Question 1 (20 points)

Supertankers off-load oil at a docking facility 2 miles offshore. The refinery is 2 miles down the coast from the point on the shore nearest the docking facility. A pipeline must be constructed connecting the docking facility with the refinery. The pipeline costs $2,000,000 per mile if constructed underwater and $1,000,000 per mile if on land. Find the minimum cost of constructing the pipeline.
Question 2 (20 points)

(i) Write out in words what the following mathematical symbols mean and then give a precise definition:

\[ \lim_{x \to \infty} f(x) = L. \]

(ii) Use your definition in (i) to prove

\[ \lim_{x \to \infty} \frac{x^2 - 2x + 2}{x^2 - 2x + 1} = 1. \]
Question 3 (20 points) Fill in the blanks

\[
\frac{d \log x^2}{dx} = \text{__________________________}
\]

\[
\frac{d^2 \log x^2}{dx^2} = \text{__________________________}
\]

\[
\frac{d^3 \log x^2}{dx^3} = \text{__________________________}
\]

\[
\frac{d^n \log x^2}{dx^n} = \text{__________________________}
\]

Explain (but do not prove) why your last answer is correct. (Full points for a well written, concise and logical answer.)
Question 4 (20 points) Find the derivative of: \( y = \sin \left( \frac{2}{x} + \frac{5}{x^3} \right) \)

Find \( \frac{d}{dx} \sqrt{1 + \tan(f(x))} = \)

Find \( \frac{d^2}{dx^2} \frac{5}{\sqrt{1-x^3}} = \)

Find \( g'(t) \) if \( g(t) = e^{0t} \sin^5 2t \)
**Question 5 (20 points)**

Below is the graph of the velocity, in feet per second, of a particle at time $t$ seconds.

Using the graph of the velocity above plot the acceleration of the particle at time $t$.

Given that the particle starts at zero at $t = 0$ and ten seconds later is at a position three feet below the origin, that is $p(0) = 0$ and $p(10) = -3$. Graph the position of the particle versus time below.