1. (a) Put the following equation into standard form (hint: complete the square in $x, y, z$)

$$x^2 + 4y^2 + 4z^2 - 4x - 8z = -4$$

$$\frac{(x-2)^2}{4} + 4 \frac{y^2}{4} + 4 \frac{(z-1)^2}{4} = -4$$

$$\frac{(x-2)^2}{4} + \frac{y^2}{4} + \frac{(z-1)^2}{4} = \frac{-4}{4}$$

(b) What type of surface is given in part (a)?

Ellipsoid

(c) Sketch the $(x, y)$ trace at $z = 1$ of the surface on the axes below.

$$z = 1 \Rightarrow \frac{(x-2)^2}{4} + \frac{y^2}{4} = 1$$

Ellipse centered at $(2, 0)$.
2. Find the general solution to the following differential equations. You must show your work and you must check your answer.

(a) \( \frac{dx}{dt} = 2x - 6 \)

\[ x' = 2(x-3) \]

\[ \int \frac{dx}{x-3} = \int 2\,dt \]

\[ \ln|x-3| = 2t + C \]

Check: \( x' = 2Ae^{2t} \)

\[ x = 3 + Ae^{2t} \]

(b) \( \frac{dy}{dx} + y = e^{-x} \)

\( p(x) = 1 \) \( u(x) = e^x \)

\( e^x y = e^x \left[ \int e^{-x} \, dx \right] \)

\[ = e^{-x} [x + C] = xe^{-x} + Ce^{-x} \]

Check: \( y' = e^{-x} - xe^{-x} \)

\[ y' + y = e^{-x} - xe^{-x} + Ce^{-x} \]

\[ = 2(3 + Ae^{2t} - 3) \]

\[ = 2Ae^{2t} \checkmark \]

(c) \( z + 4xz = x \)

\( \int p(x) = 4x \)

\( u(x) = e^{2x^2} \)

\[ \int e^{2x^2} \, dx \rightarrow \int e^{a} \, da \]

\[ a = 2x^2 \]

\[ da = 4x \, dx \]

\[ = \frac{1}{4} e^{2x^2} \]

Check: \( z' = -4Cxe^{-2x^2} \)

\[ z' + 4xz = -4Cxe^{-2x^2} + \frac{4x}{4} + 4Cxe^{-2x^2} \]

\[ = \frac{1}{4} 4x = x \checkmark \]
3. The rate of increase in sales ($S$) (in thousands of units) of a product is proportional to the current level of sales and inversely proportional to the square of the time ($t$). What differential equation represents this relationship? Circle the correct answer, no justification is necessary.

(a) $\frac{dS}{dt} = kSt^2$

(b) $\frac{dS}{dt} = k \frac{t^2}{S}$

(c) $\frac{dS}{dt} = k \frac{S}{t^2}$

(d) $\frac{dS}{dt} = k \frac{t}{S}$

(e) None of the above

0 points
4. During a chemical reaction substance \( A \) is converted into substance \( B \) at a rate proportional to the square of the amount of \( A \). Initially, 100 grams of substance \( A \) are present, and after one hour 50 grams remain.

(a) Find an equation which can predict how many grams of substance \( A \) is present at \( t \) hours.

\[
\frac{dA}{dt} = kA^2
\]

\[
\int \frac{dA}{A^2} = \int kdt
\]

\[
A(0) = 100 = \frac{1}{C-k(0)} = \frac{1}{C}
\]

\[
\Rightarrow C = \frac{1}{100}
\]

\[
A(t) = \frac{100}{1-kt}
\]

\[
A^{-1} = C - kt
\]

\[
A = \frac{1}{C-kt}
\]

(b) How much of substance \( A \) is present after 2 hours?

\[
A(2) = \frac{100}{1+2} = \frac{100}{3}\text{ grams}
\]

(c) When will only 10 grams of substance \( A \) remain?

\[
A(t) = 10 = \frac{100}{1+t}
\]

\[
10 + 10t = 100
\]

\[
1 + t = 10
\]

\[
t = 9\text{ hours}
\]
5. Find the domain and range of the following functions. Also evaluate the functions at the specified point.

7 pts  (a) \( f(x,y) = \sqrt{4-x^2-y^2} \) \( (x,y) = (0,0) \)

3 pts  Domain  \( 4-x^2-y^2 \geq 0 \) all reals such that \( x^2+y^2 \leq 4 \)

3 pts  Range: \( 0 \leq z \leq 2 \) for \( z = \sqrt{4-x^2-y^2} \)

\[
\f(0,0) = 2 \quad 1 \text{ pt}
\]

7 pts  (b) \( f(x,y) = \ln(x^2+y^2) \), \( (x,y) = (-1,-1) \)

3 pts  Domain: \( x^2+y^2 > 0 \)

3 pts  Range: all reals

1  \( f(-1,-1) = \ln((-1)^2 + (-1)^2) = \ln(2) \)
6. For each of the following quadric surfaces, describe the \((x,y)\) trace, \((x,z)\) trace, and \((y,z)\) traces.

(a) \(12x^2 + 4y^2 - 3z^2 = 0\)

\[2 \ (x,y) \text{ trace } z = 0 \quad 12x^2 + 4y^2 = 0 \quad \text{ellipse}\]

\[2 \ (x,z) \text{ trace } y = 0 \quad 12x^2 - 3z^2 = 0 \quad \text{hyperbola}\]

\[2 \ (y,z) \text{ trace } x = 0 \quad 4y^2 - 3z^2 = 0 \quad \text{hyperbola}\]

(b) \(x^2 + y^2 - z = 1\)

\[2 \ (x,y) \text{ trace } t = 0 \quad x^2 + y^2 = 1 \quad \text{circle}\]

\[2 \ (x,z) \text{ trace } y = 0 \quad x^2 - z = 1 \quad \text{parabola}\]

\[2 \ (y,z) \text{ trace } x = 0 \quad y^2 - z = 1 \quad \text{parabola}\]

(c) \(12 + 12x^2 + 3y^2 - 4z^2 = 0\)

\[2 \ (x,y) \text{ trace } z = 0 \quad 12 + 12x^2 + 3y^2 = 0 \quad \text{hyperbolae}\]

\[2 \ (x,z) \text{ trace } y = 0 \quad 12 + 12x^2 - 4z^2 = 0 \quad \text{hyperbola}\]

\[2 \ (y,z) \text{ trace } x = 0 \quad 12 + 3y^2 - 4z^2 = 0 \quad \text{hyperbola}\]

No graphs necessary.