Homework #4

1. Problem #11, page 155 of Stein & Shakarchi

2. Problem #13, page 155 of Stein & Shakarchi

3. Problem #3, page 157 of Stein & Shakarchi.

Remark: There are probably many ways to do this problem, but here is a hint in the direction I took: I first look at the case $0 < \alpha < 1$. For $n$ sufficiently large and all $\beta > \alpha$ we have the inequality

$$\Gamma(n\alpha + 1) \leq (n!)^\alpha \leq \Gamma(n\beta + 1)$$

where $\Gamma(z)$ is the gamma function. If we define (the Mittag-Leffler) function

$$E_a(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(na + 1)}$$

then it is known that for $|\arg z| \leq \frac{1}{2}a\pi$,

$$E_a(z) = \frac{1}{a} e^{z^{1/a}} + O(|z|^{-1})$$

You can either prove this last fact directly or consult some books about its proof. Now lift the restriction on $\alpha$.

4. Problem #4, page 157 of Stein & Shakarchi.