Last Initial ______________________________

Section ______________________________

Name ______________________________

Social Security Number ______________________________

MIDTERM I

21C-B, 11:00-11:50 am, Wednesday October 17, 2001

Declaration of honesty: I, the undersigned, do hereby swear to uphold the highest standards of academic honesty, including, but not limited to, submitting work that is original, my own and unaided by notes, books, calculators, mobile phones, bread machines, or any other electronic device.

Signature ____________________ **Date** _______________

Question 1 *(0 points)* When was Galileo born?

Question 2 *(5 points)* Explain, in 35 words or less, why you think the study of multivariate calculus is important?

The rest of this exam will involve the following function

$$S(t, x) = -t^2 + x^2 \,.$$

Question 3 *(15 points)* Draw the three traces of the surface

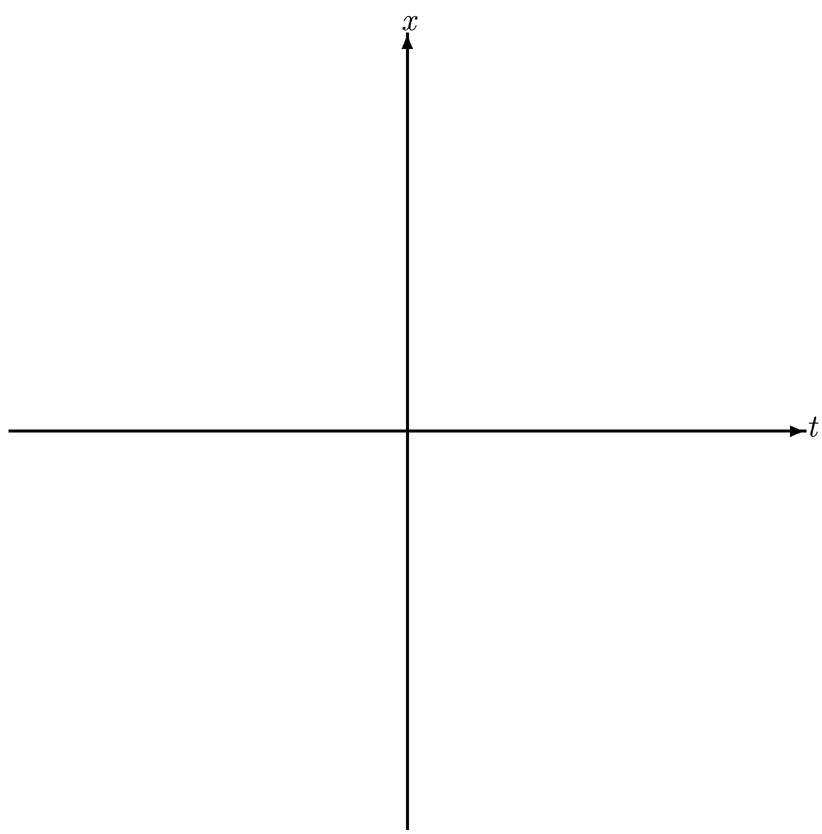
$$z = S(t, x)$$

when $x = 0$, $t = 0$ and $z = 0$. Sketch the graph of the function $S(t, x)$.

Question 4 *(10 points)* Draw level curves of the function $S(t, x)$ with

1. $S(t, x) = 2$,
2. $S(t, x) = 1$,
3. $S(t, x) = 0$,
4. $S(t, x) = -1$,
5. $S(t, x) = -2$.

Label the t and x intercepts of each level curve along with its value of $S(t, x)$.



Lets study the continuity of $T(t,x) = (t^2 + x^2)/S(t,x)$, *i.e.*

$$T(t,x) = \frac{t^2 + x^2}{-t^2 + x^2} \,.$$

Question 5 *(5 points)* Is $T(t,x)$ continuous at the origin $(x,y) = (0,0)$?

YES NO (Circle one)

Question 6 *(20 points)* Give reasons for your answer to question 5.

Question 7 *(20 points)* Compute the following

$$\frac{\partial S(t,x)}{\partial t}\,, \qquad \frac{\partial S(t,x)}{\partial x}\,,$$

$$\frac{\partial^2 S(t,x)}{\partial t^2}\,, \qquad \frac{\partial^2 S(t,x)}{\partial x \partial t}\,, \qquad \frac{\partial^2 S(t,x)}{\partial x^2}\,.$$

For which values of t and x do all all single partial derivatives vanish ($\partial_t S = 0$ and $\partial_x S = 0$)? Examine the Hessian[1] to determine whether this is a maximum, minumum or saddle point.

[1]For the absentminded, recall that the Hessian was $\Delta \equiv S_{tt}S_{xx} - S_{tx}^2$.

Question 8 *(20 points)* Suppose that both t and x are functions of a new variable τ

$$t(\tau) = \cosh(\tau)\,,$$

$$x(\tau) = \sinh(\tau)\,.$$

Use the chain rule[2] to compute

$$\frac{dS(t(\tau), x(\tau))}{d\tau}\,.$$

[2]Remember that the total differential was $dS = \partial_t S\, dt + \partial_x S\, dx$ and you only need to divide by $d\tau$ to get the chain rule. By the way, the hyperbolic cosine and sine functions obey similar identities to cos and sin. If you forgot even this, just use $\cosh(\tau) = \frac{1}{2}(e^{\tau} + e^{-\tau})$ and $\sinh(\tau) = \frac{1}{2}(e^{\tau} - e^{-\tau})$ which will be slower but better than nothing!

Question 9 *(20 BONUS points)* Check your answer to question 8 by computing $S(t(\tau), x(\tau))$ explicitly as a function of τ.