1.1 Rules of Roots

The following rules are true for all $x, y$ except where specified.

- $\sqrt{x^2} = |x|
- \sqrt{-1} = i
- \sqrt{-x} = i \cdot \sqrt{x}
- \sqrt{x} \cdot \sqrt{y} = \sqrt{x \cdot y}
- \frac{\sqrt{x}}{\sqrt{y}} = \frac{x}{y}$ when $y \neq 0$
- $\sqrt{x^3} = x \cdot \sqrt{x}$
- $\sqrt[3]{x^3} = x$
- $\sqrt[4]{x^4} = x \cdot \sqrt[4]{x}$

In general:
- $\sqrt{x} = x^{\frac{1}{2}}$
- $\sqrt[n]{x} = x^{\frac{1}{n}}$
- $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Things to be careful of:

- $\sqrt{x + y} \neq \sqrt{x} + \sqrt{y}$
- $\sqrt{x - y} \neq \sqrt{x} - \sqrt{y}$

1.2 Quadratic Formula

- Standard Form for quadratics is:

$$ax^2 + bx + c$$

- Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

A general rule for plugging in the $a, b, c$ in the quadratic formula is to put parenthesis around each value when you plug it in. This will help to keep the signs straight.
1.3 Examples

1. Solve \(2x^2 - 8x + 6 = 0\) using the quadratic formula.

   **Solution 1:** First, we notice and write that \(a = 2\), \(b = -8\), and \(c = 6\). So,
   \[
x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(6)}}{2(2)} = \frac{8 \pm \sqrt{64 - 48}}{4} = \frac{8 \pm \sqrt{16}}{4} = \frac{8}{4} \pm \frac{\sqrt{16}}{4} = 2 \pm 1
   \]
   So, \(x = 2 + 1 = 3\) or \(x = 2 - 1 = 1\).

2. Find the zeros of the function \(5x^2 + x - 2 = 0\).

   **Solution:** We want to find where \(5x^2 + x - 2 = 0\). Notice that this is not easily factorable, so we have to use the Quadratic Formula. So, using \(a = 5\), \(b = 1\), \(c = -2\),
   \[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{(1)^2 - 4(5)(-2)}}{2(5)} = \frac{-1 \pm \sqrt{1 + 40}}{10} = \frac{-1 \pm \sqrt{41}}{10}
   \]
   These aren’t very nice numbers, but we can’t do anything about that. So, \(x = -\frac{1}{10} + \frac{\sqrt{41}}{10}\) or \(x = -\frac{1}{10} - \frac{\sqrt{41}}{10}\).

3. Solve \(x^2 + 2x + 2 = 0\).

   **Solution:** Since this is not easily factorable, we need to use the quadratic formula. First, we identify \(a = 1\), \(b = 2\), and \(c = 2\). So,
   \[
x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)} = \frac{-2 \pm \sqrt{4 - 8}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i
   \]
   So \(x = 1 \pm i\). Thus, we have complex roots. Hence, \(x = 1 + i\) or \(x = 1 - i\).

4. Find the roots of the following polynomial using the Quadratic Formula
   \[
P(x) = -2x^2 + 4x - 2
   \]

   **Solution:** Notice that \(a = -2\), \(b = 4\), and \(c = -2\). So,
   \[
x = \frac{-(4) \pm \sqrt{(4)^2 - 4(-2)(-2)}}{2(-2)} = \frac{-4 \pm \sqrt{16 - 16}}{-4} = \frac{-4 \pm 0}{-4} = 1 \pm 0 = 1
   \]
   So \(x = 1\) is the only root of this polynomial.

5. Solve \(x^2 + 8x + 17 = 0\).

   **Solution:** Notice that \(a = 1\), \(b = 8\), and \(c = 17\). So,
   \[
x = \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(17)}}{2(1)} = \frac{-8 \pm \sqrt{64 - 68}}{2} = \frac{-8 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2} = -4 \pm i
   \]
   So \(x = -4 + i\) or \(x = -4 - i\).