EXAMPLES OF USING MATLAB TO ANALYZE DIFFERENTIAL EQUATIONS (u'=f(u))

```
>> c=0; a=0.2;
>> u=[-0.1:0.01:1.1];
>> f=-u*(u-a)*(u-1)+c;
??? Error using ==> *
Inner matrix dimensions must agree.

>> f=-u.*(u-a).*(u-1)+c;
>> plot(u,f)
>> axis([-0.1,1.1,-0.05,0.15]);
>> xlabel('u','fontsize',14);
>> ylabel('f(u)','fontsize',14);
>> plot([-0.1,1.1],[0,0],'r--')


% plot f(u) and y=0:
>> clf
>> plot(u,f,'linewidth',2)
>> axis([-0.1,1.1,-0.05,0.15]);
>> xlabel('u','fontsize',14);
>> ylabel('f(u)','fontsize',14);
>> hold on
>> plot([-0.1,1.1],[0,0],'r--')


% another way of plotting line y=0
% ---------------------------------------
>> n=length(u);
>> z=zero(n,1);
>> plot(u,f)
>> hold on
>> plot(u,z,'k:');


% another way of plotting f(u)
% ----------------------------------
>> clear
>> clf
>> fplot('-x*(x-0.2)*(x-1)+0',[-0.1,1.1]);
```

```
% … and still another way of plotting f(u)
% -----------------------------------------------

>> global a c
>> c=0; a=0.2;
>> clf
>> fplot('fcubic',[-0.1,1.1])
>> hold on
>> plot([-0.1,1.1],[0,0],'k:')
```

```
% solving for fixed points of u'=f(u)
% ----------------------------------------
>>clear
>>global a c
>>c=-0.05; a=0.2;
>>clf
>>subplot(211)
>>plotfcubic;

>>ustar1=fzero('fcubic',0)
>>ustar2=fzero('fcubic',a)
>>ustar3=fzero('fcubic',1)

% solve u'=f(u)
% ----------------
>> tspan=[0 10];  solve 0DE for t=0 to 10
>> u0=0.1;
>> [t1,u1]=ode45(@fcubic,tpsan,u0);
>> subplot(212)
>> plot(t1,u1);
>> hold on
>> u0=0.22;
>> [t1,u1]=ode45(@fcubic,tpsan,u0);
>> subplot(212)
>> plot(t1,u1);
```

FILE fcubic.m

```
% cubic function
% ---------------------------------------
        function [dxdt] = fcubic(x)
        global a c
        dxdt=-x*(x-a)*(x-1)+c;
```

FILE plotfcubic.m

```
% plot cubic function

u=[-0.1:0.01:1.1];
f=-u.*(u-a).*(u-1)+c;
plot(u,f)
hold on
axis([-0.1,1.1,-0.05,0.15]);
xlabel('u','fontsize',14);
ylabel('f(u)','fontsize',14);
subplot(211)
plot([-0.1,1.1],[0,0],'k:')
```

FILE plotfcubica.m

```
% plot cubic function for various values of a (animated)
% --------------------------------------------------------------
a=0.2; c=0.0;
u=[-0.1:0.01:1.1];
for a=0.3:-0.01:-0.2
    f=-u.*(u-a).*(u-1)+c;
    clf;
    plot(u,f);
    hold on;
    axis([-0.3,1.1,-0.05,0.15]);
    xlabel('u','fontsize',14);
    ylabel('f(u)','fontsize',14);
    plot([-0.3,1.1],[0,0],'k:');
    str1=sprintf('fcubic2: a= %6.4g, c= %6.3g',a,c);
    title(str1);
    pause(0.5);
end;
```

FILE plotfcubica.m

```
% plot cubic function for various values of c
% ------------------------------------------------
a=0.2;
u=[-0.1:0.01:1.1];
for c=-0.15:0.01:0.15
    f=-u.*(u-a).*(u-1)+c;
    clf;
    plot(u,f);
    hold on;
    axis([-0.1,1.1,-0.05,0.15]);
    xlabel('u','fontsize',14);
    ylabel('f(u)','fontsize',14);
    plot([-0.1,1.1],[0,0],'k:');
    str1=sprintf('fcubic2: a= %6.4g, c= %6.3g',a,c);
    title(str1);
    pause(1);
end;
```

FILE fcubicbif.m

```
% compute and plot bifurcation diagrams

clear
a=0.2;
clf;
for c0=-0.15:0.005:0.15
    c=c0;
    for u0=-0.1:0.1:1.1
        ustar=fzero('fcubic',u0);
        plot(c,ustar,'.');
        hold on
    end;
end;
xlabel('c','fontsize',16)
ylabel('u*','fontsize',16)
pause;

cstar2=u.*(u-a).*(u-1);
plot(cstar2,u,'g')
pause;

figure(2)
clf
c=0.0;
clf;
for a0=-0.15:0.01:0.35
    a=a0;
    for u0=-0.1:0.1:1.1
        ustar=fzero('fcubic',u0);
        plot(a,ustar,'.');
        hold on
    end;
end;
axis([-0.15,0.35,-0.2,1.2]);
xlabel('a','fontsize',16)
ylabel('u*','fontsize',16)
pause;

astar1=u;
plot(astar1,u,'g');
plot([-0.15,0.35],[0,0],'g');
plot([-0.15,0.15],[1,1],'g');
axis([-0.15,0.35,-0.2,1.2]);
```

FILE vectorfld.m

```
% ==============================================
% plot the phase plane for
%        x'= x(a-bx-cy)
%        y'= y(q-rx-sy)
%  Lotka-Volterramodel of competition
%   see Strogatz section 6.4 (pp.155-159).
% ==============================================

global a b c q r s

% set parameters
a=3; b=1; c=2;
q=2; r=1; s=1;


% compute and plot vector feild
% ----------------------------

% define grid and calculate v-fld on grid
xmin=-0.2; xmax=3.1; dxv=0.2;
ymin=-0.2; ymax=2.1; dyv=0.2;
[x,y] = meshgrid(xmin:dxv:xmax, ymin:dyv:ymax);
dxdt = x.*(a - b*x - c*y);
dydt = y.*(q - r*x - s*y);

% plot v-fld
figure(1)
clf
quiver(x,y,dxdt,dydt,'m');
hold on
axis([xmin,xmax,ymin,ymax]);
%axis equal
xlabel('x');
ylabel('y');

pause;


% plot nullclines
% ---------------

% x-nullcines
plot([0,0],[ymin,ymax],'k');
xg=[xmin:0.01:xmax];
```

```
xn=(a-b*xg)/c;
plot(xg,xn,'k');

% y-nullcines
plot([xmin,xmax],[0,0],'k--');
yn=(q-r*xg)/s;
plot(xg,yn,'k--');
axis([xmin,xmax,ymin,ymax]);

pause;


% solve system od ODEs and plot trajectories
% ------------------------------------------

tspan=[0 10];   %time = 0 to 10
x10=[0.2; 0.1]; % initial conditions (x(0),y(0))
[t1,x1]=ode45(@lvcomp,tspan,x10); % solve system of ODEs in lvcomp.m
plot(x1(:,1),x1(:,2));

pause

% corresponding plot for x and y vs time
figure(2)
clf
plot(t1,x1(:,1))
hold on
plot(t1,x1(:,2),'--')

pause;

% more trajctories
figure(1)

tspan=[0 10];   %time = 0 to 10
x20=[2.0; 1.5]; % initial conditions (x(0),y(0))
[t2,x2]=ode45(@lvcomp,tspan,x20); % solve system of ODEs in lvcomp.m
plot(x2(:,1),x2(:,2));

pause

for x0=0.0:0.1:xmax
   x30=[x0; 0.05];
   [t3,x3]=ode45(@lvcomp,tspan,x30);
   plot(x3(:,1),x3(:,2));
end;
```

```
for x0=0.0:0.2:xmax
    x30=[x0; ymax];
    [t3,x3]=ode45(@lvcomp,tspan,x30);
    plot(x3(:,1),x3(:,2));
end;

for y0=0.0:0.1:ymax
    x30=[0.05; y0]; % initial conditions (x(0),y(0))
    [t3,x3]=ode45(@lvcomp,tspan,x30);
    plot(x3(:,1),x3(:,2));
end;

for y0=0.0:0.2:ymax
    x30=[xmax; y0];
    [t3,x3]=ode45(@lvcomp,tspan,x30);
    plot(x3(:,1),x3(:,2));
end;
```

FILE lvcomp.m

```
% Lotka-Volterra equation for competition
% ---------------------------------------
        function [dxdt] = lvcomp(t,x)
        global a b c q r s
        dxdt = [ x(1)*(a - b*x(1) - c*x(2));
                 x(2)*(q - r*x(1) - s*x(2)) ];
```