Partridge

The n-fold rotational symmetry (or the $\tau_k = 2\pi n$ in Math's solution)

or that there are $n$-pts we take successive midpoints of (a 'fixed' order)

means there is underlying representation theory.

Here, it feels more like finding an eigenbasis.

1. For a fixed $0 < \theta < 2\pi$, what are the possible limits of $(\cos \theta)^k$ as $k \to \infty$?

2. For $(e^{i\theta} + e^{-i\theta})^k$ as $k \to \infty$?

3. If $A, B \in M_n \mathbb{C}$ and $AB = BA$ and $C$ has a basis consisting of eigenvectors of $A$ (that have distinct eigenvalues), show then any eigenvector of $A$ is also an eigenvector of $B$.

(This is related to Schur's Lemma. Think how/why.)
4) Consider the representation matrix $A$ corresponding to the $n$-cycle $\sigma = (12\ldots n)$:

$$A = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
-1 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & 1 \\
0 & 0 & \cdots & -1 & 0
\end{bmatrix}$$

Find $A$'s eigenvalues and corresponding eigenvectors.

This has something to do with decomposing the regular representation of $G = C_n \cong \mathbb{Z}/n\mathbb{Z}$. It also has something to do with understanding irreducible representations of $G = S_n$.

5) If $\mathbb{N} x_k$ grains of porridge the $k$th knight has in his bowl

Consider the vector $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{C}^n$

After one step, what is the new vector $x'$?

6) Writing $x' = Bx$, find the matrix $B$.

Lie, the porridge steaming is linear.
Or you can think of Markov processes.
(7) Explain why $AB = BA$.

At most, multiply them and verify. Better, explain why one expects it from the symmetry of the problem.

(8) Find an eigenbasis for $B$.

At worst, start from scratch. Better use #3 and realize this problem has cornerstones, parts by design.

(9) Rewrite $B$ in this basis. Call it $B'$

(10) What is the limit $(B')^k$ as $k \to \infty$?

(11) What is the limit $B^k$ as $k \to \infty$?

(12) What about $(B')^k\mathbf{x}$ and $B^k\mathbf{x}$ as $k \to \infty$?

(13) When $n$ is odd, notice

$B^k\mathbf{x} \to \mathbf{0}$

when $x = \frac{1}{n} \mathbf{e}_i$

(14) When $n$ is even and $n \mod 4 = 2$, #2

(15) When $n$ is even and $n \mod 4 = 3$, #2
With this viewpoint one can see that representation theoretically as saying the parridge stealing has $C_h$-rotational symmetry. It commutes with the action of $C_h$.

So if we break down space into the irreps of $C_h$, the partridge stealing must respect that. Decomposing, we can look at its pieces one irrept at a time.

This gives a hint for the case $S_4$ acts as rotations of the cube $C^3$. Mathematician partridge stealing commutes with that. It also preserves the irreps of $S_4$, (of how it acts on $C^3$? $\chi$?)

You can try to code it this way.

The fact that there is a homomorphism $S_4 \to C_3$ and $C_3 \to C_2 \to C_1$ explains some of Matt's (Robb's) reductions.