Math 280: Topics in Hyperbolic Geometry

Time: TR 10:40 am -12 in MSB 2112

Instructor: Anastasiia Tsvietkova, tsvietkova@math.ucdavis.edu

Office hours: after the class or by appointment

Grading: the grades will be based on the assigned HW (turned in within two weeks), presentations in class, and a take-home final exam/project.

Description

In 1976, W. Thurston suggested the Geometrization Conjecture that allowed studying manifolds from a new perspective: using geometry. In particular, Thurston demonstrated that many 3-manifolds have hyperbolic metrics or can be decomposed into pieces with hyperbolic metric. Soon it was noticed that hyperbolic manifolds formed the largest and the least understood class of 3-manifolds. Such manifolds will be the main subject of the course.

We will start by looking at 2-dimensional hyperbolic space and its properties, since many geometric techniques used here can be carried to 3-dimensional hyperbolic space. Then we will proceed to discussing 3-dimensional hyperbolic geometry, hyperbolic 3-manifolds with finite volume, and related topics. The topics will include the Geometrization Conjecture and eight possible geometries for 3-manifolds, geometric triangulations of hyperbolic 3-manifolds, SnapPea algorithm for computing hyperbolic structure of a 3-manifold, Menasco’s decomposition of a link complement into two ideal polyhedra, hyperbolic volume and other geometric invariants, intrinsic geometry of hyperbolic links and related open questions.

Suggested literature

W. Thurston, The Geometry and Topology of Three-Manifolds, Chapters 1-4 and 7, http://library.msri.org/books/gt3m/
P. Scott, Geometries of 3-manifolds, Chapters 4-6
J. Weeks, Computation of hyperbolic structures in knot theory
Three-dimensional geometry and topology by W. Thurston, edited by S. Levi
W. Menasco, Polyhedra representation of link complements
C. Adams, Hyperbolic knots

Other textbooks and lecture notes that might be of help

Lectures on Hyperbolic Geometry by R. Benedetti and C. Petronio
P. Callahan, A. Reid, Hyperbolic structures on knot complements
Low-dimensional Geometry: from Euclidean surfaces to hyperbolic knots by F. Bonahon
Foundations of Hyperbolic Manifolds by John G. Ratcliffe
Dehn surgery and 3-manifolds by C. Gordon