Guidelines for Using the Fundamental Theorem of Calculus

1. The Fundamental Theorem of Calculus describes a way of evaluating a definite integral, not a procedure for finding antiderivatives.

2. In applying the Fundamental Theorem, it is helpful to use the notation
   \[ \int_a^b f(x) \, dx = F(x) \bigg|_a^b = F(b) - F(a). \]

3. The constant of integration \( C \) can be dropped because
   \[ \int_a^b f(x) \, dx = \left[ F(x) + C \right]_a^b = [F(b) + C] - [F(a) + C] = F(b) - F(a) + C - C = F(b) - F(a). \]

In the development of the Fundamental Theorem of Calculus, \( f \) was assumed to be nonnegative on the closed interval \([a, b]\). As such, the definite integral was defined as an area. Now, with the Fundamental Theorem, the definition can be extended to include functions that are negative on all or part of the closed interval \([a, b]\). Specifically, if \( f \) is any function that is continuous on a closed interval \([a, b]\), then the definite integral of \( f(x) \) from \( a \) to \( b \) is defined to be
   \[ \int_a^b f(x) \, dx = F(b) - F(a) \]
   where \( F \) is an antiderivative of \( f \). Remember that definite integrals do not necessarily represent areas and can be negative, zero, or positive.

Properties of Definite Integrals

Let \( f \) and \( g \) be continuous on the closed interval \([a, b]\).

1. \( \int_a^b kf(x) \, dx = k \int_a^b f(x) \, dx \), \( k \) is a constant.

2. \( \int_a^b [f(x) \pm g(x)] \, dx = \int_a^b f(x) \, dx \pm \int_a^b g(x) \, dx \)

3. \( \int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx \), \( a < c < b \)

4. \( \int_a^a f(x) \, dx = 0 \)

5. \( \int_a^b f(x) \, dx = -\int_b^a f(x) \, dx \)