Math 185A

Type II Homework #4—Due February 3, 2012

In Exercise #11 (b), page 107, you were asked to find the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{z^n}{1-z^n}$$

Let this radius of convergence be denoted by R.

Let $d(n)$ equal the number of positive divisors of the positive integer n. For example, the positive divisors of the integer $n = 42$ are

$$1, 2, 3, 6, 7, 14, 21, 42$$

Thus

$$d(42) = 8$$

since there are eight divisors of 42. Clearly, if $n = p =$ prime, $d(p) = 2$ since the only divisors are 1 and p.

Prove that for $|z| < R$

$$\sum_{n=1}^{\infty} \frac{z^n}{1-z^n} = \sum_{n=1}^{\infty} d(n) z^n$$