Beyond the unitarity bound in AdS/CFT

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in collaboration with
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Introduction 1

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**Symmetries:**

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2} \eta_{ij} dx^i dx^j$$

Isometries of AdS $\Leftrightarrow$ global sym. group of the QFT $SO(d, 2)$. 

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Precise dictionary of various quantities in each side

$$e^{-I_{AdS}[\phi^{(0)}]} = \langle e^{\int \phi^{(0)} O} \rangle_{\text{CFT}}$$
For a scalar field of mass $m^2$ in $AdS_{d+1}$, there are in principle two dual operators of dimension

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If $m^2 > m_{BF}^2 + 1$ we have $\Delta_- < \frac{d}{2} - 1$ in conflict with unitarity bound for scalar operators, so we expect “something” to go wrong.
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But what exactly happens in the bulk if we go below the bound? arXiv:1105.6337 (DM and TA)
Outline

- Set-up bulk theory with $\Delta < d/2 - 1$
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- Inner product
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- UV modification
- Conclusions
Set up

Take a scalar field with mass \( m^2 = m_{BF}^2 + \nu^2 \) with \( 1 < \nu < 2 \) in \( \text{AdS}_{d+1} \)

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l_0 = -\frac{1}{2} \int_M \sqrt{g} [g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2],
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Impose the (N) boundary condition \( \phi^{(\nu)} = 0 \), so the dynamical operator has dimension \( \Delta_- = d/2 - \nu \).
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Impose the (N) boundary condition $\phi^{(\nu)} = 0$, so the dynamical operator has dimension $\Delta_\nu = d/2 - \nu$. We take the action to be

$$l_N = l_0 + \int_{\partial M} \sqrt{\gamma} \left[ \rho_\mu \partial^\mu \phi \phi - \frac{1}{2} (d/2 - \nu) \phi^2 + \frac{1}{4(\nu - 1)} \gamma^{ij} \partial_i \phi \partial_j \phi \right],$$

which satisfies $\delta l_N = \int_{\partial M} \phi^{(0)} \delta \phi^{(\nu)}$. 
Inner product

For $\nu > 1$, the usual KG inner product

$$(\phi_1, \phi_2)_{\text{bulk}} = -i \int_{\Sigma} \sqrt{g_{\Sigma}} n^\mu \phi_1^* \leftrightarrow_\mu \phi_2$$
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$$ (\phi_1, \phi_2)_{ren} = (\phi_1, \phi_2)_{bulk} - \frac{1}{2(\nu - 1)} (\phi_1, \phi_2)_{bndy} $$

Not manifestly positive definite! expect ghosts in the bulk as a result of $\Delta - \frac{d}{2} - 1$.

To do: find spectrum, compute norms and look for ghosts. Include D results for comparison.
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For N bc’s, becomes normalizable when $\nu > 1$.

Pure gauge

For N bc’s and $\nu > 1$ do not find negative norms...how is this consistent? The operator is not gauge invariant, so UB does not apply.
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Neumann 2-point function

We expand the field as

\[
\phi(x, r) = \int_{V^+} d^d k \left[ a^\dagger(k) u_k(x, r) + a(k) u_k^*(x, r) \right] \quad u = e^{ik \cdot x} \psi
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Try to fix it adding terms that are relevant in the IR.
Deformed theory

AdS/CFT dictionary: adding deformations is dual to modifying the boundary conditions.
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\[ I_{\kappa,\lambda} = I_N - \nu \int_{\partial M} d^d x \sqrt{g^{(0)}} [\kappa \partial_i \phi^{(0)} \partial^i \phi^{(0)} + \lambda (\phi^{(0)})^2], \]

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\( \kappa \) and \( \lambda \) are dimensionful and the theory flows to \( \Delta_+ \) in the IR, i.e.

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This solves the IR issue, but...
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now we do find tachyon-ghosts for all values of \( \kappa, \lambda \).
Extensions

Neumann boundary conditions are pathological for

- Scalar in global AdS: find ghosts
- Maxwell fields: odd $d^{2}$-pt function ill-defined; even $d^{2}$ ghosts. Exception: AdS $^{4}$ (analog of scalar in BF window)
- Gravitons: odd $d^{2}$-pt function ill-defined; even $d^{2}$ ghosts. [GC and DM]
- Maxwell-Chern-Simons AdS $^{3}$, even when bulk $\Omega$ can be used [with JJ and RL]
- Holographic CFT (A) $^{dS}$: find ghosts (with CU)
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where \( \partial M \) corresponds to \( r = r_0 \).
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where $\partial M$ corresponds to $r = r_0$.

$$\kappa > \frac{1}{4\nu(\nu - 1)r_0^{2(\nu - 1)}}$$
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It seems that it is possible to remove the ghosts by adding a “UV cut-off”
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Bulk theories with holographic duals that violate the unitarity bound are indeed pathological.

The specific pathologies depend on where the CFT lives.

It seems that it is possible to remove the ghosts by adding a “UV cut-off”

It would be interesting to find a physical realization of the cut-off.
Thank you!
Extra Slides
Maxwell fields $\nabla_\mu F^{\mu\nu} = 0$

In the radial gauge, for $d > 2$ (there are logs in even d)

$$A_i = A_i^{(0)} + r^{d-2} A_i^{(d-2)} + \ldots, \quad \partial^i A_i^{(d-2)} = 0$$

$A^{(0)} \Leftrightarrow$ gauge field (res. $U(1)$) ; $A^{(d-2)} \Leftrightarrow$ $U(1)$ current

N bc’s allow $A^{(0)}$ to fluctuate. Gauge invariant operator $F_{ij}^{(0)}$, $\Delta_F = 2$. $\Delta_{UB} = \max(d-2, 2)$.

Conflict with UB for $d > 4$: find ghosts (even $d$) or IR divergence (odd $d$)

$d = 4$: $F^{(0)}$ saturates UB. But $\partial^i F_{ij}^{(0)} \neq 0$ and ghosts appear.

$d = 3$: $F^{(0)}$ dual to $j = \star F^{(0)}$, which saturates UB. But! $dj = 0$ (Bianchi), so no ghosts are expected and indeed they do not arise.

$d = 2$: Neumann allows $j$ to fluctuate, so satisfies “UB” (conformal sym. is lost). Find ghosts. Implications for HSC.
Gravitons $G_{\mu \nu} = \Lambda g_{\mu \nu}$

(Most of this is in arXiv:0805.1902 [GC and DM]).

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2} (g^{(0)}_{ij} + r^d g^{(d)}_{ij}) dx^i dx^j$$

$g^{(0)} \Leftrightarrow$ metric in the CFT ; $g^{(d)}_{ij} \sim T_{ij}$

N bc’s allow $g^{(0)}$ to fluctuate and bndy diff are gauge. Gauge invariant operator transverse part of $R^{(0)}_{ij}$, which has $\Delta_R = 2$ for $d > 2$. In this case $\Delta_{UB} = d$.

Conflict with UB for $d > 2$: find ghosts (even $d$) or IR divergence (odd $d$)

For $d = 2$, no obvious conflict with UB but still find ghosts.
MCS in $AdS_3$

$$I_0 = -\frac{1}{4} \int_M d^3x \sqrt{g}(F^2 + \alpha \epsilon^{\mu
\nu\lambda} A_\mu F_{\nu\lambda})$$

For $0 < \alpha < 1$, AdS asymptotics are preserved. In addition, it turns out that $\Omega$ can be take to be simply the bulk expression. The asymptotic expansion reads

$$A_i = A_i^{(0)} + r^{-\alpha} A_i^{(-)} + r^{\alpha} A_i^{(+)}$$

where $F_{ij}^{(0)} = 0$, $A_v^{(+)} = 0$, $A_u^{(-)} = 0$.

$A_i^{(-)}$ is a vector operator of dimension $\Delta_- = 1 - \alpha < \Delta_{UB} = 1$. Accordingly, find ghosts for bc’s that allow $A^{(-)}$ to fluctuate.