Using *Mathematica* in Calculus

To start a *Mathematica* session when you are at a UNIX level you type “Mathematica” (without the quotes). If you have the software installed on your own machine you probably just click on the *Mathematica* icon. Here are some examples from calculus

**Differentiation:** Suppose you want to differentiate the function $\sin x$ with respect to $x$. In *Mathematica* you type

$$D[\sin[x],x]$$

and hit the return button. Note that the sine function begins with a capital letter and the dependence on $x$ is given with square brackets. Here is what it looks like in a *Mathematica* session

In[1]:= D[Sin[x], x]

Out[1]= Cos[x]

Suppose we want to define a more complicated function, say

$$f(x) = e^{-2x} \cos(3x) + a e^{-x} \sin(x)$$

and differentiate it with respect to $x$ treating the number $a$ as a constant. Here is how it is done in *Mathematica*

In[2]:= f[x_] := Exp[-2*x]*Cos[3*x] + a*Exp[-x]*Sin[x]

In[3]:= D[f[x], x]


except it looks better on my screen. Note how the function $f(x)$ is defined in *Mathematica*. Here is another example of defining a function now of the two variables $x$ and $y$: Suppose

$$f(x, y) = \sin(x) \cos(y) + x^2 + \sqrt{y}$$

In *Mathematica* we type

$$f[x_, y_] := \sin[x]*\cos[y] + x^2 + \sqrt{y};$$

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**Integration:** Suppose we want to do the integral

\[ \int_0^\infty \frac{1}{1 + x^2} \, dx \]

We type

\textbf{In}[5]:= \text{Integrate}\left[\frac{1}{1 + x^2}, \{x, 0, \infty\}\right] \]

Hitting the return key we get \( \frac{\pi}{2} \) as output. Suppose instead we wanted

\[ \int_0^\infty \frac{1}{1 + x^{100}} \, dx \]

Then we simply change the \textit{Mathematica} input to

\textbf{Integrate}\left[\frac{1}{1+x^{100}}, \{x,0,\infty\}\right] \]

and \textit{Mathematica} returns the result

\( \frac{\pi}{100} \csc\left(\frac{\pi}{100}\right) \)

\textit{Mathematica} can do the general case

\[ \int_0^\infty \frac{1}{1 + x^a} \, dx \]

but we have to tell it that \( a > 1 \):

\textbf{Integrate}\left[\frac{1}{1 + x^a}, \{x, 0, \infty\}, \text{Assumptions} \rightarrow \{a > 1\}\right] \]

\textit{Mathematica} returns the result

\( \frac{\pi}{a} \csc\left(\frac{\pi}{a}\right) \)

**Taylor (Power) Series:** Suppose we are asked to write the Taylor (power) series for

\[ f(x) = \sqrt{\frac{\sin x}{x}} \]

about \( x = 0 \) to order \( x^5 \). In \textit{Mathematica} we type and get the result

\textbf{In}[11]:= \text{Series}\left[\text{Sqrt}\left[\frac{\sin x}{x}\right], \{x,0,5\}\right] \]
\textbf{Out}[11]= 1-x^2/12+x^4/1440+O[x]^6
Graphing Suppose we wish to graph the function

\[ f(t) = e^{-t} \cos(6t) \]

as a function of \( t \) in the interval \( 0 \leq t \leq 5 \). In *Mathematica* one writes

\[
\text{Plot[Exp[-t]*Cos[6t],\{t,0,5\},PlotRange->All]}
\]

Then the plot returned is shown at the top of this page.