Last Initial ______________________________

Section ______________________________

Name ______________________________

Student ID ______________________________

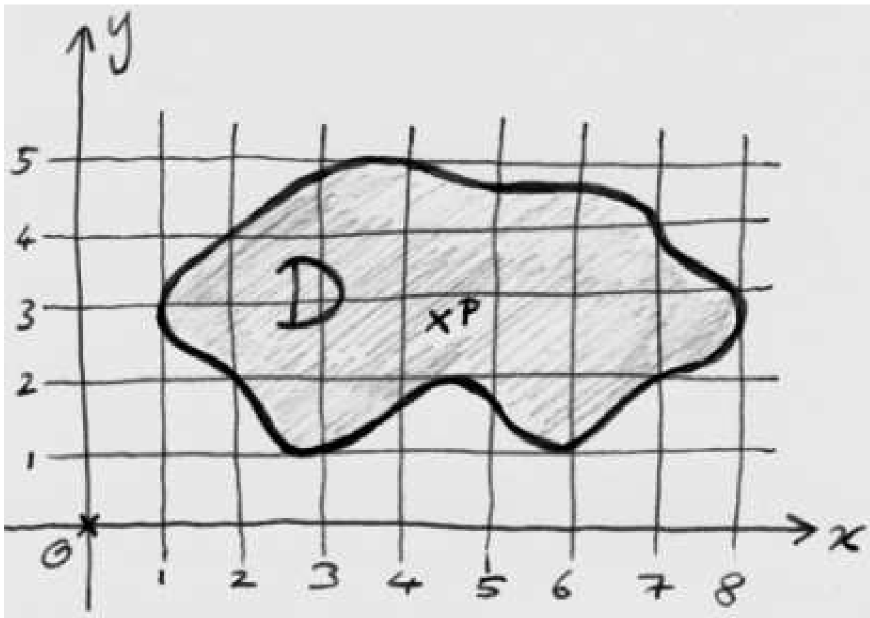
MIDTERM

21C, Monday May 16, 2005

Declaration of honesty: I, the undersigned, do hereby swear to uphold the highest standards of academic honesty, including, but not limited to, submitting work that is original, my own and unaided by notes, books, calculators, mobile phones, bread machines, IPODs, fluffy toys or any other electronic device.

Signature ____________________ **Date** _______________

Question 1 *(30 points)* Consider the domain D depicted below:



Let $\overline{\mathcal{O}P}$ be the distance from the point $P \in D$ to the origin $\mathcal{O}$. Define a function

$$f : D \longrightarrow \mathbb{R}$$

by

$$f(P) = \overline{\mathcal{O}P}.$$

Now answer the following questions:

(i) Sketch ∂D.

(ii) Give (crude) upper and lower bounds on the area A of D.

(iii) Write down, but do not compute, an integral equaling A.

(iv) Propose numbers a and b such that

$$a < \int_D dA\, f(P) < b\,.$$

Briefly explain your choices.

(Working space for Question 1)

Question 2 *(50 points)* Let the function f be the same as in Question 1 but extend its domain to the plane $\mathbb{R}^2$. Now answer the following questions:

(i) Let (x, y) be Cartesian coordinates for $\mathbb{R}^2$. Write down a formula for $f(x, y)$.

(ii) Calculate $\frac{\partial f}{\partial x}$. Does $\lim_{(x,y)\to(0,0)} \partial_x f(x, y)$ exist?

(iii) Compute the limit

$$\lim_{(x,y)\to(0,0)} f(x, y) .$$

Prove that your proposed limit is correct.

(iv) Let (r, θ) be polar coordinates for $\mathbb{R}^2$. Write down a formula for $f(r, \theta)$.

(v) Let D be a unit disc centered at $\mathcal{O}$. Compute

$$\int_D dA\, f(P).$$

(vi) Let Σ be the surface

$$z = f(x, y) .$$

Sketch the surface Σ.

(vii) Copy out your sketch from part (vi). Indicate an interpretation of the integral in part (v) on this second sketch. (An English sentence may help too!)

(viii) Let R be the solid region given in Cartesian coordinates (x, y, z) by

$$0 \leq z \leq f(x, y) , \qquad x^2 + y^2 \leq 1 .$$

Describe this region using spherical coordinates.

(Working space for Question 2)

(Working space for Question 2)

(Working space for Question 2)

Question 3 *(20 points)* Compute the moment of inertia I of a homogeneous disc of mass M, radius R, about the perpendicular axis through its center.