Mixing

• Add passive particles to flow
  – Look at where particles go...
    • Need some way to identify where they started... color scheme – simple for 4-roll mill geometry, pretty but not very quantitative
  – Effective diffusion
    • Allow particles to roam freely in flow, quantify diffusion or spreading rate
  – Finite time Lyapunov exponents
    • Measure stretching rates of trajectories

• Add passive scalar to flow
  – Look at some norm which quantifies mixing
    • Thiffeault, “Using multiscale norms to quantify mixing and transport” Review from Nonlinearity, 2011, paper on website

4-roll mill Wi = 10, stable vortex
Generically instability yields one dominant vortex

Vorticity $t = 2000$

Particle distribution $t = 500$

Particle distribution $t = 2000$

$\text{trS } t = 2000$
4-roll mill $Wi = 10$, rotating vortex
Consequences for mixing:
“cycling” vortex will mix fluid more “globally”

Particle distribution $t = 2600$

Fraction of particles from each quadrant in the
Upper right quadrant $0 < t < 2600$
How does the polymer stress affect diffusion in the flow? Tile the plane with periodic copies of the underlying flow field and let the particles be advected by the flow.

$\text{Wi} = 10$, initial data for polymer stress yielding one stable vortex, well into flow evolution.
How does the polymer stress affect diffusion in the flow?

Tile the plane with periodic copies of the underlying flow field and let the particles be advected by the flow

$Wi = 10$, initial data for polymer stress yielding one stable vortex, well into flow evolution

- Calculate average displacement of particles over time
- Over long times slope of displacement gives rate of enhanced diffusion

![Graph showing the effect of polymer stress on diffusion](image_url)
Wi = 10, initial data for polymer stress yielding rotating vortex, well into flow evolution
Wi = 10, initial data for polymer stress yielding rotating vortex, well into flow evolution.

Diffusion along $y = x \approx 0.171$

Diffusion along $y = -x \approx 0.011$

Effective diffusion = 0.18
Compute finite-time Lyapunov exponents in “mixing region”
Using Wi=10 flow from polymer stress initial conditions which
Result in single dominant vortex
Choose small region of flow outside dominant vortex
Allow particles to diffuse for some time and “fill up” mixing region

Initial Conditions for FLOW:

Initial particle locations:
Statistics of finite-time Lyapunov Exponents:

Mean of largest eigenvalue is converging to $\text{mean}(\lambda_1) \sim 0.014$

$\bar{\lambda}_1$

fitted curve

$0.014 + 0.136 \ t^{-1/2}$

Mean of largest eigenvalue is converging to $\text{mean}(\lambda_1) \sim 0.014$

$\sigma = (\alpha/t)^{1/2} \approx (0.03/t)^{1/2}$

$fitted\ curve\ 2145 + 31.33t$

$\frac{\bar{\lambda}_1}{\alpha} = 0.4667 < 1$ implies that the variance decay rate is dominated by zero-stretching orbits.

The variance decay rate is given by

$\gamma = \frac{\bar{\lambda}_1^2}{2\alpha} \approx 0.014^2 / (2 \times 0.03) = 0.0033$

variance decays on $1 / \gamma \approx 300$ time units
Update passive scalar in flow to quantify mixing, initialize to 4x4-rollers after onset of transition
Use mixing norm $W$, (see Mathew et al. Physica D ‘05) equivalent to $H^{-1/2}$ Sobolev norm, to quantify mixing, mixing rate $b$ assumes exponential decay of $W \sim e^{-bt}$, time scaled by $Wi$. 

![Graph showing mixing norm $W$ and mixing rate $b$ as functions of time and $Wi$.]