21C Homework 7

Due Friday May 20

Steinellos ≡ "Calculus and Analytic Geometry", 5th Edition,
S.K. Stein and A. Barcellos

Question 1 Steinellos, §10.1, pp 573-575, qq 2, 10, 12, 14

Question 2 Steinellos, §10.2, pp 582-583, qq 2, 6, 8, 12, 14, 28, 29, 30

Question 3 Steinellos, §10.3, pp 590-591, qq 2, 4, 6, 14, 20, 23, 25, 30

Question 4 *Asymptotic Series:* Consider the integral

$$I(R) = \int_0^\infty dx\, \frac{e^{-x/R}}{1+x}\,.$$

In this question we are interested in I when R is small. For example, when $R = .01$,

$$I(.01) = 0.009901942288\ldots\,.$$

Your mission:

(i) Suggest a useful series expansion for

$$\frac{1}{1+x}\,.$$

(ii) Think more carefully and explain why this series can't help evaluating the above integral.

(iii) Throw caution to the wind and use the above series as well as the identity

$$\int_0^\infty dx(-x)^n e^{-x/R} = n!(-1)^n R^{n+1}$$

to develop a series expansion for $I(R)$.

(iv) Does your series converge or diverge?

(v) Disregarding your previous answer, work out the first 5 partial sums when $R = .01$. How accurate are your results? Ask your TA to explain this to you!

Question 5 Prove $\lim_{n\to\infty} \frac{x^n}{n!} = 0$.