Last Initial ________________________________

Section ________________________________

FULL Name ________________________________

Social Security Number ________________________________

MIDTERM II

21C-B, 11:00-11:50 am, Wednesday November 14, 2001

Declaration of honesty: I, the undersigned, do hereby swear to uphold the highest standards of academic honesty, including, but not limited to, submitting work that is original, my own and unaided by notes, books, calculators, mobile phones, bread machines, mp3 players or any other electronic device.

Signature _____________________ **Date** ________________

Question 1 *(0 points)* When *was* Galileo born?

Question 2 *(5 points)* Explain, in 35 words or less, why you think the study of infinite series is important?

Question 3 *(20 points)* The height of Mt Midtermarama is given by the function

$$h(x,y) = e^{-x^2-y^2-2x-2y}.$$

Compute the height at the origin $(x,y) = (0,0)$. *Estimate* the height at the point $(x,y) = (0.1, 0.2)$ using the total differential. For which values of x and y does Mt Midtermarama attain its maximum height[1] (you don't need to prove this is a maximum)?

[1]Notice this problem asks for 3 separate things: the correlation between the people leaving early in midterm I and those who missed parts of questions was VERY high!!

Question 4 *(40 points)* Lets find the moment of inertia of a homogeneous, solid, right circular cone, height h, base radius a, total mass M, about its axis of symmetry.

a) Set the problem up in spherical coordinates, *i.e.*, draw a diagram as follows: Put the origin at the tip of the cone and the z-axis along its middle. Specify the coordinate ranges for ρ (the distance from the origin), ϕ (the angle from the z-axis) and θ (the angle around the z-axis). (Hint: specify the range for ρ last)

b) Recall that the moment of inertia of an infinitesimal mass dM a distance r from the axis of rotation (here the z-axis) is

$$dI = r^2 dM = r^2 \delta dV.$$

Express δ (the mass density per unit volume) in terms of M, h and a. Write out r^2 as a function of ρ and ϕ and the infinitesimal volume dV in terms of ρ, ϕ and θ and differentials thereof. Put these together to make a single formula for dI.

c) Integrate dI over the ranges you found in part *a)* above to find the total moment of inertial I as a function of M, h and a. (*Full credit will be given to any student who computes the correct answer with working using any coordinate system. Steps a) and b) are just guidelines but can be completed for partial credit.*)

Question 5 *(0 points)* Warm-up:

YES NO (Circle one)

Question 6 *(15 points)* The sequence a_n satisfies $a_n > a_{n+1} > 0 \; \forall n$.

a) Does $\sum_{n=1}^{\infty} a_n$ converge?

YES NO (Circle one)

b) Does $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converge?

YES NO (Circle one)

c) If you answered YES to part *b)*, suggest a weaker condition on a_n that would make you change your mind. For those who answered NO, propose an additional requirement on a_n to make the series converge.

Question 7 *(20 points)* The density of information encoded on an elliptical DVD is

$$I(x,y) = \exp\Big(-x^2 - \frac{y^2}{4}\Big)\,.$$

The DVD, manufactured by Midtermarama *inc.*, is described by the lamina given by the region in the (x,y)-plane

$$x^2 + \frac{y^2}{4} < 1\,.$$

Introduce elliptical coordinates

$$\begin{aligned} x &= \rho\cos\theta \\ y &= 2\rho\sin\theta\,. \end{aligned}$$

a) Show that $x^2 + \frac{y^2}{4} = \rho^2$. Express the function $I(x,y)$ as a function of ρ and θ.

b) Write ranges for the coordinates ρ and θ on the DVD. Using these, compute the total information $I = \int_{\rm DVD} dA\; I(P)$. (Hint: in these coordinates $dA = 2\rho d\rho d\theta$.)

Question 8 *(20 points)* Decide, using any test you think is appropriate whether the following series converge or diverge.

a) $\sum_{n=1}^{\infty} \frac{1}{n}$

b) $\sum_{n=0}^{\infty} e^{-n}$

c) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$

d) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

Question 9 *(10 points)* Consider the sequence $a_n = \frac{1}{n^2}$. Prove that $\lim_{n \to \infty} a_n = 0$ by proposing a value N such that $0 < |a_n| < \epsilon$ for all $n > N$.

Question 10 *(20 points)* Evaluate the iterated integral

$$\int_0^1 dx \int_0^x dy \int_0^3 dz (x^2 + y^2) \, .$$

Sketch the volume you just integrated over.