- 50 million light-years \(\approx\) Separation between clusters of galaxies

10 million light-years \(\approx\) diameter of a cluster

- 1 million light-years \(\approx\) separation between galaxies in a cluster

100 thousand light-years \(\approx\) distance across Milky Way

- 28 thousand light-years \(\approx\) distance to galactic center
Standard Model of Cosmology

1922 (Alexander Friedmann):
Derived FRW solutions of the Einstein equations:
3-space of constant curvature expanding in time:

\[ ds^2 = -dt^2 + R(t)^2 \left\{ \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right\} \]

The Big Bang theory based on the FRW metric was worked out by George Lemaître in the late 1920’s leading to Hubble’s confirmation of redshift vs luminoscity consistent with an FRW spacetime

Hubble’s Constant \( \equiv H \equiv \frac{\dot{R}}{R} \)
In 1935: Howard Robertson and Arthur Walker derived Friedmann spacetime from the

Copernican Principle:
“Earth is not in a special place in the Universe”

- R-W: Friedmann uniquely determined by condition
  Homogeneous and Isotropic about every point
  Any point can be taken as $r = 0$
  Each $t=\text{const}$ surface is a 3-space of constant scalar curvature
Observations of the micro-wave background IMPLY
\[ k = 0 \]

“Critical expansion to within about 2-percent”
The Friedmann metric when $k=0$:

\[ ds^2 = -dt^2 + R(t)^2 \left\{ dr^2 + r^2 d\Omega^2 \right\} \]

The universe is infinite flat space $\mathbb{R}^3$ at each fixed time:

(Assumed to Apply on the Largest Length Scale)
Standard Model of Cosmology

- FRW metric, $k=0$:

$$ds^2 = -dt^2 + R(t)^2 \left\{ dr^2 + r^2 d\Omega^2 \right\}$$

- $D = Rr$ Measures distance between galaxies at each fixed $t$

- Conclude:

$$\dot{D} = \dot{R}r = \frac{\dot{R}}{R} Rr = H D$$

Hubble’s Constant $\equiv H \equiv \frac{\dot{R}}{R}$
Standard Model of Cosmology

Hubble’s Law:

\[
\dot{D} = HD
\]

Conclude--

``The universe is expanding like a balloon''
The inverse Hubble Constant estimates the Age of the Universe

\[ \frac{1}{H_0} \approx 10^{10} \text{ years} \approx \text{age of universe} \]

\[ \frac{c}{H_0} \] is the distance of light travel since the Big Bang, a measure of the size of the visible universe

\[ \frac{c}{H_0} = \text{Hubble Length} \approx 10^{10} \text{ lightyears} \]
Measuring the Hubble Constant

Measures distance from Earth to distant galaxy at present time $t_0$

$H_0D = \dot{D}$

Hubble’s Law

$D \approx d_\ell \equiv$ luminosity distance

$\dot{D} \approx z \equiv$ redshift factor $= \frac{\lambda_0 - \lambda_e}{\lambda_e}$

$H_0d_\ell = z + \frac{1}{4}z^2 - \frac{1}{8}z^3 + O(z^4)$

Friedmann $k = 0$
Up until 1999, we could only measure the leading linear term:

\[ H_0 d_\ell = z + \frac{1}{4} z^2 - \frac{1}{8} z^3 + O(z^4) \]

Friedmann

\[ k = 0 \]

\[ z \ll 1 \]

\[ H_0 \approx h_0 \frac{100 \text{ km}}{s \text{ mpc}} \]

\[ h_0 \approx 0.68 \]

\[ \text{mpc} \approx 3.2 \text{ million light years} \]

``A galaxy at a distance of one mega-parsec is receding at about 68 kilometers per second..."
The 1999 supernova data tested the dependence of the Hubble constant on time, and the results don’t fit standard model...

“Anomalous Acceleration of Galaxies”

Introduction of “Cosmological Const” and “Dark Energy”

Dark energy is non-classical

Negative pressure $\rightarrow$ Anti-gravity effect
Recent supernova data have tested the dependence of the Hubble constant on time, and the results don’t fit standard model...

\[ H_0 d_\ell = z + \frac{1}{4} z^2 - \frac{1}{8} z^3 + O(z^4) \]

Friedmann
\[ k = 0 \]

This is measured at about \( 0.425 \) not \( 0.25 \)
Recent supernova data have tested the dependence of the Hubble constant on time, and the results don’t fit standard model...

This is usually interpreted in terms of a Best Fit to Friedmann Universes with the Cosmological Constant

\[(k, \Omega_\Lambda) \rightarrow k = 0, \Omega_\Lambda \approx .7\]
Thanks to Philip Hughs

UM-Astronomy

Standard Model

k=0 FRW

Supernova Data

“Not a Good Fit”

Thanks to Philip Hughs

UM-Astronomy
That is: To preserve the **Copernican Principle**, that the Universe on the Largest Length Scale is evolving according to a **Uniform Friedmann Spacetime** with $p=0, k=0$. A **Cosmological Constant** must be added to Einstein’s Equations.

The **Physical Interpretation** is **Dark Energy**
Thanks to Philip Hughs
UM-Astronomy

Best Fit:
70% Dark Energy
30% Classical Energy
Einstein Equations for Friedmann:

- **Einstein Equations (1915):** \[ G_{ij} = \kappa T_{ij} \]

\[ G_{ij} = \text{Einstein Curvature Tensor} \]

\[ T_{ij} = (\rho + p)u_iu_j + pg_{ij} = \text{Stress Energy Tensor (perfect fluid)} \]

- **Einstein Equations for k=0 Friedmann metric:**

\[ H^2 = \frac{\kappa}{3}\rho \]

\[ \dot{\rho} = -3(\rho + p)H \]

**Solutions determined by equation of state:** \[ p = p(\rho) \]
Incorporating Dark Energy into Friedmann

- Assume Einstein equations with a cosmological constant:
  \[ G_{ij} = 8\pi T_{ij} + \Lambda g_{ij} \]

- Assume \( k = 0 \) FRW:
  \[ ds^2 = -dt^2 + R(t)^2 \left\{ dr^2 + r^2 d\Omega^2 \right\} \]

- Leads to:
  \[ H^2 = \frac{\kappa}{3} \rho + \frac{\kappa}{3} \Lambda \]

- Divide by \( H^2 = \frac{\kappa}{3} \rho_{\text{crit}} \)
  \[ 1 = \Omega_M + \Omega_\Lambda \]

- Best data fit leads to \( \Omega_\Lambda \approx .7 \) and \( \Omega_M \approx .3 \)

- Implies: The universe is 70 percent dark energy
m - M = "Distance Modulus"

M = absolute Magnitude

m = apparent magnitude

d = distance in parsecs:

m - M = 5 log(d) - 5

z = redshift factor

1 + z = \frac{\lambda_{emit}}{\lambda_{obs}}

\Omega_m + \Omega_\Lambda = 1\text{ for a flat } (k = 0)\text{ universe.}

Best Fit:
70% Dark Energy
30% Classical Energy
Standard Model
Composition of Universe

- Heavy Elements: 0.03%
- Neutrinos: 0.3%
- Stars: 0.5%
- Free Hydrogen and Helium: 4%
- Dark Matter: 25%
- Dark Energy: 70%

Courtesy of NASA
The Question we Explore:

“Could the **Anomalous Acceleration of the galaxies** be due to the fact that we are looking outward into an expansion wave that formed during the **Radiation Epoch of the Big Bang**?”
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“Could the Anomalous Acceleration of the galaxies be due to the fact that we are looking outward into an expansion wave that formed during the Radiation Epoch of the Big Bang?”

🌟 The Einstein equations have been confirmed without the cosmological constant in every setting except cosmology...
The Question we Explore:

“Could the Anomalous Acceleration of the galaxies be due to the fact that we are looking outward into an expansion wave that formed during the Radiation Epoch of the Big Bang?”

🌟 The Einstein equations have been confirmed without the cosmological constant in every setting except cosmology...

Note: A general expansion wave has a center of expansion...
Summary of our results for the Wave Theory
Hubbles Law:

\[ H_0 \ d_\ell = z \]

(1929)

Hubble’s Constant  Luminosity Distance  Redshift Factor

Measured value:

\[ H_0 = h_0 \ \frac{100km}{s \ mpc} \]

\[ h_0 \approx 0.68 \]
The 1999 Supernova data was refined enough to measure the quadratic correction to Hubble’s Relation:

\[ H_0 d_\ell = z + Q z^2 \]
Einstein’s Equations: \[ G = \kappa T + \Lambda g \]

\[ \Omega_M + \Omega_\Lambda = 1 \]

\[ H_0 d_\ell = z + 0.425z^2 + O(z^3) \]

\[ H_0 d_\ell = z + 0.25z^2 + O(z^3) \]

Anomalous Acceleration

Friedmann
\[ \Omega_\Lambda = 0 \]

Friedmann
\[ \Omega_\Lambda = 0.7 \]
A small wave perturbation at the end of radiation will expand to create a large region of accelerated uniform expansion at the Center of the Wave.
WE PROVE: The Friedmann Universe is UNSTABLE

A small wave perturbation at the end of radiation will expand to create a large region of accelerated uniform expansion at the Center of the Wave.

This induces exactly the same range of quadratic corrections to redshift vs luminosity as does Dark Energy.
WE PROVE: The Friedmann Universe is UNSTABLE

The self-similar perturbations we identified at the end of the radiation epoch TRIGGER this instability when $p=0$

This induces exactly the same range of quadratic corrections to redshift vs luminosity as does Dark Energy
WE PROVE: The Friedmann Universe is UNSTABLE

The self-similar perturbations we identified at the end of the radiation epoch TRIGGER this instability when $p=0$

This induces exactly the same range of $Q$ as does Dark Energy:

$$H_0d_L = z + Qz^2 + O(z^3)$$
\[
H_0 d_\ell = z + 0.25 (1 + \Omega_\Lambda) z^2 - 0.125 \left(1 + \frac{2}{3} \Omega_\Lambda - \Omega_\Lambda^2\right) z^3 + O(z^4)
\]

\[
0.25 \leq Q \leq 0.5
\]

\[
\Omega_M + \Omega_\Lambda = 1
\]

\[
0 \leq \Omega_\Lambda \leq 1
\]

In the case \( \Omega_M = 0.3, \Omega_\Lambda = 0.7 \) this gives

\[
H_0 d_\ell = z + 0.425 z^2 - 0.1804 z^3 + O(z^4)
\]
Our Wave Theory

\[ H_0 d_\ell = z + Q(z_2, w_0)z^2 + C(z_2, w_0, w_2)z^3 + O(z^4) \]

\[ 0.25 \leq Q \leq 0.5 \]

as

\[ z'_2 = -3w_0 \left( \frac{4}{3} + z_2 \right) \]

\[ w'_0 = - \left( \frac{1}{6} z_2 + \frac{1}{3} w_0 + w^2_0 \right) \]

Orbit evolves to a NEW STABLE REST POINT

- A Wave with Underdensity:

\[ \frac{\rho_{SM} - \rho_{ssw}}{\rho_{SM}} = 7.45 \times 10^{-6} \]

\[ H_0 d_\ell = z + 0.425z^2 + 0.3591z^3 + O(z^4) \]
Conclusion: The cubic correction is of the same order, but of a different sign, from Dark Energy… …A Testable Prediction!

\[ H_0 d_\ell = z + 0.425 z^2 - 0.1804 z^3 \]  
\[ H_0 d_\ell = z + 0.425z^2 + 0.3591z^3 \]
The $k = 0$ Friedmann spacetimes admit self-similar expressions when $\rho = \sigma^2 \rho$

\[
ds^2 = -B(\xi)dt^2 + \frac{1}{A(\xi)}dr^2 + r^2d\Omega^2
\]

$\xi = \frac{r}{ct}$  “Fractional Distance to Hubble Length”

$\rho r^2 = z(\xi)$  “Dimensionless Density”

$\frac{v}{\xi} = w(\xi)$  “Dimensionless Velocity”
\[ ds^2 = -B_F(\xi) \, d\tilde{t}^2 + \frac{1}{A_F(\xi)} \, d\tilde{r}^2 + \tilde{r}^2 \, d\Omega^2 \]

\[ A_F(\xi) = 1 - \frac{4}{9} \xi^2 - \frac{8}{27} \xi^4 + O(\xi^6) \]
\[ D_F(\xi) \equiv \sqrt{A_F B_F} = 1 - \frac{1}{9} \xi^2 + O(\xi^4). \]

\[ z_F(\xi) = \frac{4}{3} \xi^2 + \frac{40}{27} \xi^4 + O(\xi^6) \]
\[ \omega_F \equiv \frac{\nu}{\xi} = \frac{2}{3} + \frac{2}{9} \xi^2 + O(\xi^4) \]

The \( p=0 \) Friedmann Universe in Self-Similar Coordinates
Self-similar coordinates for Friedmann with Pure Radiation

\[ p = \frac{c^2}{3} \rho \]
\[ \sigma = \frac{1}{\sqrt{3}} \]

\[ z_{1/3} = \frac{3}{4} \bar{\xi}^2 + \frac{9}{16} \bar{\xi}^4 + O(\bar{\xi}^6) \]
\[ \nu_{1/3} = \frac{1}{2} \bar{\xi} + \frac{1}{8} \bar{\xi}^3 + O(\bar{\xi}^5) \]
\[ A_{1/3} = 1 - \frac{1}{4} \bar{\xi}^2 - \frac{1}{8} \bar{\xi}^4 + O(\bar{\xi}^6) \]
\[ D_{1/3} = 1 + O(\bar{\xi}^4) \]
The Friedmann Universe extends to 1-parameter family of Self-Similar spacetimes that perturb the Standard Model during the Radiation Epoch:

\[ p = \frac{c^2}{3} \rho \]

The Friedmann Universe DOES NOT admit Self-Similar perturbations!

(Something has to give when \( p \) drops to zero!)

(The topic of our PNAS and MEMOIR)
A 1-parameter family of solutions depending on the **Acceleration Parameter** $0 < a < \infty$

\[ z_{1/3}^a = \frac{3a^2}{4} \bar{\xi}^2 + \frac{3a^2(2+a^2)}{16} \bar{\xi}^4 + O(\bar{\xi}^6) \]

\[ v_{1/3}^a = \frac{1}{2} \bar{\xi} + \frac{2-a^2}{8} \bar{\xi}^3 + O(\bar{\xi}^5) \]

\[ A_{1/3}^a = 1 - \frac{a^2}{4} \bar{\xi}^2 + \frac{a^2(1-3a^2)}{16} \bar{\xi}^4 + O(\bar{\xi}^6) \]

\[ D_{1/3}^a = 1 + O(\bar{\xi}^4) \]
The **ANSATZ** that triggers the instability when \( p=0 \):
The ANSATZ that triggers the instability:

\[
\begin{align*}
    z(t, \xi) &= \left( \frac{4}{3} + z_2(t) \right) \xi^2 + \left\{ \frac{40}{27} + z_4(t) \right\} \xi^4 + O(\xi^6), \\
    w(t, \xi) &= \left( \frac{2}{3} + w_0(t) \right) + \left\{ \frac{2}{9} + w_2(t) \right\} \xi^2 + O(\xi^4),
\end{align*}
\]
The ANSATZ:

\[ z(t, \xi) = \left( \frac{4}{3} + z_2(t) \right) \xi^2 + \left\{ \frac{40}{27} + z_4(t) \right\} \xi^4 + O(\xi^6), \]

\[ w(t, \xi) = \left( \frac{2}{3} + w_0(t) \right) + \left\{ \frac{2}{9} + w_2(t) \right\} \xi^2 + O(\xi^4), \]

\[ \xi = \frac{r}{ct} \]

"Fractional Distance to Hubble Length"
Theorem: The $p = 0$ waves take the asymptotic form

\[ z(t, \xi) = \left( \frac{4}{3} + z_2(t) \right) \xi^2 + \left\{ \frac{40}{27} + z_4(t) \right\} \xi^4 + O(\xi^6), \]

\[ w(t, \xi) = \left( \frac{2}{3} + w_0(t) \right) + \left\{ \frac{2}{9} + w_2(t) \right\} \xi^2 + O(\xi^4), \]

The ANSATZ: $\xi = \frac{r}{ct}$

"Fractional Distance to Hubble Length"

\[ z(t, \xi) = \rho r^2 \]

"Dimensionless Density"
THEOREM: The \( p = 0 \) waves take the asymptotic form

\[
\begin{align*}
z(t, \xi) &= \left( \frac{4}{3} + z_2(t) \right) \xi^2 + \left\{ \frac{40}{27} + z_4(t) \right\} \xi^4 + O(\xi^6), \\
w(t, \xi) &= \left( \frac{2}{3} + w_0(t) \right) + \left\{ \frac{2}{9} + w_2(t) \right\} \xi^2 + O(\xi^4),
\end{align*}
\]

The ANSATZ:

\[
\dot{\xi} = \frac{r}{ct} \quad \text{“Fractional Distance to Hubble Length”}
\]

\[
z(t, \xi) = \rho r^2 \quad \text{“Dimensionless Density”}
\]

\[
w(t, \xi) = \frac{v}{\xi} \quad \text{“Dimensionless Velocity”}
\]
The ANSATZ:

\[ z(t, \xi) = \left( \frac{4}{3} + z_2(t) \right) \xi^2 + \left\{ \frac{40}{27} + z_4(t) \right\} \xi^4 + O(\xi^6), \]

\[ \rho(t) \sim \frac{\left( \frac{4}{3} + z_2(t) \right)}{t^2} = \frac{f(t)}{t^2} \]

Uniform Density out to errors \( \xi^4 \)

\[ z(t, \xi) = \rho r^2 \]
THEOREM: The $p = 0$ waves take the asymptotic form

\[
z(t, \xi) = \left( \frac{4}{3} + z_2(t) \right) \xi^2 + \left\{ \frac{40}{27} + z_4(t) \right\} \xi^4 + O(\xi^6),
\]

\[
w(t, \xi) = \left( \frac{2}{3} + w_0(t) \right) + \left\{ \frac{2}{9} + w_2(t) \right\} \xi^2 + O(\xi^4),
\]

where $z_2(t), z_4(t), w_0(t), w_2(t)$ evolve according to the equations

\[
-t \dot{z}_2 = 3 w_0 \left( \frac{4}{3} + z_2 \right),
\]

\[
-t \dot{z}_4 = -5 \left\{ \frac{2}{27} z_2 + \frac{4}{3} w_2 - \frac{1}{18} z_2^2 + z_2 w_2 \right\}
\]

\[
-t \dot{w}_0 = \frac{1}{6} z_2 + \frac{1}{3} w_0 + w_0^2,
\]

\[
-t \dot{w}_2 = \frac{1}{10} z_4 + \frac{4}{9} w_0 - \frac{1}{3} w_2 + \frac{1}{24} z_2^2 - \frac{1}{3} z_2 w_0
\]

\[
-\frac{1}{3} w_0^2 + 4 w_0 w_2 - \frac{1}{4} w_0^2 z_2.
\]
Our Wave Theory

\[ H_0 d_\ell = z + Q(z_2, w_0)z^2 + C(z_2, w_0, w_2)z^3 + O(z^4) \]

\[ \begin{align*}
0.25 & \leq Q \leq 0.5 \\
\text{as} & \\
\frac{z'}{r} & = -3w_0 \left( \frac{4}{3} + z_2 \right) \\
\frac{w'}{s} & = -\left( \frac{1}{6}z_2 + \frac{1}{3}w_0 + w_0^2 \right)
\end{align*} \]

Orbit evolves to a NEW STABLE REST POINT
Our Wave Theory

\[ H_0 d_\ell = z + Q(z_2, w_0)z^2 + C(z_2, w_0, w_2)z^3 + O(z^4) \]

\[ .25 \leq Q \leq .5 \]

as

\[ z' = -3w_0 \left( \frac{4}{3} + z_2 \right) \]

\[ w'_0 = -\left( \frac{1}{6}z_2 + \frac{1}{3}w_0 + w_0^2 \right) \]

Orbit evolves to a NEW STABLE REST POINT

- A Wave with Underdensity:
  \[ \frac{\rho_{SM} - \rho_{ssw}}{\rho_{SM}} = 7.45 \times 10^{-6} \]

\[ H_0 d_\ell = z + .425z^2 + .3591z^3 + O(z^4) \]
Stable Rest Point

Unstable Rest Point

1-Parameter Family of $a$-waves, $a < 1$
Strategy: Use our equations to evolve the initial data for a-waves at the end of radiation to determine \((a, T_*)\) that gives the correct anomalous acceleration.

I.e., \((a, T_*)\) that give the observed quadratic correction to redshift vs luminosity at present time.
In the Standard Model $p=0$ at about

\[ t_\star \approx 10,000-30,000 \text{ yrs} \]

\[ T_\star \approx 9000^0 K \]

(Depending on theories of Dark Matter)

Our simulation turns out to be entirely insensitive to the initial $t_\star, T_\star$

I.e., we need only compute the value of the acceleration parameter that gives the correct anomalous acceleration.
THE ANSWER: The value of the acceleration for the wave perturbation of SM that produces a quadratic correction of .425 at the present value of $H_0$ is:

$$a = 0.99999957 = 1 - (4.3 \times 10^{-7})$$

$$H_0d_\ell = z + .425z^2 + .3591z^3$$

This corresponds to an relative under-density of

$$\frac{\rho_{SM} - \rho_{ssw}}{\rho_{SM}} = 7.45 \times 10^{-6}$$
$w, z' = -3w(\frac{4}{3} + z)$

$w' = -\left(\frac{1}{6z} + \frac{1}{3w} + w^2\right)$

Stable Rest Point

$w_0$ Creates…

$\alpha = \alpha$

($\approx 0.00000057$)

- Same Hubble Constant
- Same $0.425$ Acceleration

As Dark Energy
\[ w' = -3w(\frac{4}{3} + z) \]
\[ w' = -(\frac{1}{6z} + \frac{1}{3w} + w^2) \]

- Stable Rest Point
- Present Universe
- Same Hubble Constant
- Same .425 Acceleration

As Dark Energy
The relative underdensity at the end of radiation:

$$\frac{\rho_{SM} - \rho_{ssw}}{\rho_{SM}} = 7.45 \times 10^{-6}$$

The relative underdensity at present time:

$$\frac{\rho_{ssw}(t_0)}{\rho_{SM}(t_0)} = 0.1438 \approx \frac{1}{7}.$$
Conclude:

An under-density of one part in $10^6$ at the end of radiation produces a seven-fold under-density at present time…
CONCLUDE:

The **Standard Model** is **Unstable** to Perturbation by this 1-parameter family of Waves.
Comparison with Dark Energy:

\[ H_0 d_\ell = z + 0.425 z^2 - 0.1804 z^3 \]  
Dark Energy

\[ H_0 d_\ell = z + 0.425 z^2 + 0.3591 z^3 \]  
Wave Theory

A prediction:
The wave contributes MORE to the Anomalous Acceleration 
far from the center

\[ z \sim \frac{d_\ell}{H_0} \sim \frac{r}{ct} \sim \xi \]

Measures Fractional Distance to Hubble Length
\[ z << 1 \]
Neglecting $O(\xi^4)$ errors:
The spacetime near the center evolves toward the Stable Rest Point

- The metric tends to Flat Minkowski Spacetime which is not co-moving with the fluid

- BUT: The evolution creates a uniformly expanding density near the center, which, neglecting relativistic corrections, is CENTER-INDEPENDENT (like Friedmann Spacetimes)
CONCLUDE:

The wave creates a **UNFORMLY EXPANDING SPACETIME** with an **ANOMALOUS ACCELERATION** in a **LARGE, FLAT, CENTER-INDEPENDENT** region near the center of the wave.
Neglecting errors $O(\xi^4)$:

\[ z(t, \xi) = \left( \frac{4}{3} + z_2(t) \right) \xi^2 + \left\{ \frac{40}{27} + z_4(t) \right\} \xi^4 + O(\xi^6), \]

\[ w(t, \xi) = \left( \frac{2}{3} + w_0(t) \right) + \left\{ \frac{2}{9} + w_2(t) \right\} \xi^2 + O(\xi^4) \]

\[ z \sim \text{density} \quad w \sim \text{velocity} \]

\[ \xi = \frac{r}{t} \sim \text{fractional distance to Hubble Length} \]
THEOREM: Neglecting $O(\xi^4)$ errors, as the orbit tends to the Stable Rest Point:

- The Density drops FASTER than SM:

$$\rho_{\text{WAVE}}(t) = \frac{k_0}{t^3(1 + \bar{w})}$$
$$\rho_{\text{SM}}(t) = \frac{4}{3t^2}$$

where $\bar{w}(t)$ and $k_0(t)$ change exponentially slowly.

- The metric tends to FLAT MINKOWSKI:

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$
Theorem: There exists a unique value

\[ a = 0.99999956 \approx 1 - 4.3 \times 10^{-7} \]  

such that:
Theorem: There exists a unique value $a = 0.99999956 \approx 1 - 4.3 \times 10^{-7}$ such that:

- The $p = 0$ evolution starting from this initial data evolves to $H - H_0, Q = .425$ at $t = t_0$, in agreement with Dark Energy at $t = t_{DE}$.
Theorem: There exists a unique value
\[ a = 0.99999956 \approx 1 - 4.3 \times 10^{-7} \] such that:

- The \( p = 0 \) evolution starting from this initial data evolves to \( H - H_0, Q = 0.425 \) at \( t = t_0 \), in agreement with Dark Energy at \( t = t_{DE} \).

- The cubic correction is \( C = 0.3591 \) at \( t = t_0 \), while Dark Energy is \( C = -0.1804 \) at \( t = t_{DE} \).
Theorem: There exists a unique value

\[ a = 0.99999956 \approx 1 - 4.3 \times 10^{-7} \]

such that:

- The \( p = 0 \) evolution starting from this initial data evolves to \( H - H_0, \ Q = .425 \) at \( t = t_0 \), in agreement with Dark Energy at \( t = t_{DE} \).

- The cubic correction is \( C = 0.3591 \) at \( t = t_0 \), while Dark Energy is \( C = -0.1804 \) at \( t = t_{DE} \).

- The age of the universe is:

\[ t_0 \approx 1.45 \ t_{DE} \approx 1.45 \ t_{SM} \approx 9.8 \times 10^9 \text{yr} \]
Around 2007: Other research groups began exploring the possibility that the anomalous acceleration might be due to the earth lying near the center of a large region of Under-Density.

We first saw publication in 2009.
This proposal is still taken seriously in Astrophysics
Some of the more important discrepancies are as follows:

- the $\Lambda$CDM model predicts more galactic satellites (dwarf galaxies) than what has been observed [11] (this can be in part cured by a large merger rate, see however Ref. [12]);

- the Gaussian model for the origin of Universe’s structure has difficulties in explaining the controversial large scale (dark) flow of galaxies [13] (even though the Planck satellite has not seen evidence of such flows in its data), and outliers such as the large relative speed in the Bullet Cluster collision [14];

- our Universe is supplied with a large number of voids, whose sizes and distribution may not be consistent with the $\Lambda$CDM model; moreover the voids should be filled with dwarfs and low surface brightness galaxies [15], which is not what has been observed [16];

- there are hints [17] that the structure growth rate is somewhat slower from that predicted by the $\Lambda$CDM model (alternatively we live in a universe with the equation of state parameter for dark energy $w_{de} < -1$);

- the disagreement between the Hubble Key Project and supernovae measurements of the Hubble constant [18, 19] and that obtained from the Planck data could be an indication that we live in an underdense region, whose size and magnitude would be difficult to reconcile with the standard $\Lambda$CDM with Gaussian initial perturbations (see however [20]).
\[
\begin{align*}
  z'_2 &= -3w_0 \left( \frac{4}{3} + z_2 \right) \\
  w'_0 &= - \left( \frac{1}{6} z_2 + \frac{1}{3} w_0 + w_0^2 \right)
\end{align*}
\]
3. The Initial Data determined by the Self-Similar Waves from the Radiation Epoch
Neglecting $O(\xi^4)$

$\rho(t) = \frac{k_0}{t^3(1+\bar{w})}$, 

$A \approx 1 - (0.063) \xi^2$ 

$D \approx 1 - (0.016) \xi^2$
CONCLUSIONS:

Our Proposal: The AA is due to a local under-density on the scale of the supernova data, created by a self-similar wave from the radiation epoch that triggers an instability in the SM when the pressure drops to zero.

We have made no assumptions regarding the space-time far from the center of the perturbations. The consistency of this model with other observations in astrophysics would require additional assumptions.
CONCLUSIONS:

- This is arguably the simplest explanation for the anomalous acceleration within Einstein’s original theory of GR, without requiring Dark Energy.
- It demonstrates that any local center of the Standard Model of Cosmology is unstable on the largest length scale, to perturbation by exact solutions from the Radiation Epoch.
- These perturbations are stabilized by a nearby stable rest point that generates the same accelerations as Dark Energy.
- It makes testable predictions.
QUESTIONS:

• On what scale would such waves apply?

• If these came from time-asymptotic wave patterns created in an earlier epoch, would we expect a secondary transitional waves far from the center?

• How does cosmology address the instability? Can Dark Energy help? (NO!)

• Implications of a preferred center?

• Is this more fine-tuned than Dark Energy?
There are large scale anomalies in the data indicating a lack of uniformity on the largest length scale.

The main large angular scale anomalies are [4, 5]:

- a high quadrupole-octupole alignment (if accidental, it would occur in about 3% cases);

- a low variance in the lower galactic ecliptic plane and a low skewness in the southern plane;

- a northern/southern ecliptic hemisphere asymmetry (the northern hemisphere correlation function is featureless and lacks power on large angular scales);

- phase correlations on large angular scales shown in figure 2, whose significance is more than three standard deviations and which imply that there are non-Gaussian features on large angular scales;
Prokopek...2013 (Astrophysicist, Utrecht University)

- a dipolar asymmetry, which includes a dipolar modulation and a dipolar power asymmetry;
- a parity asymmetry (which is related to the dipolar modulation) that comes in two disguises: a parity reflection asymmetry and a mirror asymmetry, both of which show significant statistical evidence for low multipoles;
- a very cold spot (on angular scale of about 5° with significance of more than four standard deviations);
- a lack of power on one hemisphere on angular scales corresponding to the multipoles $\ell \in [5, 25]$ that has statistical significance of almost three standard deviations.
Every aspect of this work came from Applied Mathematics,

Whatever its implications to Physics, it stands on its own as a self-contained model in Applied Mathematics.
Mathematics is part of physics…
…the part of physics where experiments are cheap.

—Arnold, Paris, 1997
End
Big Bang blunder bursts the multiverse bubble

Premature hype over gravitational waves highlights gaping holes in models for the origins and evolution of the Universe, argues Paul Steinhardt.

When a team of cosmologists announced at a press conference in March that they had detected gravitational waves generated in the first instants after the Big Bang, the origins of the Universe were once again major news. The reported discovery created a worldwide sensation in the scientific community, the media and the public at large (see Nature 507, 281–283; 2014).

According to the team at the BICEP2 South Pole telescope, the detection is at the 5–7 sigma level, so there is less than one chance in two million of it being a random occurrence. The results were hailed as proof of the Big Bang inflationary theory and its progeny, the multiverse. Nobel prizes were predicted and scores of theoretical models spawned. The announcement also influenced decisions about academic appointments and the rejections of papers and grants. It even had a role in governmental planning of large-scale projects.

The BICEP2 team identified a twisty (B-mode) pattern in its maps of polarization of the cosmic microwave background, concluding that this was a detection of primordial gravitational waves. Now, serious flaws in the analysis have been revealed that transform the sure detection into no detection. The search for gravitational waves must begin anew.

The problem is that other effects, including light scattering from dust and the synchrotron radiation generated by electrons moving around galactic magnetic fields within our own Galaxy, can also produce these twists.

The BICEP2 instrument detects radiation at only one frequency, so cannot distinguish the cosmic contribution from other sources. To do so, the BICEP2 team used measurements of galactic dust collected by the Wilkinson Microwave Anisotropy Probe and Planck satellites, each of which operates over a range of other frequencies. When the BICEP2 team did its analysis, the Planck dust map had not yet been published, so the team extracted data from a preliminary map that had been presented several months earlier.

Now a careful reanalysis by scientists at Princeton University and the Institute for Advanced Study, also in Princeton, has concluded that the BICEP2 announcement already insists that the theory fails. Such is the nature of normal science.

Yet some proponents of inflation who celebrated the BICEP2 announcement already insist that the theory is equally valid whether or not gravitational waves are detected. How is this possible?

The answer given by proponents is alarming: the inflationary paradigm is so flexible that it is immune to experimental and observational tests. First, inflation is driven by a hypothetical scalar field, the inflaton, which has properties that can be adjusted to produce effectively any outcome. Second, inflation does not end with a universe with uniform properties, but almost inevitably leads to a multiverse with an infinite number of bubbles, in which the cosmic and physical properties vary from bubble to bubble. The part of the multiverse that we observe corresponds to a piece of just one such bubble.

Scanning over all possible bubbles in the multiverse, everything that can physically happen does happen an infinite number of times. No experiment can rule out a theory that allows for all possible outcomes. Hence, the paradigm of inflation is unfalsifiable.

This may seem confusing given the hundreds of theoretical papers on the predictions of this or that inflationary model. What these papers typically fail to acknowledge is that they ignore the multiverse and that, even with this unjustified choice, there exists a spectrum of other models which produce all manner of diverse cosmological outcomes. Taking this into account, it is clear that the inflationary paradigm is fundamentally untestable, and hence scientifically meaningless.

Cosmology is an extraordinary science at an extraordinary time. Advances, including the search for gravitational waves, will continue to be made and it will be exciting to see what is discovered in the coming years. With these future results in hand, the challenge for theorists will be to identify a truly explanatory and predictive scientific paradigm describing the origin, evolution and future of the Universe.