Sample Problems, 218, 1972

1. Let \( R \) be the region bounded by \( y = x^3, \ y = 8, \ x = 0 \).
   A. Sketch \( R \).

   B. Find the volume of the solid generated by revolving \( R \) about the line \( x = -2 \) using the shell method.

   C. Find the volume of the solid generated by revolving \( R \) about the line \( y = 8 \) using the shell method.

2. Find the length of the curve \( y(x) = \ln x - \frac{x^2}{8} \) from \( x = 1 \) to \( x = 2 \).

3. Find the area of the surface obtained by revolving the curve \( y = \sqrt{2x-x^2}, \ 0.5 \leq x \leq 1.5 \), about the \( x \)-axis.

4. A vertical right-circular cylindrical tank is 30 ft. high and 20 ft. in diameter. It is half-full of olive oil weighing 57 lb/ft\(^3\). How much work does it take to pump the oil to the rim of the tank?

5. A semi-circular plate 2 ft. in diameter sticks straight down into freshwater (weight-density 62.4 lb/ft\(^3\)) with the diameter along the surface. Find the force exerted by the water on one side of the plate.

6. Find the center of mass of a thin plate of constant density \( s = 1 \) bounded by the curves \( g(x) = x^2(x+1), \ f(x) = 2, \ x = 0 \).

7. The half-life of plutonium-210 is 139 days. How many days after it arrives in the lab will 95% of it be gone?
8. Integrate:
   A. $\int 4x \sec^2 2x \, dx$
   B. $\int xe^x \, dx$
   C. $\int e^x \cos x \, dx$
   D. $\int x2^{x^2} \, dx$
   E. $\int \tan x \ln (\cos x) \, dx$
   F. $\int \sin^3 x \cos^3 x \, dx$
   G. $\int \ln x \, dx$

9. Solve the differential equations:
   A. $\sqrt{2xy} \; \frac{dy}{dx} = 1$
   B. $\frac{dy}{dx} = \frac{e^{2x-y}}{e^{x+y}}$