Math 205A

Homework #1

#1–#5: Work the following exercises in Stein & Shakarchi: (1.4.7),[1] (1.4.9), (1.4.10), (1.4.19), (1.4.21) only first part.[2]

#6 (partitions): Let $p(n)$ denote the number of partitions of the integer n. For example, $p(5) = 7$ since 5 can be partitioned in 7 ways:

$$5,\ \ 4+1,\ \ 3+2,\ \ 3+1+1,\ \ 2+2+1,\ \ 2+1+1+1,\ \ 1+1+1+1+1$$

By definition, $p(0) = 1$.

1. Prove Euler's identity

$$P(z) := \sum_{n=0}^{\infty} p(n) z^n = \prod_{n=1}^{\infty} \frac{1}{1-z^n} \tag{1}$$

2. What is the radius of convergence of (1)?

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[1] This refers to Chapter 1, §4, exercise #7 of Stein & Shakarchi.

[2] Hint: Write

$$\begin{aligned} \frac{z}{1-z} &= \sum_{n=1}^{\infty} z^n \\ &= \sum_{n=1}^{\infty} z^{2n-1} + \sum_{n=1}^{\infty} z^{2n} \\ &= \frac{z}{1-z^2} + \sum_{n=1}^{\infty} z^{2n} \end{aligned}$$

and proceed recursively on the last term in the above equation.