The forms of the graphs in Figure 4.2 are typical of the graphs of the exponential functions \( y = a^{-x} \) and \( y = a^x \), where \( a > 1 \). The basic characteristics of such graphs are summarized in Figure 4.3.

**Graph of \( y = a^{-x} \)**
- Domain: \((-\infty, \infty)\)
- Range: \((0, \infty)\)
- Intercept: \((0, 1)\)
- Always decreasing
- \( a^{-x} \to 0 \) as \( x \to \infty \)
- \( a^{-x} \to \infty \) as \( x \to -\infty \)
- Continuous
- One-to-one

**Graph of \( y = a^x \)**
- Domain: \((-\infty, \infty)\)
- Range: \((0, \infty)\)
- Intercept: \((0, 1)\)
- Always increasing
- \( a^x \to \infty \) as \( x \to \infty \)
- \( a^x \to 0 \) as \( x \to -\infty \)
- Continuous
- One-to-one

**Figure 4.3** Characteristics of the Exponential Functions \( y = a^{-x} \) and \( y = a^x \) \((a > 1)\)

**Example 4**

**Graphing an Exponential Function**

Sketch the graph of

\[
f(x) = 3^{-x} - 1.
\]

**Solution**

Begin by creating a table of values, as shown below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>( 3^2 - 1 = 8 )</td>
<td>( 3^1 - 1 = 2 )</td>
<td>( 3^0 - 1 = 0 )</td>
<td>( 3^{-1} - 1 = -\frac{2}{3} )</td>
<td>( 3^{-2} - 1 = -\frac{8}{9} )</td>
</tr>
</tbody>
</table>

From the limit

\[
\lim_{x \to \infty} (3^{-x} - 1) = \lim_{x \to \infty} 3^{-x} - \lim_{x \to \infty} 1
\]

\[
= \lim_{x \to \infty} \frac{1}{3^x} - \lim_{x \to \infty} 1
\]

\[
= 0 - 1
\]

\[
= -1
\]

We can see that \( y = -1 \) is a horizontal asymptote of the graph. The graph is shown in Figure 4.4.

**Try It 4**

Complete the table of values for \( f(x) = 2^{-x} + 1 \). Sketch the graph of the function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
</tr>
</tbody>
</table>

\[
\begin{array}{cccc}
 x & 1 & 2 & 3 \\
 f(x) & \ & \ & \\
\end{array}
\]

**Take Another Look**

**Finding a Pattern**

Use a graphing utility to investigate the function \( f(x) = a^x \) for \( 0 < a < 1 \), \( a = 1 \), and \( a > 1 \). Discuss the effect that \( a \) has on the shape of the graph.