Math 141, Spring 2014, Midterm 1

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Please do not turn this page until you are told to start the exam. You are not allowed to use books, notes, or calculators. You should show all of your work. You may use a ruler and straight-edge. A correct answer with an incomplete or incorrect explanation will only get partial credit. An incorrect answer with a good explanation will get partial credit. THE POINT VALUE OF THE PROBLEM IS NOT AN INDICATOR OF ITS DIFFICULTY. MAKE SURE YOU HAVE ALL THE PAGES OF YOUR EXAM! There are 5 problems. Good Luck!
1 Problem 1

(20 points)

Definition: Let $L$ be a line and $P$ be a point not on $L$. The distance from $P$ to $L$ is the length of the perpendicular line segment from $P$ to $L$.

Proposition: If $P$ is a point on the angle bisector of $\angle ABC$, then the distance from $P$ to the line containing $A$ and $B$ is equal to the distance from $P$ to the line containing $B$ and $C$.

a. Draw a diagram to illustrate this proposition.

b. Prove the Proposition. You can use "common language" to justify your steps, for example, you can cite "side-side-side congruence".

Drop perpendicular lines $L_{1}$ and $M_{1}$ from $P$ to $\overrightarrow{AB}$ and $\overrightarrow{BC}$ respectively, to the points $R$ and $S$ respectively.

We need to show $\angle RPL = \angle SPL$.

$\angle BPR = \frac{\pi}{2} - \angle PBA$ (interior angles of a triangle sum to $\pi$).

$\angle BPS = \frac{\pi}{2} - \angle PBS$

$\angle PBA = \angle PBS$ by hypothesis.

$\Rightarrow \angle BPR = \angle BPS$

$\Rightarrow \triangle RBP \cong \triangle SBP$ (ASA congruence)

$\Rightarrow \angle RPL = \angle SPL$
2 Problem 2

(20 points)

a. Draw a carefully labeled diagram to accompany Proposition 33 from the "Elements".

   see "Elements Online"

b. Draw a carefully labeled diagram to accompany Proposition 37 from the "Elements".

   see "Elements Online"
3 Problem 3

(20 points)

Show that the diagonals of a parallelogram bisect each other. You may use any of Propositions 1-33 to justify the steps of your proof. Cite particular propositions by number to justify your steps.

\[ \text{Show } |AO| = |OC| \text{ and } |BO| = |OD|. \]

The marked angles are equal:

(Repeated applications of Proposition 29)

The marked sides are equal:

\[ \text{(Proposition 33)} \]

\[ \triangle AOB \cong \triangle COD \quad \text{(ASA; Proposition 26)} \]

\[ \Rightarrow |AO| = |CO| \]

\[ \triangle BOC \cong \triangle DOA \quad \text{(ASA; Proposition 26)} \]

\[ \Rightarrow |BO| = |OD| \]
4 Problem 4

(20 points)

Solve the following logic puzzle by Lewis Carroll. Your answer should be in the form of a single statement which uses all of the five statements below. You must explain your reasoning. Hints: The "universe" here is "poems". "Popular among people of taste" is one attribute; you can assume that "ancient" is equivalent to "not modern".

1. No interesting poems are unpopular among people of real taste
2. No modern poetry is free from affectation;
3. All your poems are on the subject of soap-bubbles
4. No affected poetry is popular among people of real taste
5. No ancient poem is on the subject of soap-bubbles

Poems:
A. Interesting
B. Popular among people of taste
C. Modern
D. Affected
E. About soap-bubbles
F. Yours

1. $A \Rightarrow B$
2. $C \Rightarrow D$
3. $F \Rightarrow E$
4. $B \Rightarrow \sim D$
5. $E \Rightarrow C$

$F \Rightarrow E \Rightarrow C \Rightarrow D \Rightarrow \sim B \Rightarrow \sim A$

Your poetry is not interesting.
5 Problem 5

(20 points)

Explain why the diagram below yields a proof of the Pythagorean Theorem, by explaining what is being illustrated at each step and justifying each step.

\[ \text{Area (ΔABC)} = \text{Area (ΔABD)} \]
\[ \text{(bases are equal and heights are equal)} \]

\[ \text{Area (ABD)} = \text{Area (AEF)} \]
\[ \text{(triangles are congruent; SAS)} \]

\[ \text{Area (AEF)} = \text{Area (AGF)} \]
\[ \text{(bases are equal and heights are equal)} \]

How do you complete the proof?

Since \( \frac{1}{2} \text{Area (□ABXC)} = \frac{1}{2} \text{Area (□FAGY)} \),
\[ \text{Area (□ABXC)} = \text{Area (□FAGY)} \]

Repeat argument on other square; conclude the sum of the areas of two upper squares is the area of lower square.