Math 141, Spring 2014, Midterm 2

NAME: printed
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STUDENT ID

Solutions

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Please do not turn this page until you are told to start the exam. You are not allowed to use books, notes, or calculators. You should show all of your work. A correct answer with an incomplete or incorrect explanation will only get partial credit. An incorrect answer with a good explanation will get partial credit. THE POINT VALUE OF THE PROBLEM IS NOT AN INDICATOR OF ITS DIFFICULTY. MAKE SURE YOU HAVE ALL THE PAGES OF YOUR EXAM! There are 6 problems. Good Luck!
1 Problem 1

(15 points)

Let
\[ A = (0, 1) \]
\[ B = (0, 0) \]
\[ C = (1, 0) \]
and let \( f \) be an isometry of the plane. State a simple test for telling whether \( f \) is a reflection, translation, rotation or glide reflection from the positions of \( f(A), f(B) \) and \( f(C) \).

Check orientation

preserving

\[ f(A) = A + \vec{v} \]

Check whether
\[ f(B) = B + \vec{v} \]
\[ f(C) = C + \vec{v} \]

yes

translation

no

rotation

reversing

check whether the reflection that takes
\[ A \rightarrow f(A) \]
also takes
\[ B \rightarrow f(B) \text{ and } C \rightarrow f(C) \]

yes

reflection

no

glide reflection
2 Problem 2

(20 points)

Let

\( r_1 \) be reflection across \( y = -1 \)
\( r_2 \) be reflection across \( y = x \)
\( r_3 \) be reflection across \( y = -x \)
\( r_4 \) be reflection across \( y = 3 \)

Let \( f(x, y) = r_4 \circ r_3 \circ r_2 \circ r_1(x, y) \).

A. Calculate (no partial credit):
\[
\begin{align*}
f(0, 1) & = (2, 3) \\
f(1, 1) & = (-1, 3) \\
f(1, 0) & = (-1, 4)
\end{align*}
\]

B. Is \( f \) a reflection, translation, or glide reflection (no partial credit, no justification required)?

\[
\text{orientation preserving} \quad \Rightarrow f \text{ is a rotation}
\]

\[\text{not a translation}\]
3 Problem 3

(20 points)

Use vectors in the plane to prove that a triangle inscribed in a semi-circle is a right triangle.

\[ (\vec{u} - \vec{v}) \cdot (-\vec{u} - \vec{v}) = -[\vec{u} \cdot \vec{u} - \vec{v} \cdot \vec{v}] \]

\[ = -[|\vec{u}|^2 - |\vec{v}|^2] \]

\[ = 0, \text{ since } |\vec{u}| = |\vec{v}|, \]

because \( \vec{u} \) and \( \vec{v} \) lie on a semi-circle.
4 Problem 4

(20 points)

Solve the following logic puzzle by Lewis Carroll. Your answer should be in the form of a single statement which uses all of the five statements below. You must explain your reasoning.

(1) There is no box of mine here that I dare open;
(2) My writing-desk is made of rose-wood;
(3) All my boxes are painted, except what are here;
(4) There is no box of mine that I dare not open, unless it is full of live scorpions;
(5) All my rose-wood boxes are unpainted.

Universe: BOXES OF MINE; point of information: a "writing desk" is a kind of box; hint: "here" is one of the categories.

\[
\begin{align*}
A & : \text{Here} & \quad 1. & A \Rightarrow \neg B \\
B & : \text{I dare to open} & 2. & C \Rightarrow E \\
C & : \text{Writing desk} & 3. & \neg A \Rightarrow D \\
D & : \text{Painted} & 4. & \neg B \Rightarrow F \\
E & : \text{Made of rosewood} & 5. & E \Rightarrow \neg D \\
F & : \text{Full of scorpions} & \quad C \Rightarrow E \Rightarrow \neg D \Rightarrow A \Rightarrow \neg B \Rightarrow F \\
\end{align*}
\]

"My writing desk is full of live scorpions."
5  Problem 5

(15 points)

The two triangles below are perspective from the line $L$. State what Desargues' theorem guarantees about these two triangles, and find the relevant point.
6 Problem 6

(10 points)

Find the vanishing point in the photo reproduced below (look familiar?). Your point must be within 1/4" of the correct answer. There is no partial credit for this problem.