1. Recall that if $X$ has geometric distribution then 
\[ P(X = k) = q^k(1 - q), \ k = 0, 1, 2, \ldots \]

(a) Compute $E(X)$.
(b) Compute $\text{var}(X)$.
(c) Compute $E(X^3)$.
(d) In a Bernoulli process, let $X$ denote the number of failures before the first success. Show that $X$ has geometric distribution.
(e) In a Bernoulli process, let $T_r$ denote the waiting time for $r$ successes. Find $E(T_r)$ and $\text{var}(T_r)$. Hint: Let $X_k$ denote the number of failures between the $(k - 1)^{st}$ and $k^{th}$ successes. Show that $X_k$ has geometric distribution and that $T_r = X_1 + X_2 + \cdots + X_r$.

2. Let $S_n$ be a random variable having binomial distribution; that is, 
\[ P(S_n = k) = \binom{n}{k} p^k(1 - p)^{n-k}, \ k = 0, 1, \ldots, n. \]

In class we proved that 
\[ P \left( \frac{S_n}{n} \geq p + \varepsilon \right) \leq \exp\left( -\frac{1}{4} n \varepsilon^2 \right), \ \varepsilon > 0. \]

Prove 
\[ P \left( \frac{S_n}{n} \leq p - \varepsilon \right) \leq \exp\left( -\frac{1}{4} n \varepsilon^2 \right), \ \varepsilon > 0. \]

3. Simulating a perfect coin. Given a biased coin such that the probability of heads is $\alpha$, we simulate a perfect coin as follows: Throw the biased coin twice. Interpret $HT$ as success and $TH$ as failure; if neither event occurs repeat the throws until a decision is reached. (a) Show that this model leads to Bernoulli trials with $p = 1/2$. 
