Homework #1

#1–#5: Work the following exercises in Stein & Shakarchi: (1.4.7), (1.4.9), (1.4.10), (1.4.19), (1.4.21) only first part.

#6 (partitions): Let $p(n)$ denote the number of partitions of the integer $n$. For example, $p(5) = 7$ since 5 can be partitioned in 7 ways:

\begin{align*}
5, \quad 4 + 1, \quad 3 + 2, \quad 3 + 1 + 1, \quad 2 + 2 + 1, \quad 2 + 1 + 1 + 1, \quad 1 + 1 + 1 + 1 + 1
\end{align*}

By definition, $p(0) = 1$.

1. Prove Euler’s identity

$$P(z) := \sum_{n=0}^{\infty} p(n)z^n = \prod_{n=1}^{\infty} \frac{1}{1 - z^n}$$

2. What is the radius of convergence of (1)?

---

1. This refers to Chapter 1, §4, exercise #7 of Stein & Shakarchi.
2. Hint: Write

$$\frac{z}{1 - z} = \sum_{n=1}^{\infty} z^n = \sum_{n=1}^{\infty} z^{2n-1} + \sum_{n=1}^{\infty} z^{2n} = \frac{z}{1 - z^2} + \sum_{n=1}^{\infty} z^{2n}$$

and proceed recursively on the last term in the above equation.