The Limit Laws

To calculate limits of functions that are arithmetic combinations of functions having known limits, we can use several easy rules.

**THEOREM 1—Limit Laws**

If $L$, $M$, $c$, and $k$ are real numbers and

\[
\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = M,
\]

then

1. **Sum Rule:**
   \[
   \lim_{x \to c} (f(x) + g(x)) = L + M
   \]

2. **Difference Rule:**
   \[
   \lim_{x \to c} (f(x) - g(x)) = L - M
   \]

3. **Constant Multiple Rule:**
   \[
   \lim_{x \to c} (k \cdot f(x)) = k \cdot L
   \]

4. **Product Rule:**
   \[
   \lim_{x \to c} (f(x) \cdot g(x)) = L \cdot M
   \]

5. **Quotient Rule:**
   \[
   \lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}, \quad M \neq 0
   \]

6. **Power Rule:**
   \[
   \lim_{x \to c} [f(x)]^n = L^n, \quad n \text{ a positive integer}
   \]

7. **Root Rule:**
   \[
   \lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{L} = L^{1/n}, \quad n \text{ a positive integer}
   \]

(If $n$ is even, we assume that $\lim_{x \to c} f(x) = L > 0$.)

In words, the Sum Rule says that the limit of a sum is the sum of the limits. Similarly, the next rules say that the limit of a difference is the difference of the limits; the limit of a constant times a function is the constant times the limit of the function; the limit of a product is the product of the limits; the limit of a quotient is the quotient of the limits (provided that the limit of the denominator is not 0); the limit of a positive integer power (or root) of a function is the integer power (or root) of the limit (provided that the root of the limit is a real number).