Math 135A: HW Assignment #8

1. Consider a one-dimensional random walk on the integer lattice $\mathbb{Z}$. Now there is *inhomogeneity* in the problem; namely if the walker is on an *even site*, i.e. $0, \pm 2, \pm 4, \ldots$, the walker jumps to the right with probability p_e and to the left with probability $q_e := 1 - p_e$; however if the walker is on an *odd site*, i.e. $\pm 1, \pm 3, \pm 5, \ldots$, the walker jumps to the right with probability p_o and to the left with probability $q_o := 1 - p_o$. In general, $p_e \neq p_o$. Each jump is independent of all the other jumps.

   (a) Assuming the walker starts at lattice site 0, let T_1 denote the *first passage time* to the lattice site $+1$. Find the generating function for T_1. Hint: Model your proof on the one done in class, but be careful—you must take into account the inhomogeneity. This will require two different first passage generating functions.

   (b) Find the expected value of T_1.

   (c) What is the generating function for the first passage time, T_r, to the site r, $r > 0$, assuming the walker starts at 0. What is the expected value of T_r?

2. Let X denote the continuous random variable with the Gaussian density function

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}, \quad \sigma > 0.$$

   (a) Show that $\mathbb{E}(X) = \mu$ and $\mathrm{var}(X) = \sigma^2$.

   (b) Define the normalized random variable

$$Y = \frac{X - \mathbb{E}(X)}{\sigma}$$

   Show that $\mathbb{E}(Y) = 0$ and $\mathrm{var}(Y) = 1$. Show that the density function for Y is

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right).$$