Dendritic Effects in Networks of Electrically Coupled Fast-Spiking Interneurons

Timothy J. Lewis, * John Rinzel

Center for Neural Science and Courant Institute of Mathematical Sciences, New York University, 4 Washington Place, New York, NY 10003

Abstract

We examine how the location of weak electrical coupling affects phase-locking in a pair of model fast-spiking interneurons. Each model neuron consists of a somatic compartment and a passive dendritic compartment. At relatively low frequencies, the phase-locking structure for somatic and dendritic coupling is qualitatively the same: below a critical frequency, stable synchronous and anti-phase activity co-exist, and only synchrony is stable above this critical frequency. At higher frequencies, the synchronous state remains stable for somatic coupling, but for dendritic coupling, the synchronous state becomes unstable and anti-phase oscillations become stable.

Key words: electrical coupling, dendrites, synchrony, anti-phase, interneurons

1 Introduction

Recently, direct electrical coupling has been found to be widespread in networks of cortical inhibitory interneurons (6; 8). The effects of electrical coupling between cortical interneurons has been the focus of much experimental and theoretical work, however the functional role that electrical coupling plays in cortical networks remains unclear. Evidence suggests that electrical coupling can help coordinate synchronous oscillatory behavior in inhibitory networks, which has been hypothesized to be important for sensory and cognitive processes.

Previous studies have systematically examined synchronization patterns between single-compartment neurons coupled by electrical coupling alone (3; 9;

* Corresponding author.
Email address: tim.lewis@nyu.edu (Timothy J. Lewis,).

10). Whether or not these studies are applicable to dynamics in cortical interneuronal networks depends on, among other things, whether or not a single-compartment description is a sufficient model of cortical interneurons. When neurons are not sufficiently electrotonically compact, a single-compartment model is not adequate in most circumstances. In this case, the response properties of neurons can be highly dependent on the site of the applied or synaptic input (2; 4; 11). Similarly, the location of electrical coupling can have substantial effects on phase-locking (1).

In this article, we begin to address the issue of effects of dendrites on network dynamics. We add a passive dendritic compartment to a single-compartment model for fast-spiking (FS) interneurons and systematically examine the effects of the location of weak electrical coupling on phase-locking in networks of neurons.

2 The Two-Compartment FS Model: Single Cell and Coupled Cells

Single cell model: We take a single model fast-spiking (FS) neuron to consist of an active somatic compartment and a passive dendritic compartment. The governing equations are

$$\begin{aligned} C\frac{dV_s}{dt} &= -I_{ionic}(V_s, m, h, n_1, n_3) + I_{applied} + g_{sd}(V_d - V_s) \\ C\frac{dV_d}{dt} &= -g_{lk}(V_d - E_{lk}) + \gamma g_{sd}(V_s - V_d) \end{aligned} \tag{1}$$

where V_s and V_d are the transmembrane potentials of the somatic and dendritic compartments, g_{sd} is the electrotonic coupling strength between the compartments, $I_{applied}$ is the bias current applied to the soma, g_{lk} and E_{lk} are the conductance and reversal potential of the dendritic leakage current, and γ is the ratio of the membrane surface area of the somatic compartment to that of the dendritic compartment. I_{ionic}, g_{lk}, E_{lk} and governing equations for the gating variables m, h, n_1, n_3 are given by a conductance-based model for cortical FS interneurons proposed by Erisir and coworkers (5). All parameters are the same as those in the original Erisir *et al* model except for the leak conductance for which we use $g_{lk} = 0.25\,\mathrm{mS/cm^2}$ (instead of $1.25\,\mathrm{mS/cm^2}$). For this work, we take $\gamma = 1$. The electrotonic coupling strength g_{sd} between the compartments is taken to be half of the leakage conductance, which yields a steady state attenuation factor $\Delta V_d/\Delta V_s$ of $1/3$ for small $I_{applied}$.

Given sufficiently large input current $I_{applied}$, the two-compartment cell can exhibit intrinsic oscillatory behavior. We define $V_s^*(t)$ and $V_d^*(t)$ to be the membrane potentials of the soma and dendrites during the periodic activity.

Examples of these are seen in the top panels of figure 1. Note that $V_s^*(t)$ and $V_d^*(t)$ depend on all of the single-cell parameters including $I_{applied}$.

Electrically Coupled Cells: Now let us consider a pair of cells. We add the subscript i to denote the variables for the i^{th} cell. Electrical coupling between cells is modeled as an ohmic resistance. If coupling is between somata of cells i and j, then the right-hand side of the differential equation for $V_{s,i}$ will have the additional term $g_c(V_{s,j} - V_{s,i})$, where g_c is the electrical coupling conductance. If coupling is between dendrites of cells i and j, then the right-hand side of the differential equation for $V_{d,i}$ will have the additional term $g_c(V_{d,j} - V_{d,i})$.

Weak coupling and phase models: When coupling is sufficiently weak, the complete state of the cells can be approximated by the phase $t + \phi_i$ of the cell in the periodic oscillations, i.e. $V_s(t) = V_s^*(t + \phi_i)$, $V_d(t) = V_d^*(t + \phi_i)$, etc. The theory of weakly coupled oscillators (7; 9) can be used to derive equations that describe the slow rate of change in the relative phases ϕ_i of the cells

$$\frac{d\phi_i}{dt} = g_c \; H_x(\phi_i - \phi_j) = \frac{1}{T} \int_0^T Z_x(\tilde{t}) \; g_c(V_x^*(\tilde{t} - (\phi_i - \phi_j)) - V_x^*(\tilde{t}))d\tilde{t}$$

where T is period of the intrinsic single-cell oscillation. When electrical coupling is between the somata, $x = s$; when electrical coupling is between dendrites, $x = d$. $Z_x(t)$ is called the infinitesimal phase response curve (iPRC). $Z_s(t)$ ($Z_d(t)$) is proportional to the phase-shift resulting from a small current perturbation rapidly delivered to the somatic (dendritic) compartment at a time $t \in [0, T)$ in the oscillation.

Note that, because H_x depends only on the difference in relative phase, a single differential equation for the phase difference between two cells, $\phi = \phi_i - \phi_j$, can be obtained

$$\frac{d\phi}{dt} = g_c \; (H_x(-\phi) - H_x(\phi)) = g_c \; G_x(\phi).$$

Phase-locked states with phase difference ϕ_{ss} are determined simply by $G_x(\phi_{ss}) = 0$. The phase-locked states are stable (unstable) if $G_x'(\phi_{ss}) < 0$ ($G_x'(\phi_{ss}) > 0$).

3 Phase-Locking in Electrically Coupled Oscillating Cells

For conductance-based models like the one we consider here, the periodic orbit $V_x^*(t)$ and iPRC $Z_x(t)$ must be found numerically for a given set of parameters describing the dynamics of the single cell. $V_x^*(t)$ is found by straight-forward numerical simulations; $Z_x(t)$ can be found by linearizing the system around the periodic orbit and solving the adjoint equations.

The somatic and dendritic membrane potentials, iPRCs and the G-functions for $I_{applied} = 8\mu\text{A/cm}^2$ and $I_{applied} = 12\mu\text{A/cm}^2$ are shown in figure 1. $I_{applied} = 8\mu\text{A/cm}^2$ induces oscillations at $55Hz$, and $I_{applied} = 12\mu\text{A/cm}^2$ produces $95Hz$ oscillations.

The filtering effects of the dendrites are apparent in both the membrane potentials and the iPRCs. V_d^* is attenuated and smoothed compared to V_s^*. There is also a delay that is manifested as a phase-shift to the right. Similar effects are seen in the iPRCs, but the delay effect in Z_d is manifested as shifted to the left when compared to Z_s. This is because a current perturbation to the dendrites must first travel through the dendrites before affecting the active processes in the somata. Note that the membrane potentials and the iPRCs for $I_{applied} = 12\mu\text{A/cm}^2$ look qualitatively similar to those for $I_{applied} = 8\mu\text{A/cm}^2$, but the phase-shifts and the smoothing effects of the dendrites are relatively greater for the higher frequency oscillations.

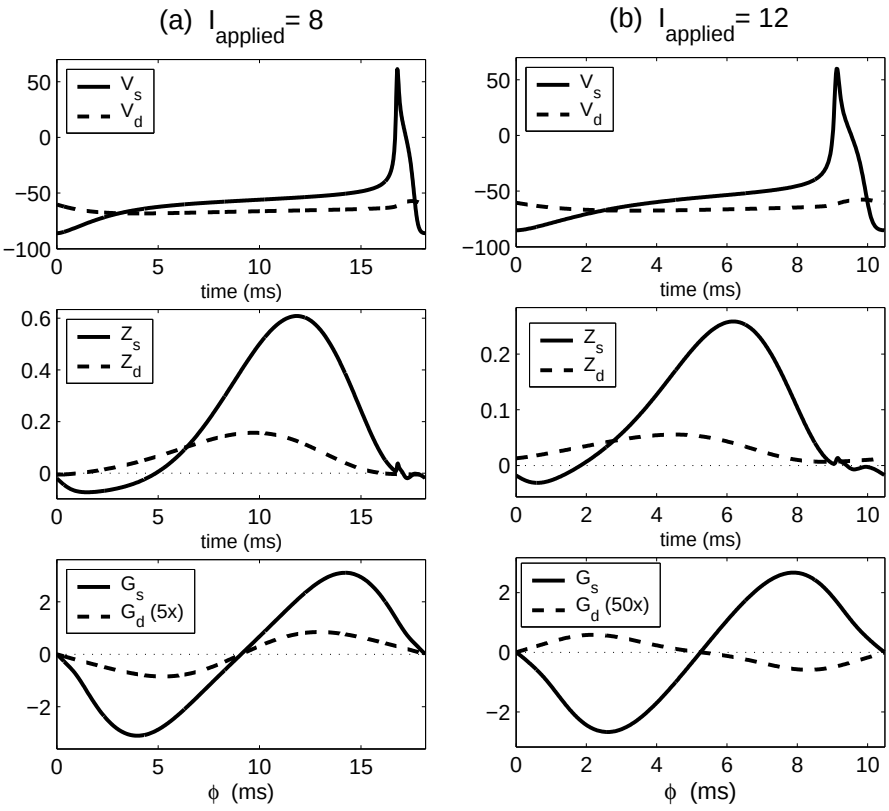

Fig. 1. (a) $I_{applied} = 8\mu\text{A/cm}^2$; (b)$I_{applied} = 12\mu\text{A/cm}^2$. **[top]** periodic oscillations in transmembrane potential (mV) vs time for somatic compartment ($V_s^*(t)$, solid line) and dendritic compartment ($V_d^*(t)$, dashed line). **[middle]** iPRCs for perturbations to the somatic compartment (Z_s, solid line) and perturbations to the dendritic compartment (Z_d, dashed line). **[bottom]** G-functions for somatic coupling (G_s, solid line) and dendritic coupling (G_d, dashed line). $\phi/T = 0, 1$ and $\phi/T = 1/2$ correspond to synchrony and anti-phase activity, respectively.

For $I_{applied} = 8\mu\text{A/cm}^2$, $G_s(\phi)$ indicates that synchronous and anti-phase phase-locked states exist when cells are electrically coupled via the somata, but only the synchronous state is stable; $G_d(\phi)$ shows that the same result holds for dendritic electrical coupling in this case. However, increased filtering effects at higher frequencies can lead to different phase-locking for the different coupling locations. $G_s(\phi)$ for $I_{applied} = 12\mu\text{A/cm}^2$ shows that somatic coupling yields stable synchronous activity and unstable anti-phase activity, whereas $G_d(\phi)$ indicates that dendritic coupling produces phase-locked states with the opposite stability: stable anti-phase and unstable synchrony.

Figure 2 depicts bifurcation diagrams that plot the phase-difference of phase-locked states as a function of constant current applied to the soma $I_{applied}$. The bifurcation diagram for somatic electrical coupling is qualitatively the same as that for the single compartment FS model (not shown). Synchronous activity is stable over the entire frequency range studied, $10 - 130Hz$ ($I_{applied} = 3.5 - 15\mu\text{A/cm}^2$); stable anti-phase activity co-exists with the stable synchrony but only at relatively low frequencies. The bifurcation diagram for dendritic coupling is qualitatively similar to that for somatic coupling at lower frequencies, but at high frequencies, the synchronous state becomes unstable and the anti-phase state becomes stable. In an intermediate frequency range ($83 - 93Hz$), only an asynchronous non-anti-phase phase-locked state is stable.

The effective electronic distance from the soma of dendritic electrical coupling is controlled by g_{sd}. When g_{sd} is increased, the dendritic filtering effects are increased and the stability transition points are shifted to lower frequencies.

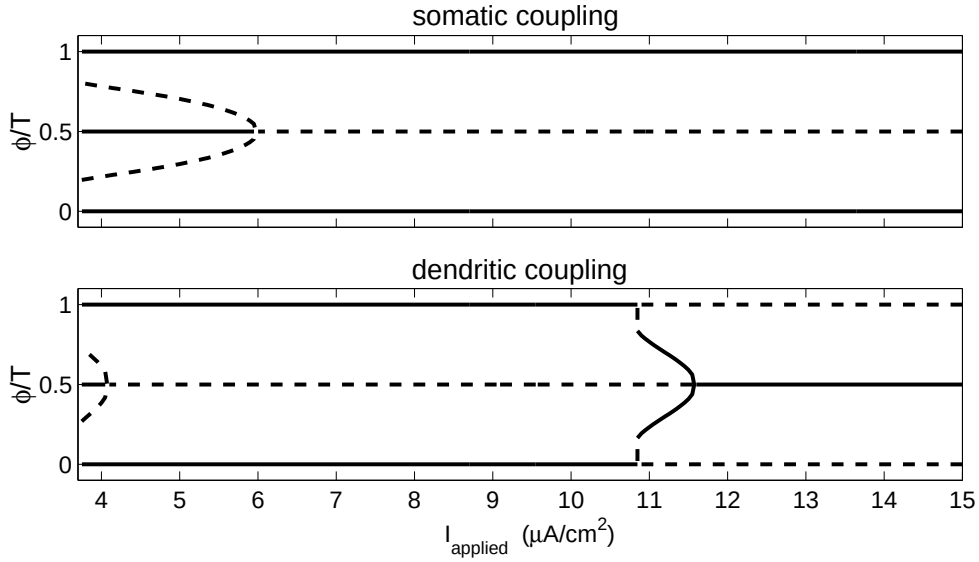

Fig. 2. Bifurcation diagrams for somatic coupling and dendritic compartment. Solid lines indicate phase-locked states that are stable; dashed lines indicate phase-locked states that are unstable. $\phi/T = 0, 1$ and $\phi/T = 1/2$ correspond to synchrony and anti-phase activity, respectively.

4 Conclusions

The results presented in this paper demonstrate that, when neurons are not electrotonically compact, the location of weak electrical coupling can have substantial effects on synchronization patterns. Oscillations at higher frequencies are particularly sensitive to the location of the coupling. The mechanisms generating these effects arise from the filtering properties (smoothing and phase shifts) of the dendrites. A better understanding of these results and the mechanisms that underlie them could be important for understanding synchronous oscillations observed in experiments.

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