21C Homework 6

Due Friday May 13

Steinellos $\equiv$ "Calculus and Analytic Geometry", 5th Edition,
S.K. Stein and A. Barcellos

Start early–this is a BIG one!!!!

Question 1 Give geometric explanations for the infinitesimal volume elements $dV = \rho\, d\rho\, d\theta\, dz$ and $dV = r^2\, dr\, d(\cos\theta)\, d\varphi$ in cylindrical and spherical coordinates, respectively.

Question 2 Consider the coordinate system (ρ, η) for the plane.

$$x = \rho \cosh \eta$$

$$y = \rho \sinh \eta.$$

What do lines of equal ρ and η look like? Does this system cover the whole plane $\mathbb{R}^2$? Compute the infinitesimal area element dA in this coordinate system. Sketch a natural domain $D = \{(\rho, \eta) : \rho_1 \leq \rho \leq \rho_2, \eta_1 \leq \eta \leq \eta_2\}$.

Question 3 Compute the volume of a cylinder, radius R, height h, using an iterated integral in cylindrical coordinates.

Question 4 Compute the volume of a sphere radius R using an interated integral in spherical coordinates.

Question 5 The gravitational force between two point particles, masses m and M, respectively, is $F = GmM/r^2$ where r is their separation and G is Newton's constant. Prove that the same formula holds even when one of the particles is replaced by a sphere of radius R and mass M. Hints: (i) set up an integral in spherical coordinates. (ii) Remember (or look up) the cosine rule for triangles.

Question 6 Show that the moment of inertia of a sphere, mass M, radius R, spinning around its axis is $I = \frac{2}{5}MR^2$. Hint: use cylindrical coordinates.

Question 7 Steinellos, §15.4, pp 907-910, qq 2, 10, 20, 28, 34, 44

Question 8 Steinellos, §15.5, pp 916-918, qq 2, 10, 16, 20, 34, 38

Question 9 Steinellos, §15.6, pp 921-922, qq 2, 12, 16, 28, 32, 38

Question 10 Steinellos, §15.7, pp 928-930, qq 2, 12, 18, 32, 38